PHYS 239 Lec2

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1 Radiative Transfer in medium

Set up: In the medium, we can write equation

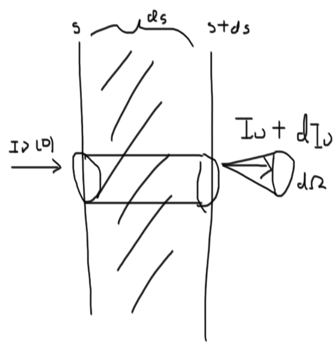
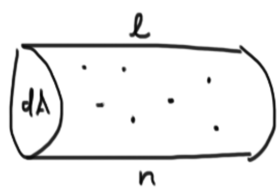


Figure 1: Set up of scattering with medium

$$dI_\nu = \underbrace{-I_\nu \kappa_\nu ds}_{\text{attenuation}} + \underbrace{j_\nu ds}_{emission}$$

1.1 Mean Free Path



Definition 1 (mean free path)**.** $\sigma_v \equiv$ cross section. Mean free path = distance through the volume with n particles before probability of collision = 1. N is the column density

$$N = nl, \ [cm^{-2}]$$

$$\frac{Area_{coll}}{dA} = 1 \Rightarrow \frac{nl \cdot dA \cdot \sigma_\nu}{dA} = 1 \Rightarrow l = \frac{1}{n\sigma_\nu}$$

Can manipulate unit of mean free path:

$$l = \frac{m}{mn\sigma_\nu} = \frac{m}{\rho\kappa_\nu}$$

where κ_v is the mass absorption coefficient $[cm^2/g]$.

1.2 Purely absorbing

$$dI_\nu = -n\sigma_\nu I_\nu ds = -\alpha_\nu I_\nu ds$$

We can write $\alpha_\nu = n\sigma_\nu = \rho\kappa_\nu$ as the absorption constant. Now we have

$$I_\nu(s) = I_\nu(0)e^{-\alpha_\nu s}$$

Define

$$\tau_\nu = \alpha_\nu s$$

as the optical depth. We can also manipulate the unit:

$$\tau_\nu = \underbrace{\kappa_\nu \rho s}_{\sum k_\nu} = \underbrace{n\sigma_\nu s}_{N\sigma_\nu}$$

We can define two regimes:

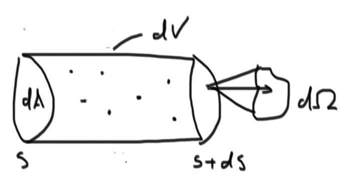
- optically thin: $\tau_\nu < 1$
- optically thick: $\tau_\nu > 1$

Relationship between optical depth and mean free path:

$$\tau_\nu = \frac{s}{l}$$

1.3 Emission

Now $I_\nu \to 0$. Assume isotropic emission



$$dE = j_\nu dV d\Omega dt d\nu$$

where j_ν is the emissivity coefficient. We can derive its unit:

$$[j_\nu] = [W/m^2/Hz/sr]$$

Now

$$dE = j_\nu dA ds d\Omega dt d\nu \Rightarrow \boxed{dI_\nu = j_\nu ds}$$

1.4 Combining Absorption and Emission

$$dI_\nu = j_\nu ds - \alpha i_\nu ds \Rightarrow \frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu$$

$$\text{divide both sides by } \alpha_\nu : \frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\alpha_\nu} - I_\nu$$

$$\boxed{\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu}$$

Where $S_\nu \equiv \frac{j_\nu}{\alpha_\nu}$ is the source function of the medium.

2 Scattering

Light entering the medium can be scattered outside of the ray. Scattering can also introduce light into the ray. **This process is no longer local**.

Starting with strict assumptions:

- isotropic scattering
- coherent/monochromatic scattering: the frequency and energy of the photon is unchanged $\Rightarrow \nu$ is unchanged
- τ_ν increases towards observers

$$j_{\nu,scatter} = n\sigma_\nu J_\nu$$

where σ_ν is the scattering cross section.

$$J_\nu \equiv \frac{\int I_\nu d\Omega}{\int d\Omega} = \frac{1}{4\pi}\int I_\nu d\Omega$$

is the mean intensity for isotropic scattering.

2.1 Pure Scattering

Source function

$$S_\nu \equiv \frac{j_\nu}{\alpha_\nu} = \frac{n\sigma_\nu J_\nu}{n\sigma_\nu} = J_\nu$$

Now the RT differential eq becomes

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + J_\nu = -I_\nu + \frac{1}{4\pi}\int I_\nu d\Omega$$

is hard to solve: depends sensitively on the set up of the problems and initial conditions .

3 Non-Scattering setup

$$\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu$$
$$\text{define } \tilde{I} \equiv I_\nu e^{\tau_\nu},\ \tilde{S} \equiv S_\nu e^{\tau_\nu}$$
$$\Rightarrow \frac{d\tilde{I}}{d\tau_\nu} = \underbrace{\frac{dI_\nu}{d\tau_\nu} e^{\tau_\nu}}_{(S_\nu - I_\nu)e^{\tau_\nu}} + \underbrace{I_\nu \frac{de^{\tau_\nu}}{d\tau_\nu}}_{I_\nu e^{\tau_\nu}}$$
$$= S_\nu e^{\tau_\nu} = \tilde{S}$$
$$\tilde{I}(\tau_\nu) = \tilde{I}(0) + \int_0^{\tau_\nu} \tilde{S}(\tau'_\nu) d\tau'_\nu$$
$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} \tilde{S}(\tau'_\nu) e^{-(\tau_\nu - \tau'_\nu)} d\tau'_\nu$$

where the first term is the background radiation field attenuated by τ_ν, and the second term is the emission from the material attenuated by absorption between where emitted and S (self absorption).

If S_ν is independent of τ_ν:

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu e^{-\tau_\nu} \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} d\tau'_\nu = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

When $\tau_\nu \gg 1$ optically thick: $I_\nu(\tau_\nu) = S_\nu$: the radiation only depends on the property of the medium.

When $\tau_\nu \ll 1$ optically thin:

$$I_\nu(\tau_\nu) = I_\nu(0)(1 - \tau_\nu) + S_\nu(1 - 1 + \tau_\nu) = I_\nu(0) - \tau_\nu I_\nu(0) + S_\nu \tau_\nu$$

Most of the radiation coming into the medium leaves the medium.

3.1 Order of magnitude estimate

What's the optical depth of a tree when you lie down beneath a tree and look up? (Write your own answer here, but $\tau_{tree} \sim 10$ sounds about right)

4 Thermal eq.

- Thermodynamic equilibrium

$$\frac{\partial T}{\partial t} = 0 \text{ or } \nabla T = 0, \ T_{rad} = T_{kin} = T_{ext}$$

- Local TE (LTE)

$$\frac{\partial T(\vec{x})}{\partial t} \approx 0, \ \nabla T(\vec{x}) \sim 0$$

Not in TE/LTE:

$$T_{rad} \neq T_{kin} \neq T_{ext}$$

4.1 Planck function

photon occupation number:

$$N(\nu) = \frac{1}{e^{h\nu/kT} - 1}$$

from Bose-Einstein distribution

number density of states per volume per frequency

$$\rho_s = \frac{4\pi g \nu^2}{c^3}$$

where $c = 2$ is the degeneracy for photon.

Can now write down the density of energy of radiation field:

$$u_\nu = \epsilon \rho_s N = \frac{4\pi g h \nu^3 / c^3}{e^{h\nu/kT} - 1}, \ \epsilon = h\nu \text{ for photons}$$

$$u_\nu(\Omega) = \frac{dE}{\underbrace{dA\ cdt}_{dV} d\Omega d\nu} = \frac{I_\nu}{c}$$

The general energy density

$$u_\nu = \int u_\nu(\Omega) d\Omega = \frac{1}{c} \int I_\nu d\Omega$$

In TE/LTE: I_ν is uniform:

$$u_\nu = 4\pi \frac{I_\nu}{c} = 4\pi J_\nu = \frac{4\pi g h \nu^3 / c^3}{e^{h\nu/kT} - 1} \Rightarrow I_\nu = \frac{2h\nu^3/c^3}{e^{h\nu/kT} - 1}$$

is the planck function.