PHYS 239 Lec 4

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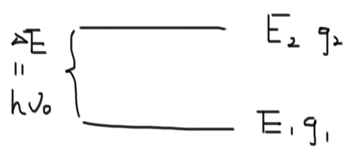
Oct, 9, 2018

For a non scattering medium

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{\tau_{nu}})$$

1 Two Level Atom Count



$$\frac{g_1 B_{12}}{g_2 B_{21}} = 1,\ \frac{A_{21}}{B_{21}} = \frac{2h\nu^3}{c^2}$$

$$j_\nu = \frac{A_{21} h\nu_0 n_2 \phi(\nu)}{4\pi},\ \alpha_\nu = \frac{h\nu_0}{4\pi}\phi(\nu)(n_1 B_{12} - n_2 B_{21})$$

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = \frac{n_2 A_{21}}{n_1 B_{12} - n_2 B_{21}} = \frac{2h\nu^3}{c^2}\left(\frac{g_2 n_1}{g_1 n_2} - 1\right)$$

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-h\nu/RT}$$

$$\Rightarrow S_\nu = B_\nu(T)$$

2 Scattering

Consider pure scattering

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + 4\pi \int I_\nu d\Omega$$

Want to do three things:

1. Probabilistic: random walker
2. Local homogeneity: quantities change slowly relative to mean free path → Rosseland Approximation
3. Eddington Approximation

3 Random Walk

Let

$$\begin{aligned}\vec{r}_i &= \text{step}\\ L &= \text{length of step}\\ \vec{R} &= \text{final position after } N \text{ steps}\end{aligned}$$

can't use $\langle\vec{R}\rangle = 0$. Instead use

$$\sqrt{\langle\vec{R}\cdot\vec{R}\rangle} = \langle r_1^2\rangle + \langle r_2^2\rangle + \langle r_3^2\rangle + 2\cancelto{0}{\langle r_1 \cdot r_2\rangle} + \ldots$$

where

$$\langle\vec{r}_i \cdot \vec{r}_j\rangle = \delta_{ij}L^2$$

Thus we have

$$\langle R^2\rangle = NL^2$$

which average out to be

$$\boxed{l_* = \sqrt{\langle R^2\rangle} = \sqrt{N}L}$$

For radiation

$$\begin{aligned}L = l_{\text{mfp}} = \frac{1}{n\sigma_\nu} &\Rightarrow l_* = \sqrt{N_{\text{steps}}}\, l_{\text{mfp}} = D\\ \tau_\nu &= \#\text{of mfp's through the medium}\\ \tau_\nu \gg 1 &\Rightarrow N_{\text{steps}} \approx \frac{D^2}{l_{\text{mfp}}^2} \approx \tau_\nu^2\\ \tau_\nu \ll 1 &\Rightarrow N_{\text{steps}} \approx \text{mean scattering} \sim 1 - e^{-\tau_\nu} \approx \tau_\nu\end{aligned}$$

Key Point: Total opacity is sum of individual Assuming isotropic scattering opacities, so the highest opacity will dominate.

Assume isotropic scattering

$$\begin{aligned}\frac{dI_s}{ds} &= -\alpha_{\nu,abs}(I_\nu - S_{\nu,abs}) - \alpha_{\nu,sca}(I_\nu - J_\nu)\\ &= (-\alpha_{\nu,abs} + \alpha_{\nu,sca})I_\nu + (\alpha_{\nu,abs}S_{|nu} + \alpha_{\nu,sca}J_\nu)\\ \alpha_{\nu,tot} &= \alpha_{\nu,abs} + \alpha_{\nu,sca})\\ S_{\nu,tot} &= \frac{\alpha_{\nu,abs}S_{\nu,abs} + \alpha_{\nu,sca}J_\nu}{a_{\nu,tot}}\\ \Rightarrow \frac{dI_\nu}{ds} &= -\alpha_{\nu,tot}(I_\nu - S_{\nu,tot})\end{aligned}$$