Physics 224
The Interstellar Medium

Lecture #10: Dust Optical Properties, Heating & Cooling

Outline

- Part I: Dust Optical Properties
- Part II: Dust Heating & Cooling
- Part III: Dust Composition

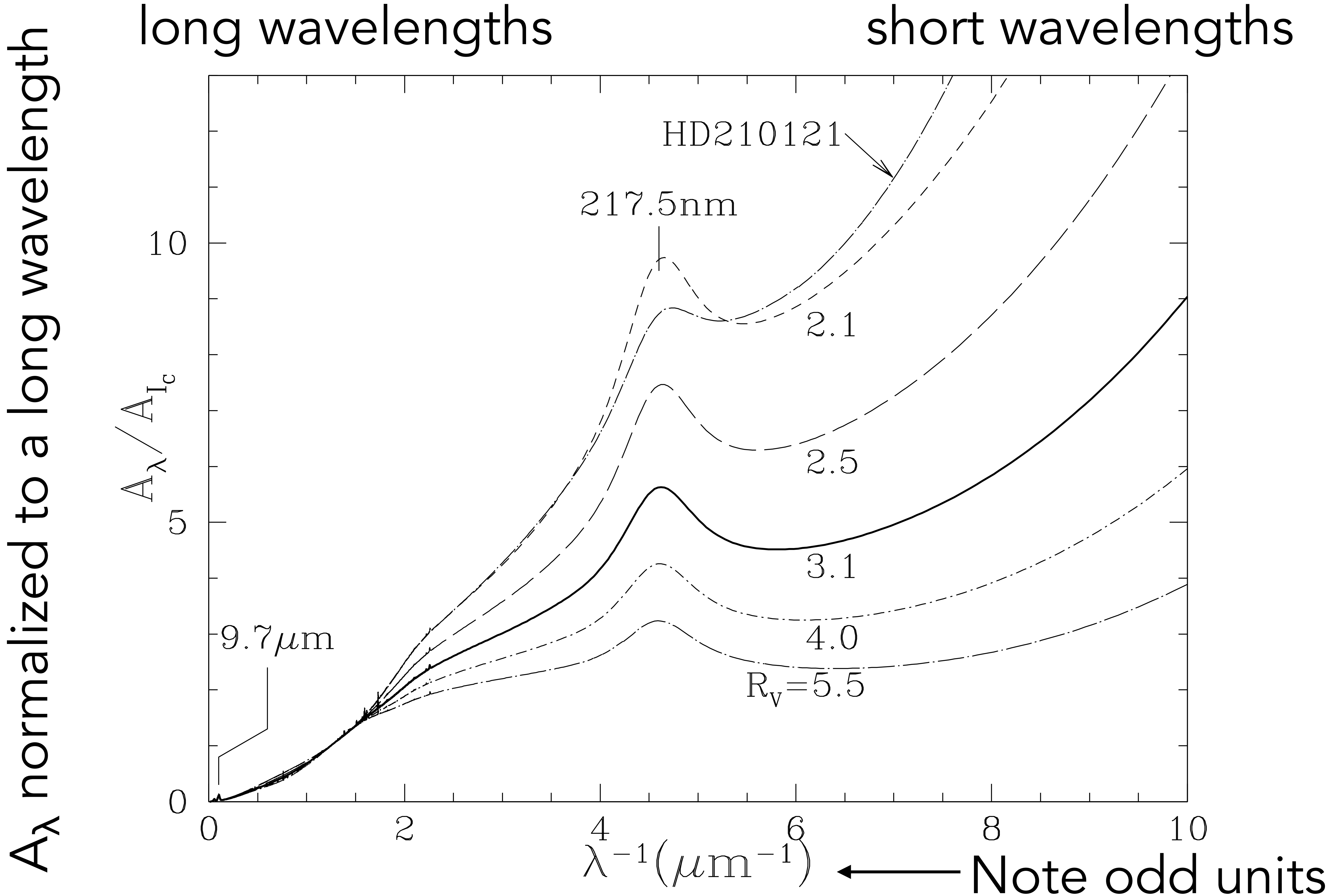
How we learn about dust

- Extinction: wavelength dependence of how dust attenuates (absorbs & scatters) light
- Polarization: of starlight and dust emission
- Thermal emission from grains
- Microwave emission from spinning small grains

Dust/Light Interaction

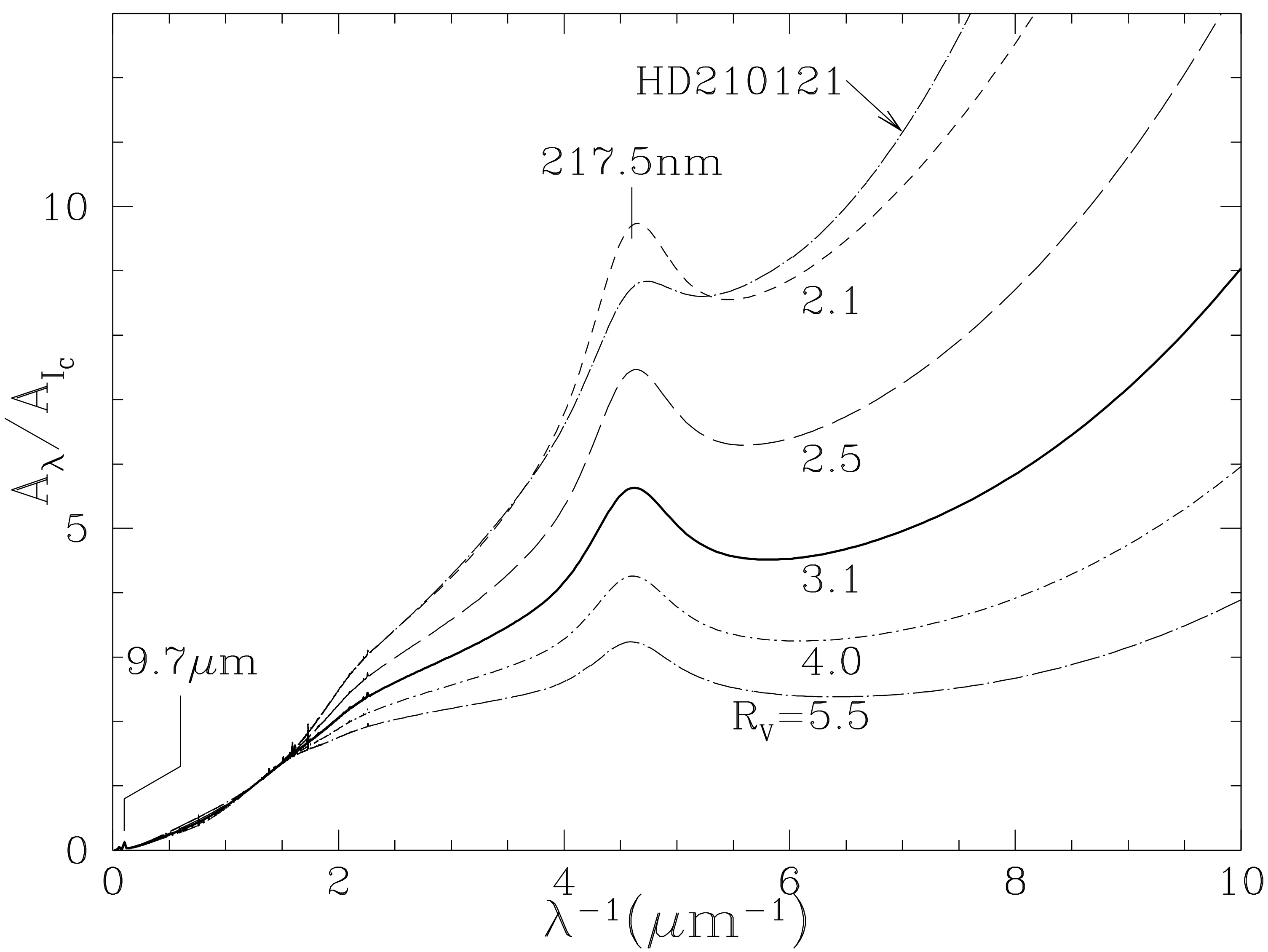
- Depletion of elements from the gas relative to expected abundance
- Presolar grains in meteorites or ISM grains from Stardust mission (7 grains!)

Milky Way Dust Extinction Curves



Selective Extinction R_V

$$R_V \equiv \frac{A_V}{A_B - A_V} \equiv \frac{A_V}{E(B-V)}$$

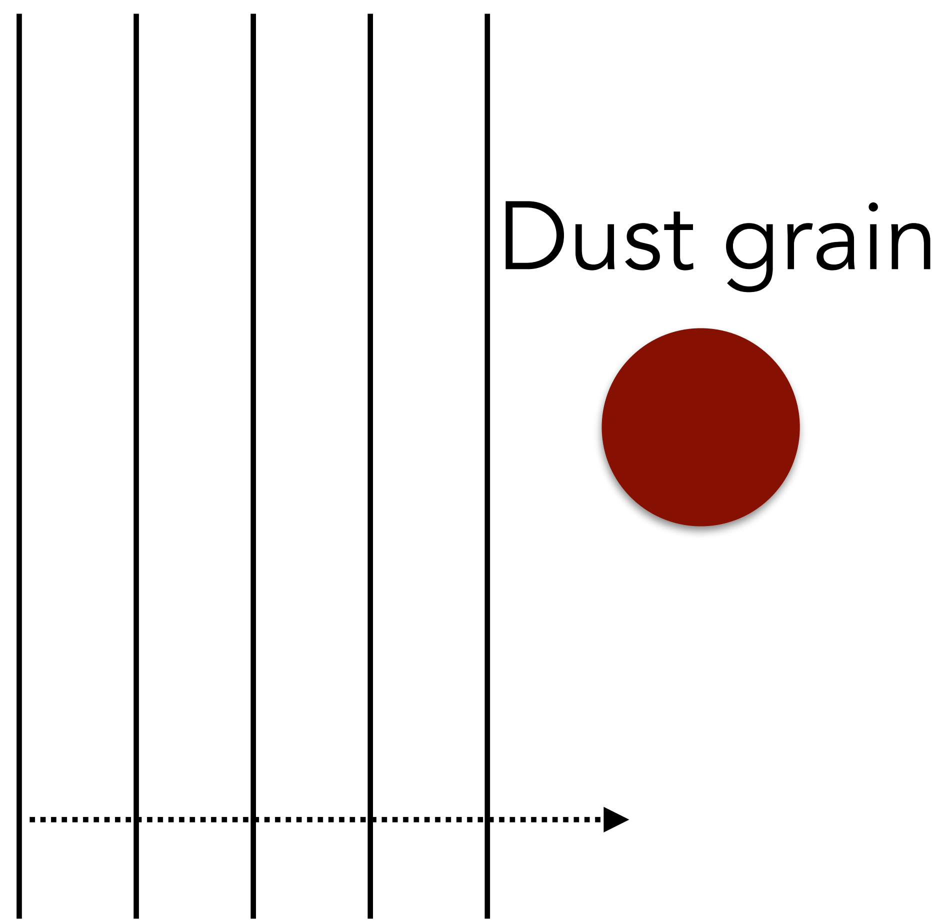


R_V = slope of extinction curve in optical B & V bands

MW average R_V = 3.1 but it varies!

Scattering & Absorption by Small Particles

key reference: Bohren & Huffman textbook



Scattering & absorption result from interaction of grain material with oscillating E & B field

when wavelength of light is < mm magnetic permeability = 1 can ignore magnetic field interaction

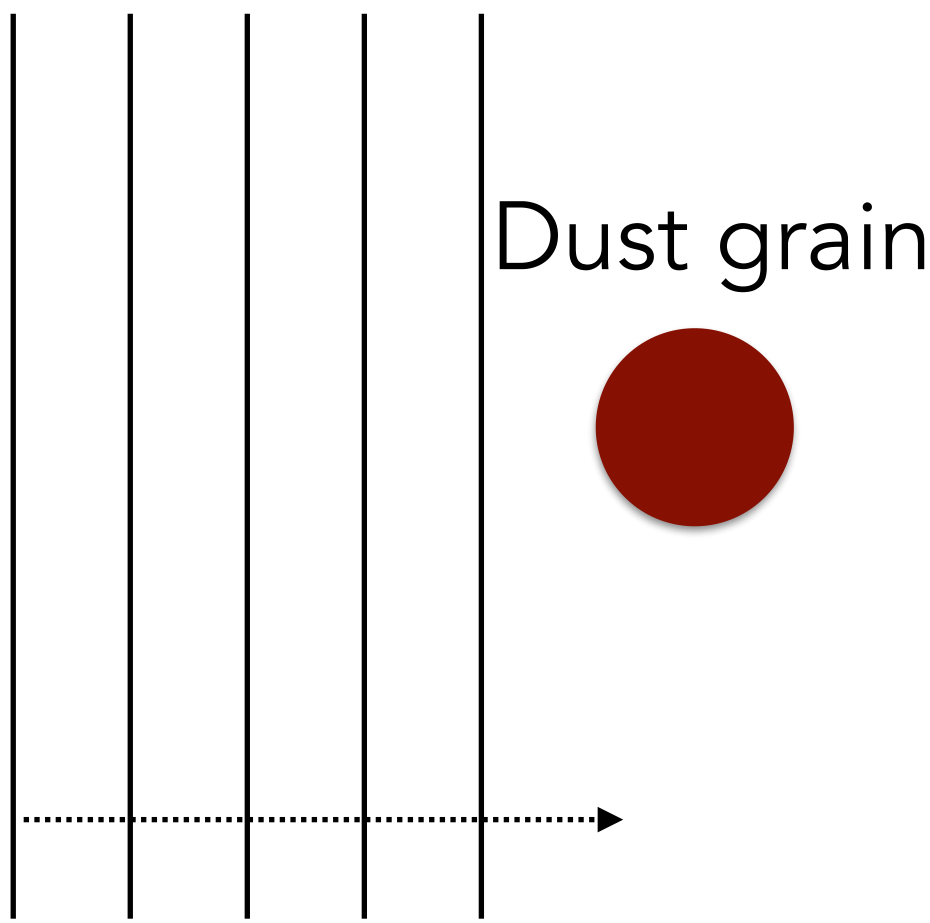
plane EM wave

$$E = E_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}$$

$$\lambda = 2\pi c/\omega$$

Scattering & Absorption by Small Particles

key reference: Bohren & Huffman textbook



plane EM wave

$E = E_0 \, e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}$

Scattering & absorption result from interaction of grain material with oscillating E & B field

response of material to E field set by *dielectric function*

$$\epsilon(\omega) = \epsilon_1 + i\epsilon_2$$

related to *refractive index*

$$m = \sqrt{\epsilon}$$

Scattering & Absorption by Small Particles

Define:

Geometrical Cross Section: πa^2

Absorption Cross Section: $C_{abs}(\lambda)$

Scattering Cross Section: $C_{sca}(\lambda)$

Extinction Cross Section: $C_{ext}(\lambda) = C_{abs}(\lambda) + C_{sca}(\lambda)$

Scattering & Absorption by Small Particles

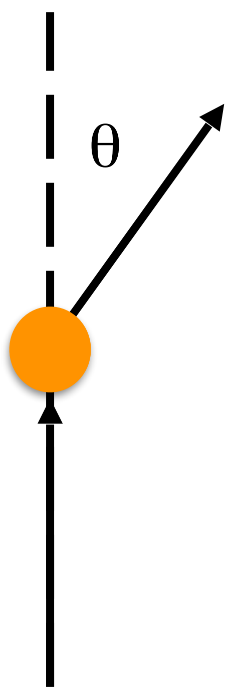
Define:

Geometrical Cross Section: πa^2

Scattering & Absorption Efficiency Factors

$Q_{abs} = C_{abs}/\pi a^2$ $\qquad$ $Q_{sca} = C_{sca}/\pi a^2$

Scattering & Absorption by Small Particles



Scattering Definitions:

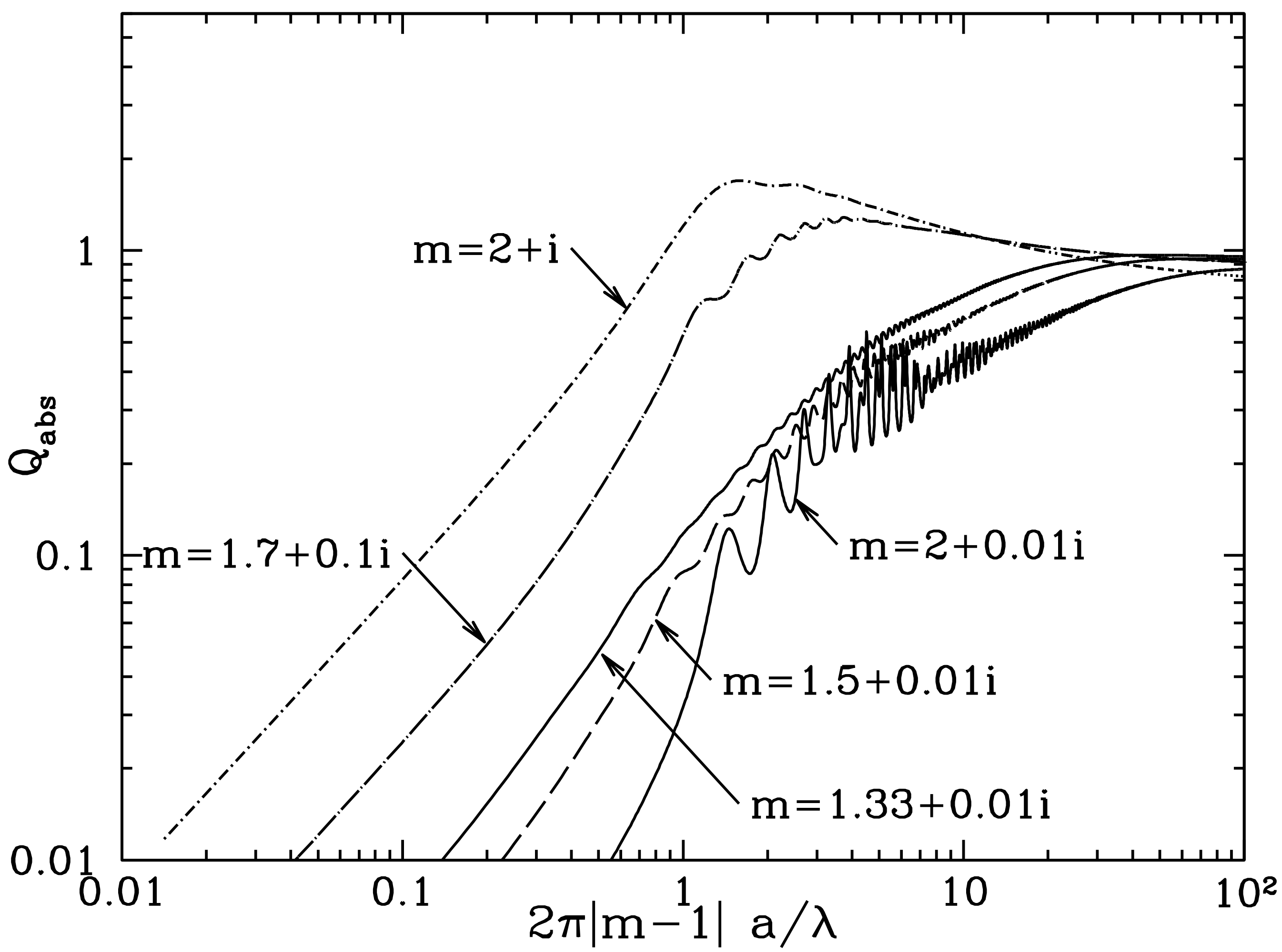
Albedo = C_{sca}/C_{ext}

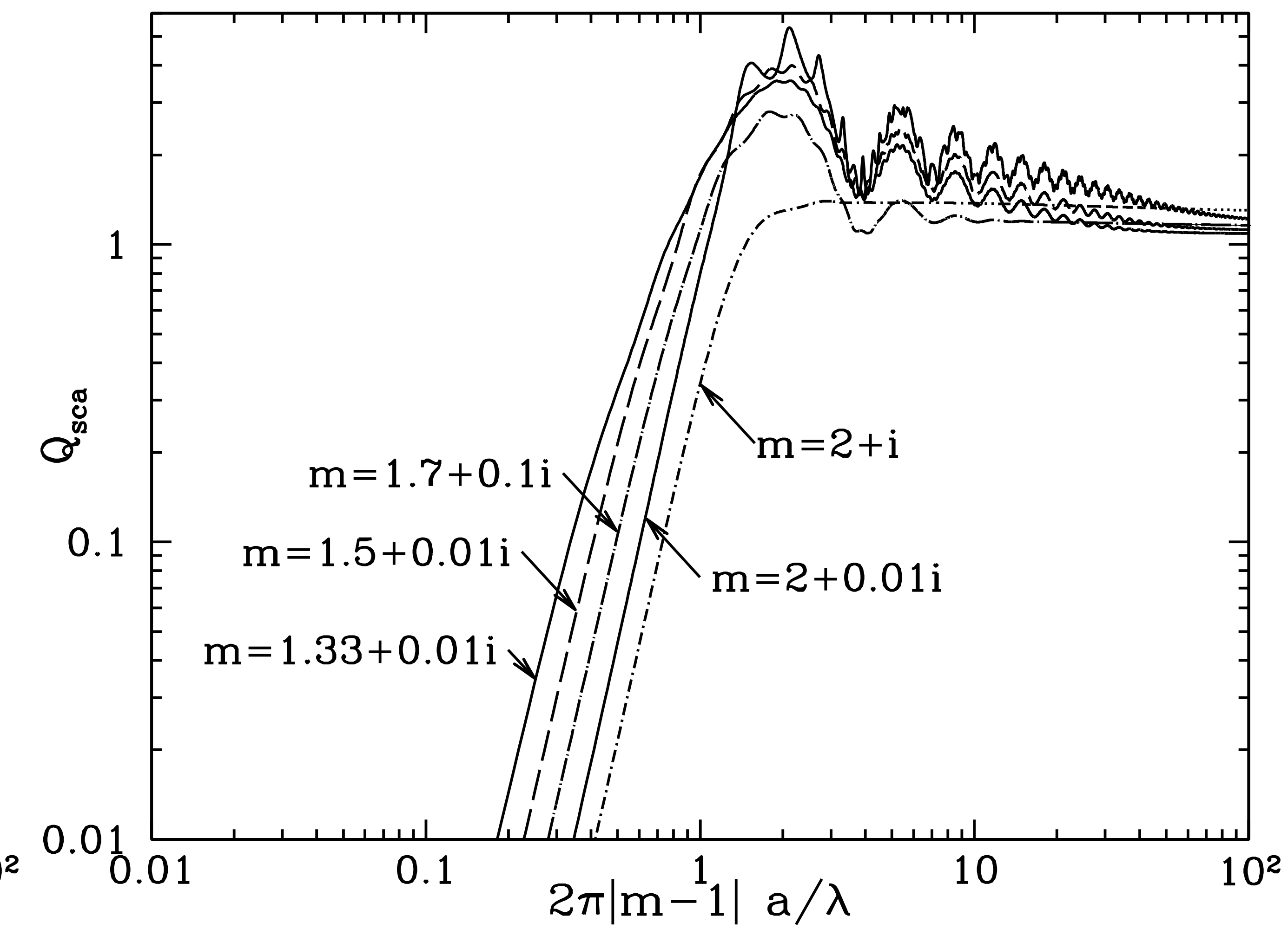
Differential scattering angle $$\frac{dC_{sca}(\theta)}{d\Omega}$$

Scattering asymmetry factor $$\langle \cos\theta \rangle = \frac{1}{C_{sca}} \int_0^{\pi} \cos\theta \frac{dC_{sca}(\theta)}{d\Omega} 2\pi \sin\theta d\theta$$

- Isotropic scattering $\langle \cos\theta \rangle = 0$
- Forward scattering $\langle \cos\theta \rangle = 1$
- Back scattering $\langle \cos\theta \rangle = -1$

Scattering & Absorption by Small Particles

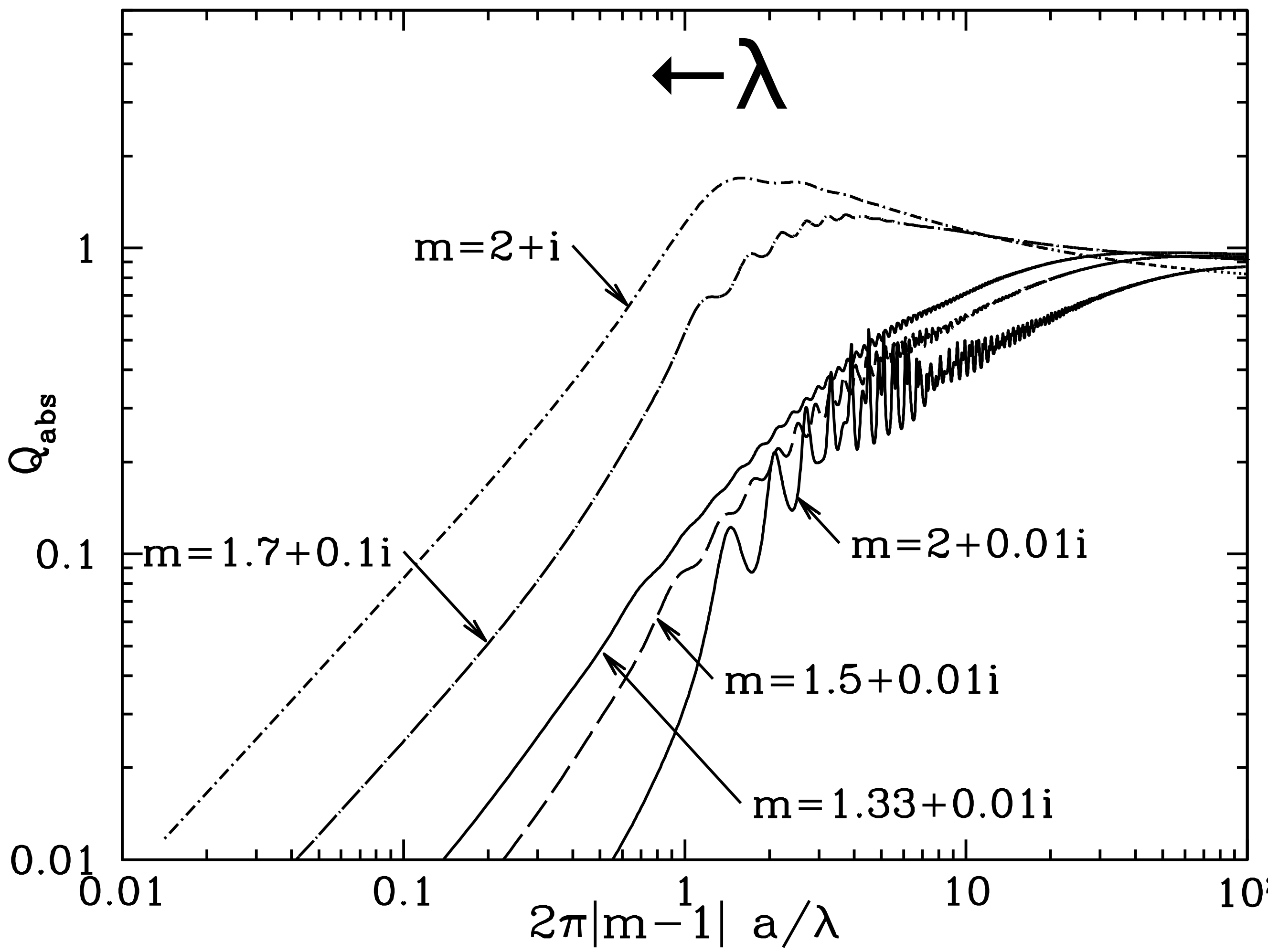




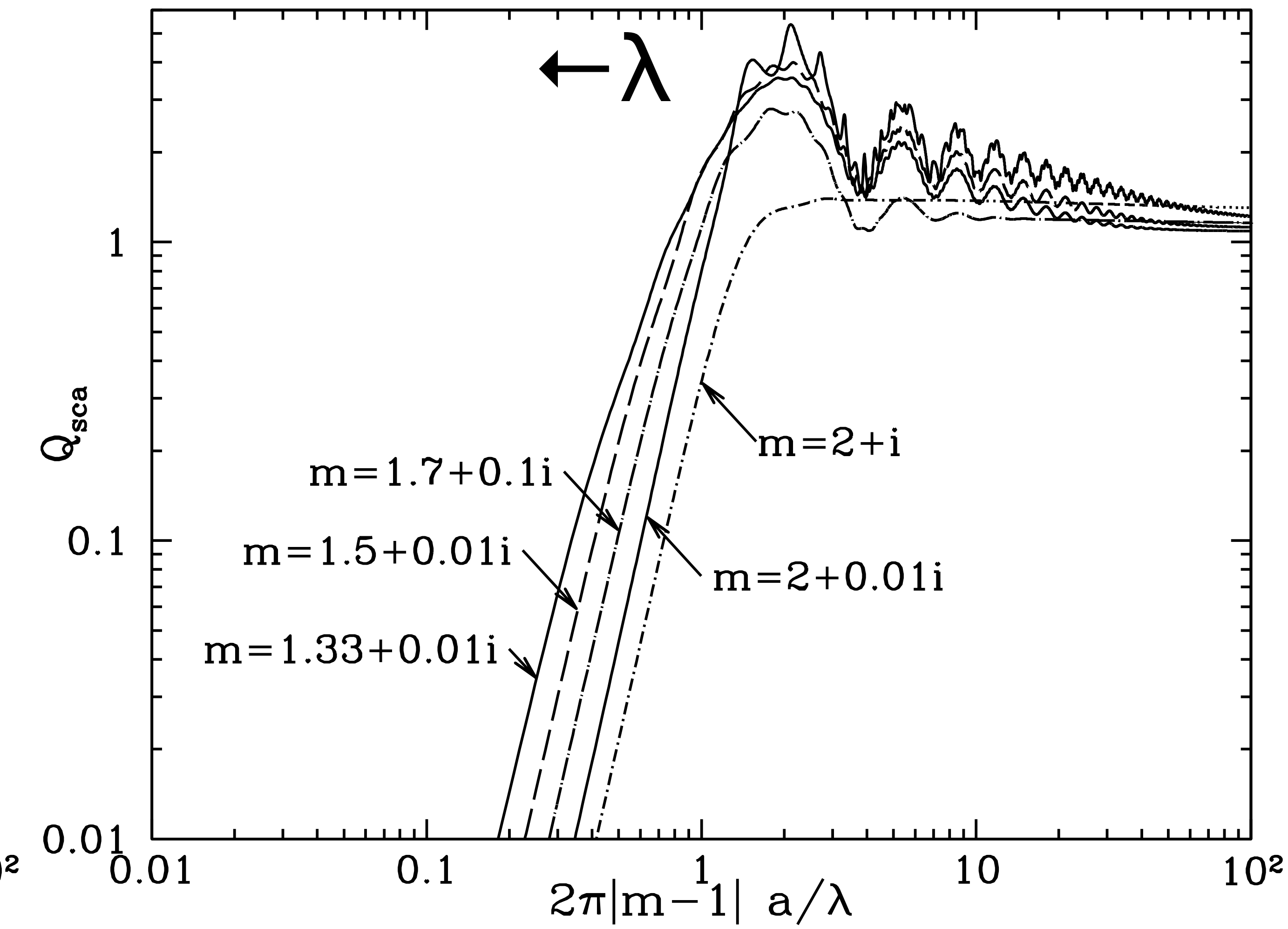
a/λ - grain size relative to wavelength of light defines different regimes

Scattering & Absorption by Small Particles

for fixed a



for fixed a



a/λ << 1 grain much smaller than wavelength - analytic soln's

$$Q_{\rm abs} = 4\frac{2\pi a}{\lambda}{\rm Im}\left(\frac{\epsilon-1}{\epsilon+2}\right) \qquad Q_{\rm sca} = \frac{8}{3}\left(\frac{2\pi a}{\lambda}\right)^4\left|\frac{\epsilon-1}{\epsilon+2}\right|^2$$

Scattering & Absorption by Small Particles

$$Q_{\mathrm{abs}} = 4\frac{2\pi a}{\lambda}\mathrm{Im}\left(\frac{\epsilon - 1}{\epsilon + 2}\right) \qquad\qquad Q_{\mathrm{sca}} = \frac{8}{3}\left(\frac{2\pi a}{\lambda}\right)^4 \left|\frac{\epsilon - 1}{\epsilon + 2}\right|^2$$

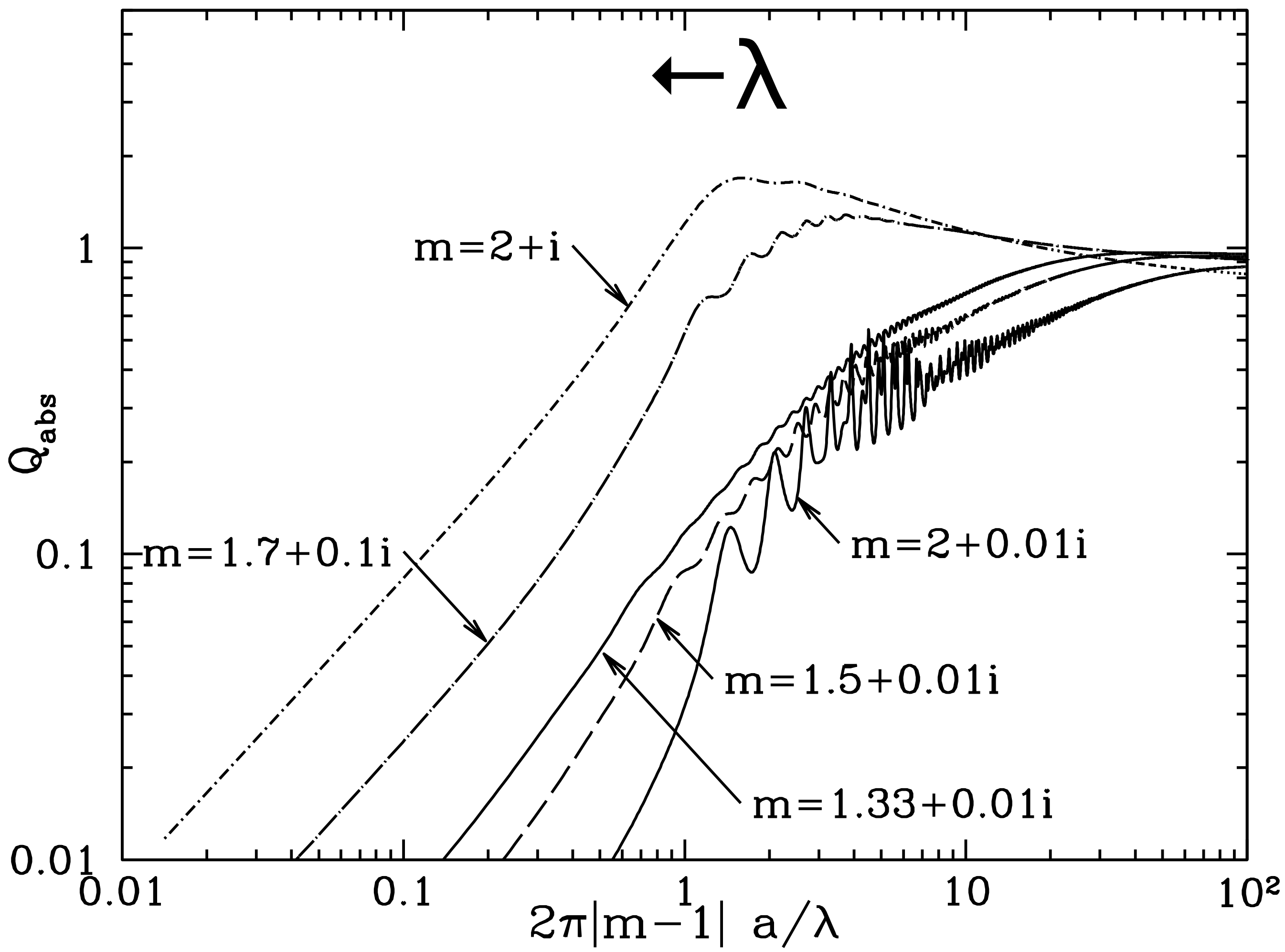
In long wavelength limit, general behavior is:

$Q_{abs} \sim V/\lambda^2$ $\qquad\qquad$ $Q_{sca} \sim V^2/\lambda^4$

where V = grain volume

Scattering & Absorption by Small Particles

for fixed a



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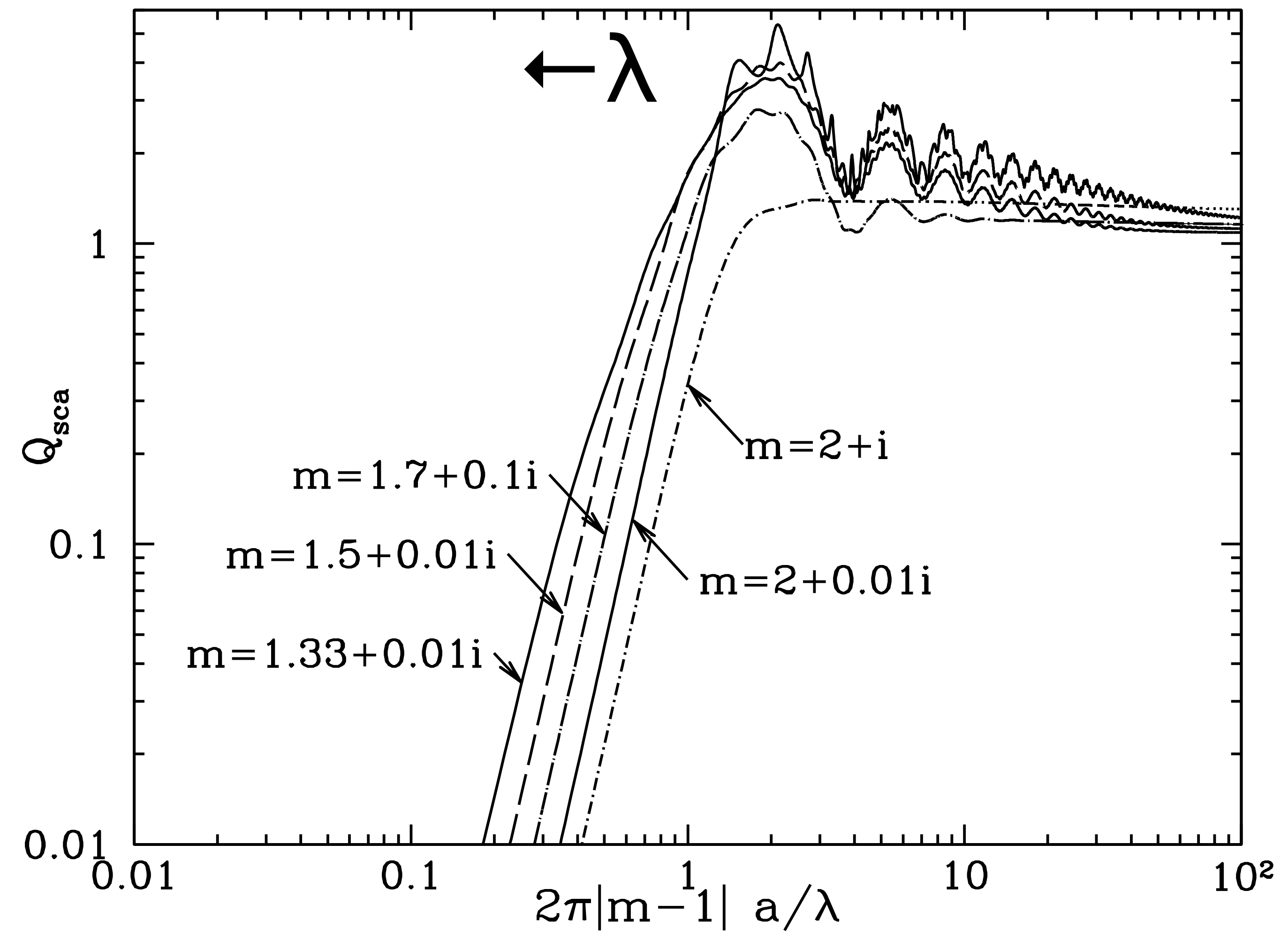
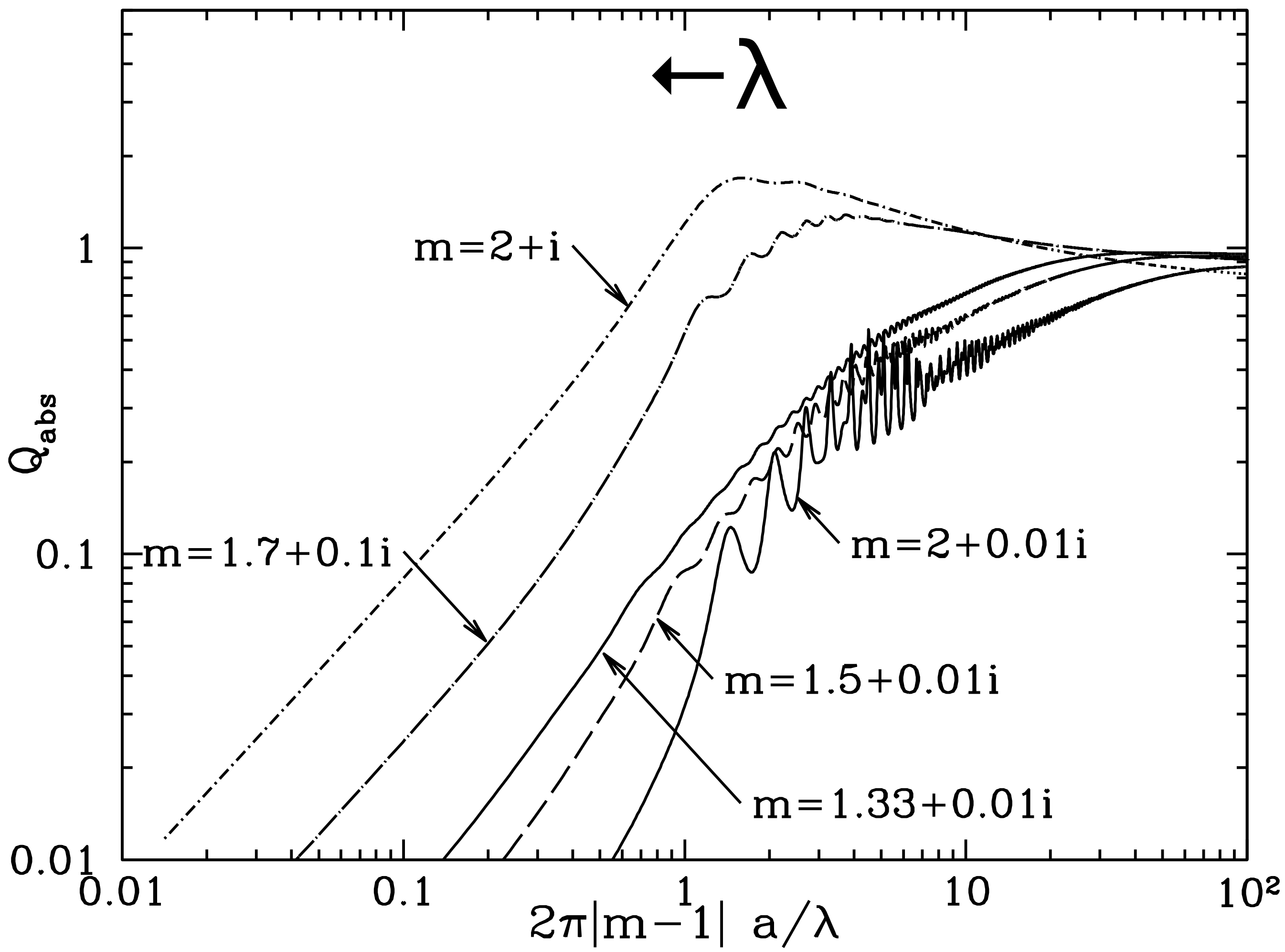


a/λ ~ 1 use Mie theory

Scattering & Absorption by Small Particles

for fixed a



$$Q_{\rm abs} = 4\frac{2\pi a}{\lambda}{\rm Im}\left(\frac{\epsilon-1}{\epsilon+2}\right)$$

<u>Absorption</u>

note that at long wavelength:

$C_{abs} = Q_{abs}\pi a^2 \propto a^3 \propto m_{dust}$

absorption efficiency when $a >> \lambda$ levels off to 1, $C_{abs} = \pi a^2$

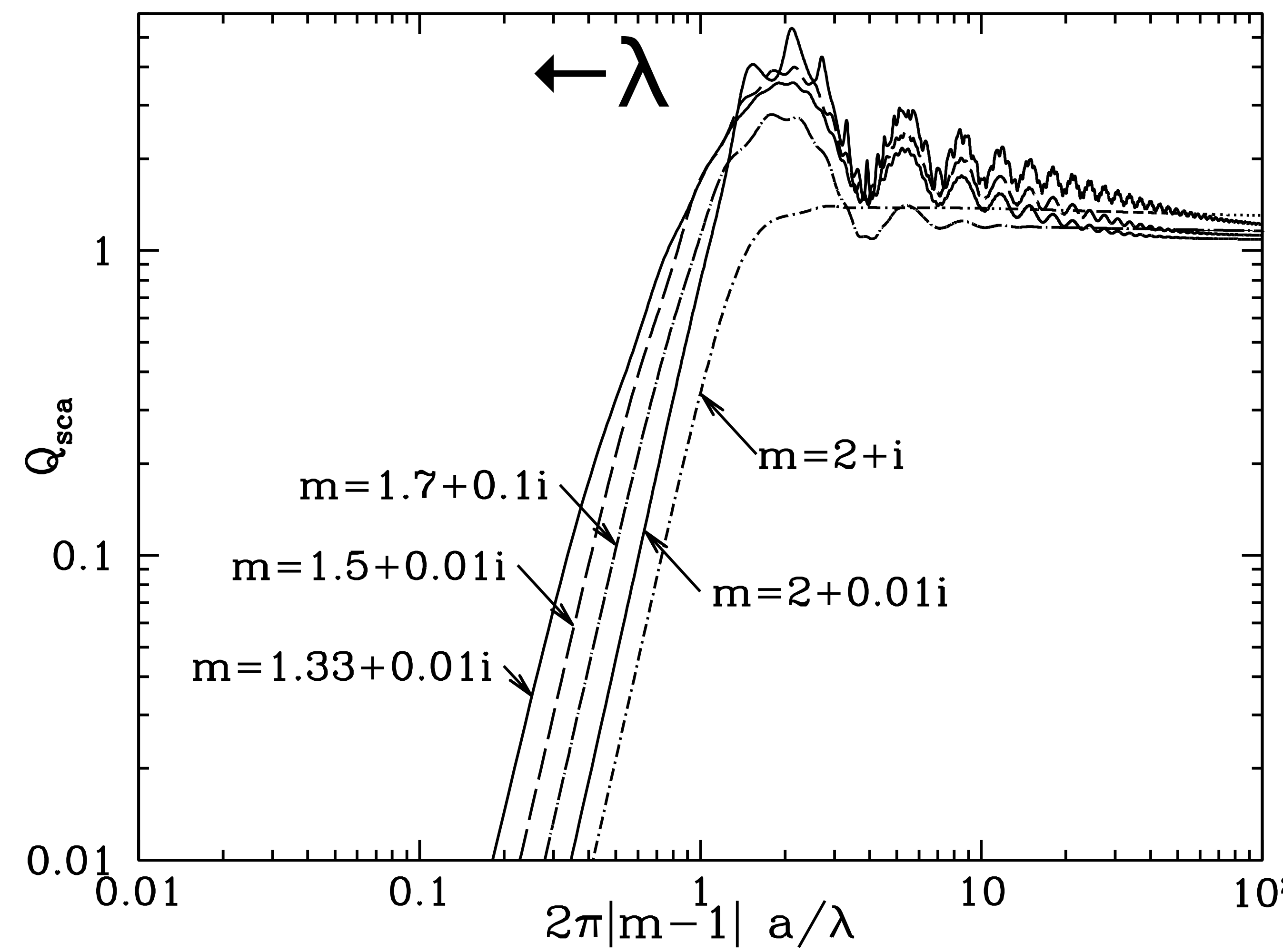
Scattering & Absorption by Small Particles

Scattering

scattering efficiency drops steeply with wavelength when $a/\lambda << 1$

Rayleigh scattering λ^{-4}

for fixed a

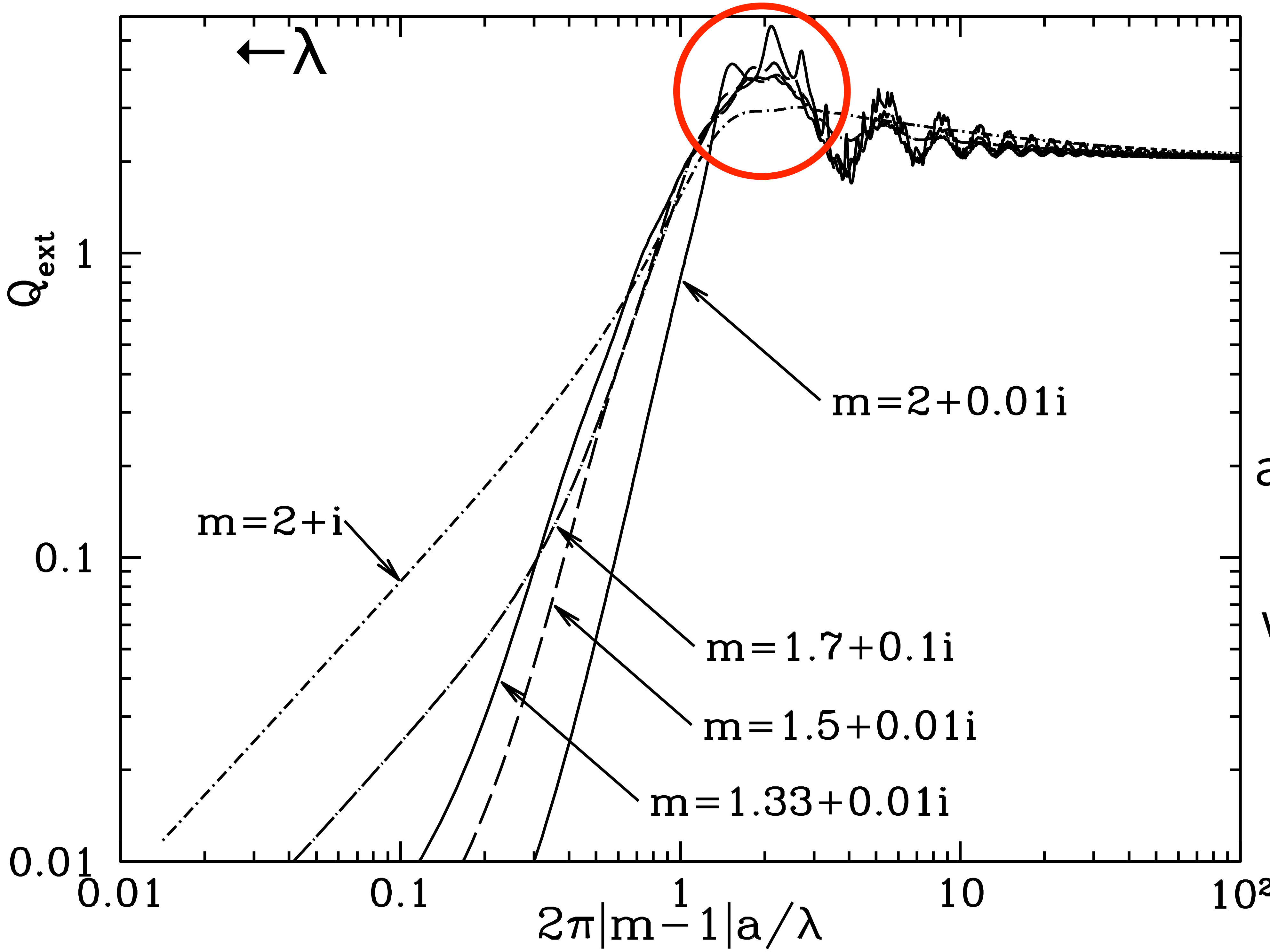


$$Q_{\rm sca} = \frac{8}{3}\left(\frac{2\pi a}{\lambda}\right)^4 \left|\frac{\epsilon - 1}{\epsilon + 2}\right|^2$$

Reflection Nebula vdB1
Image Credit & Copyright: Adam Block,
Mt. Lemmon SkyCenter, University of Arizona

Scattering & Absorption by Small Particles

for fixed a

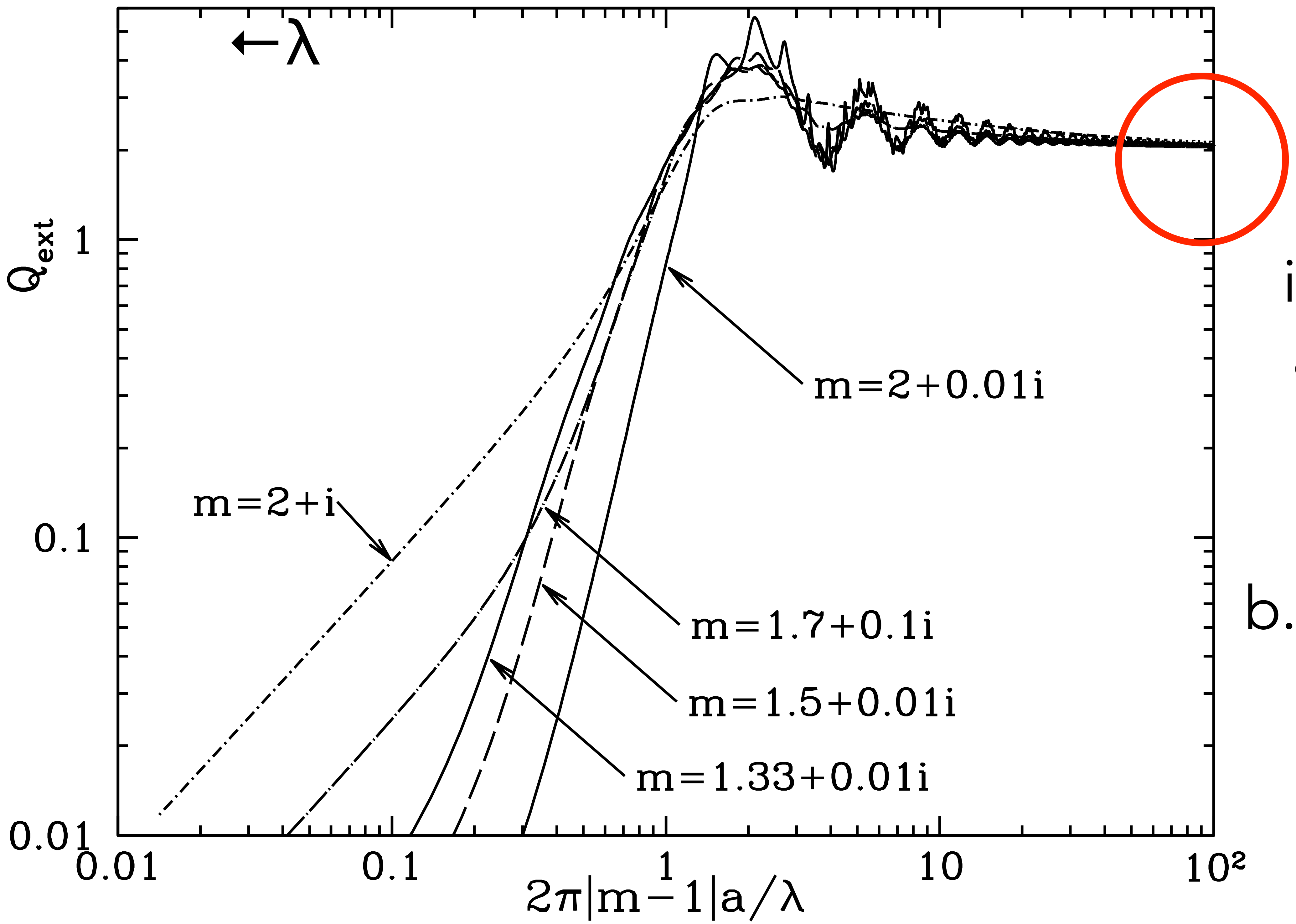


Maximum Q_{ext} occurs where $\lambda \sim a$

i.e. dust grains are most effective at blocking light with wavelengths close to their sizes

Scattering & Absorption by Small Particles

for fixed a



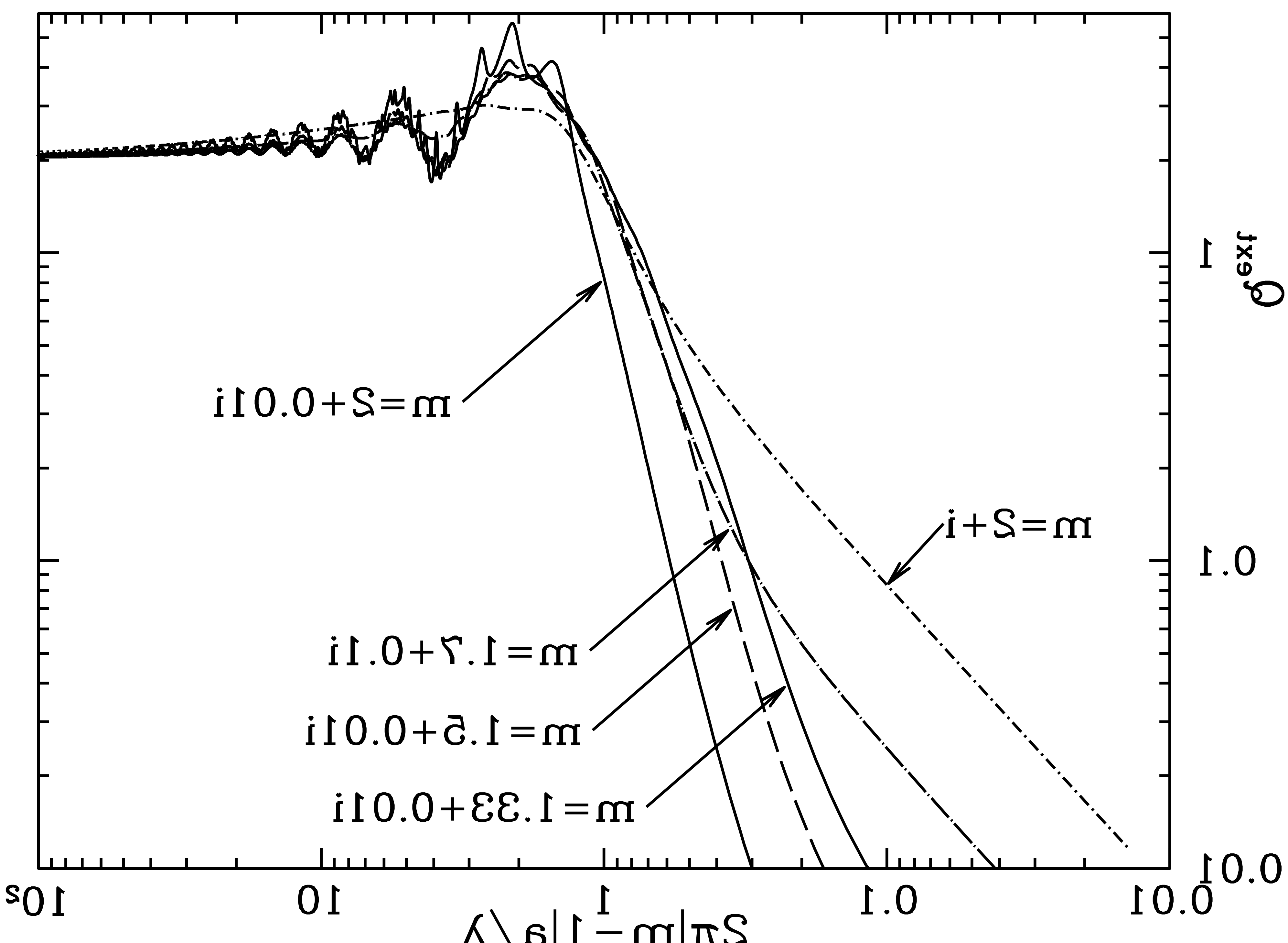
when a>>λ

Q_{ext} ~ 2

i.e. twice the geometrical cross sec

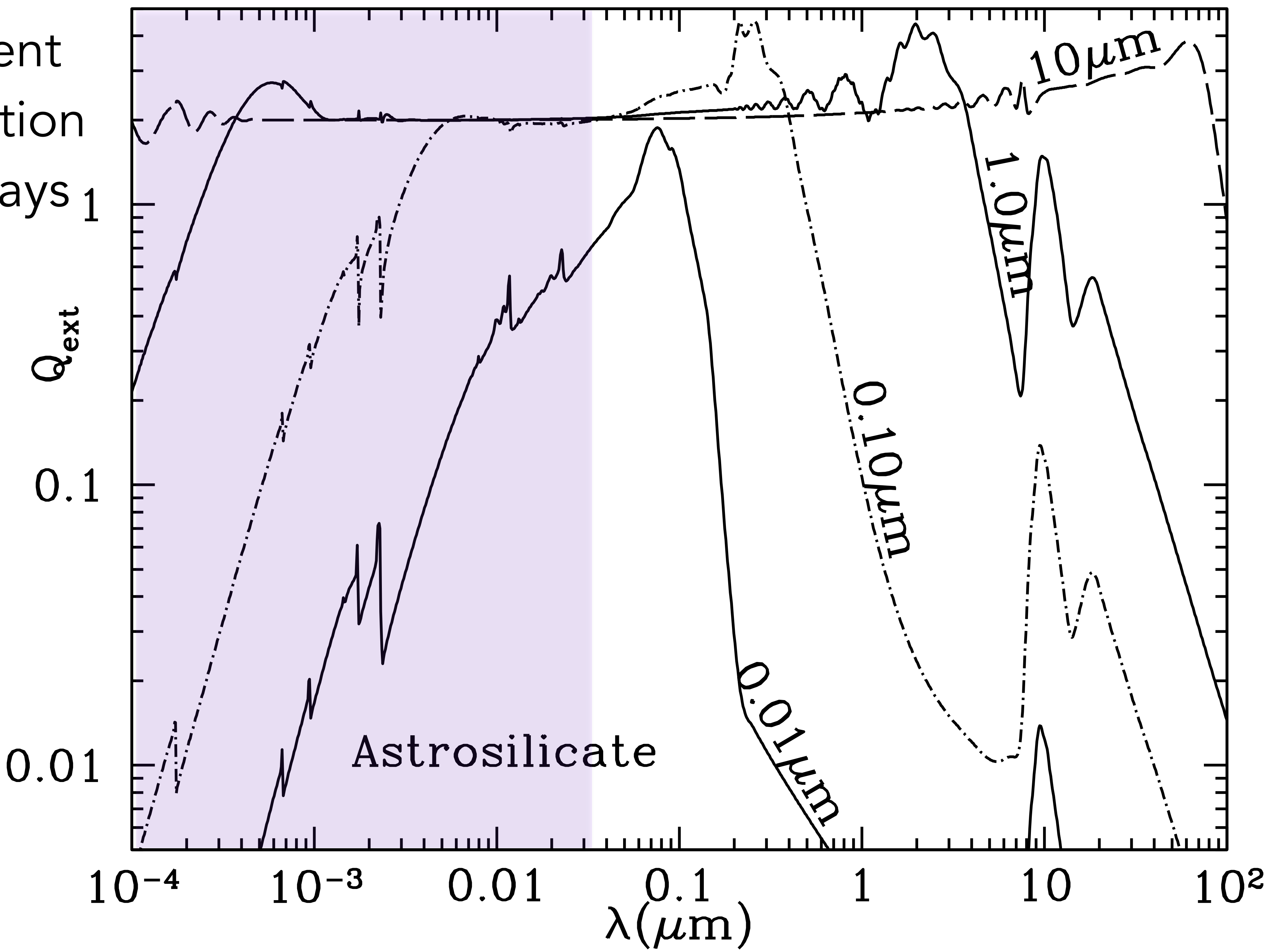
b.c. diffraction!!

Scattering & Absorption by Small Particles

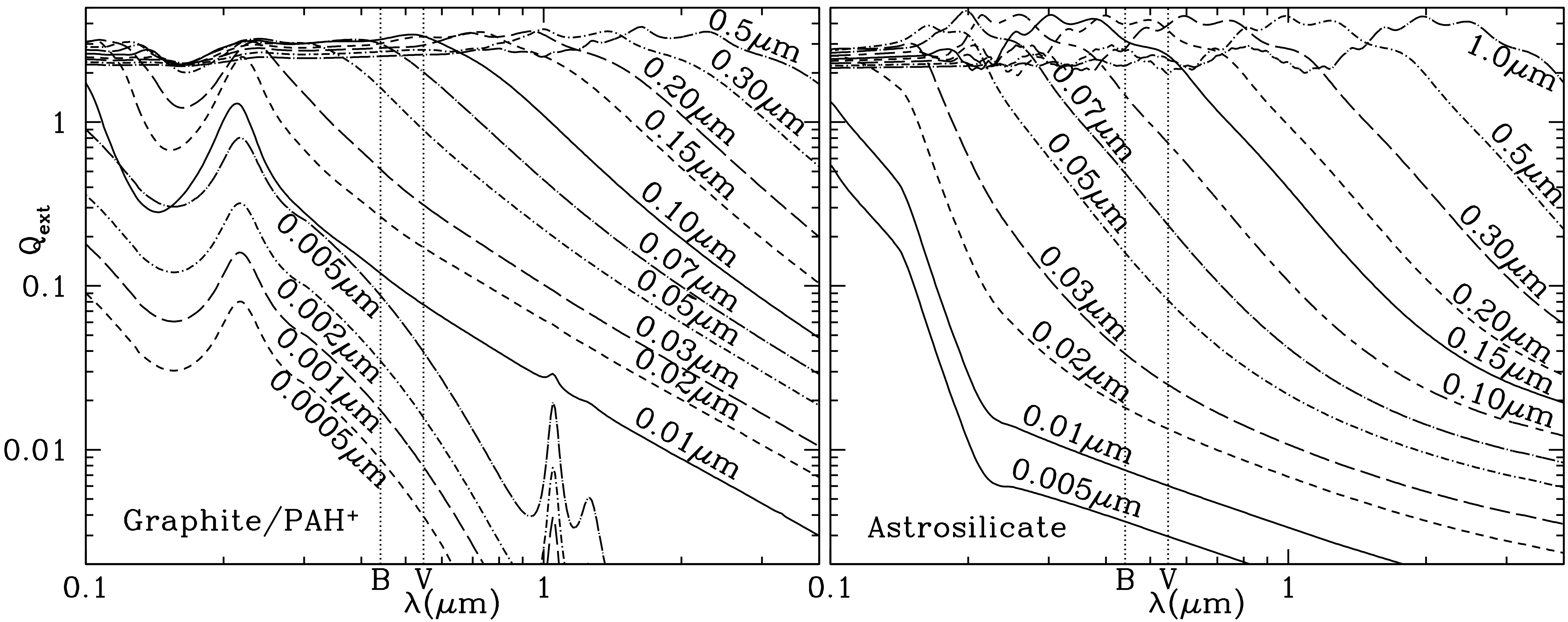


Astronomical Dust

Need different calculation for x-rays

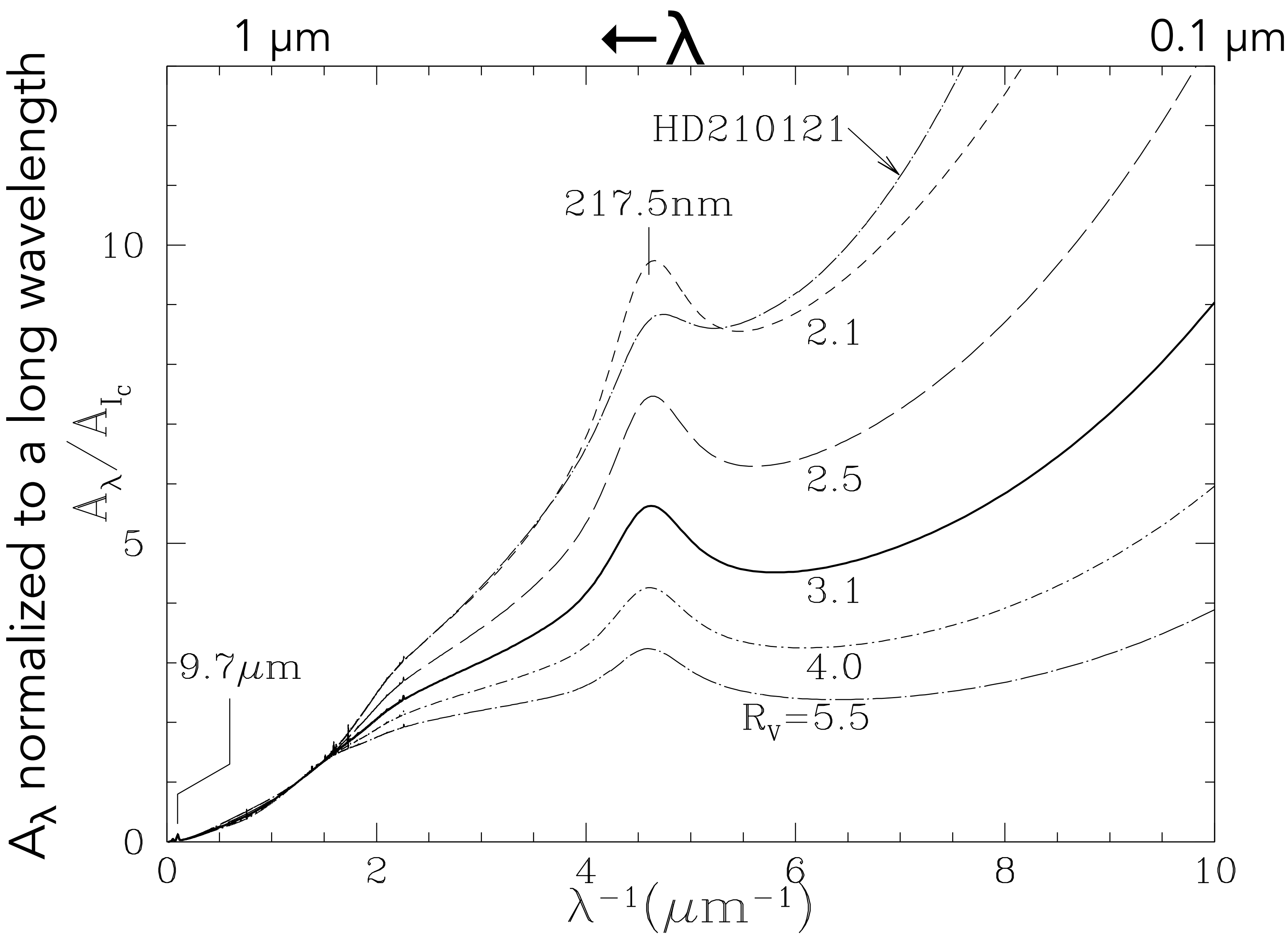


Astronomical Dust



Q_{ext} for astronomical dust analogs

Extinction Curve



This does not look like the Q_{ext} plots from before - why?

There is a range of grain sizes!

Extinction Curve

$$\frac{A_\lambda}{\mathrm{mag}} = 2.5 \log \left[e^{\tau_\lambda}\right] = 1.086 \tau_\lambda$$

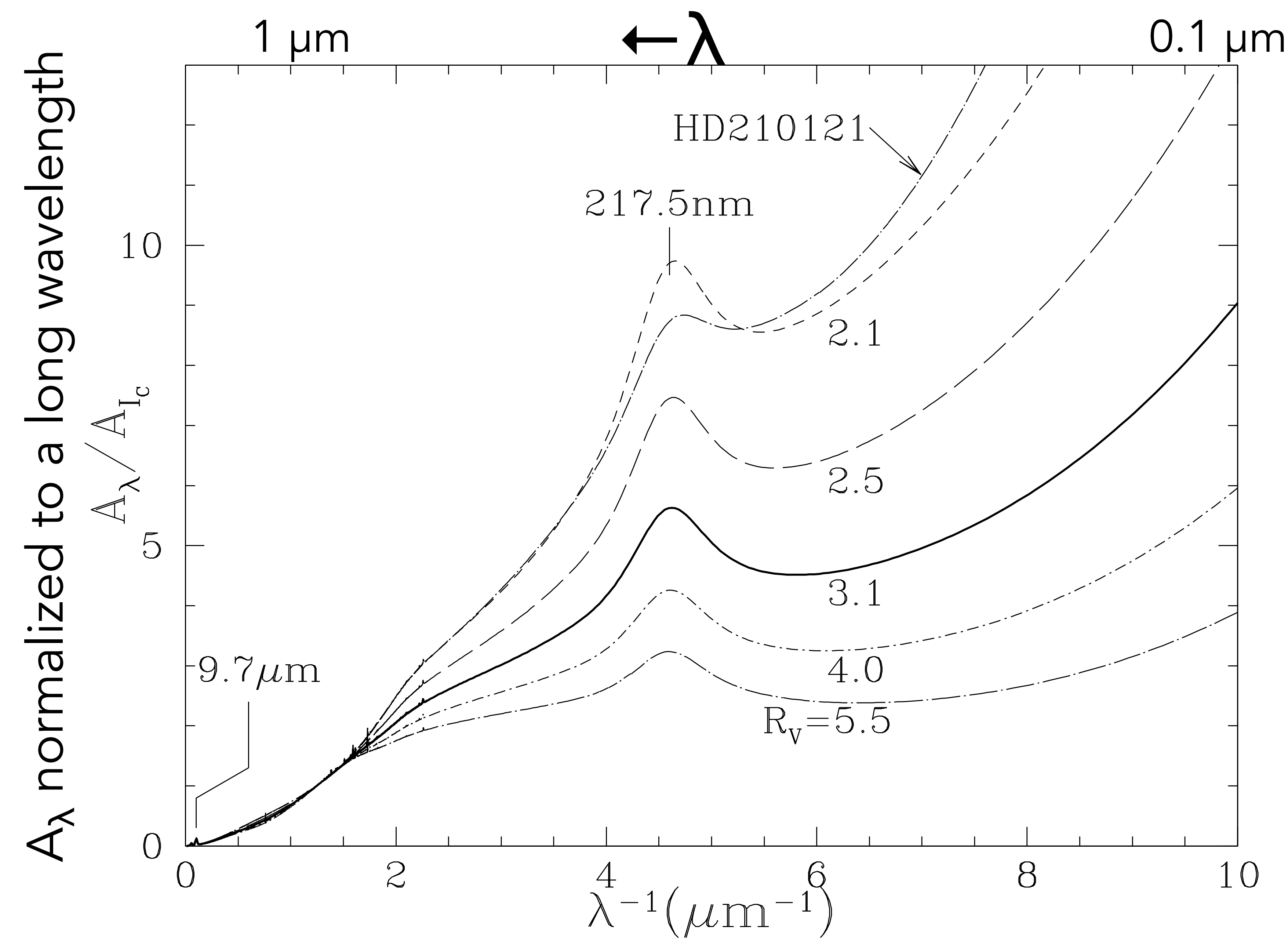
For a given grain size:

$$\tau_\nu(a) = N_d(a)\ Q_{ext}(a)\ \pi a^2$$

Rearrange units to get Weingartner & Draine 2001 eq 7:

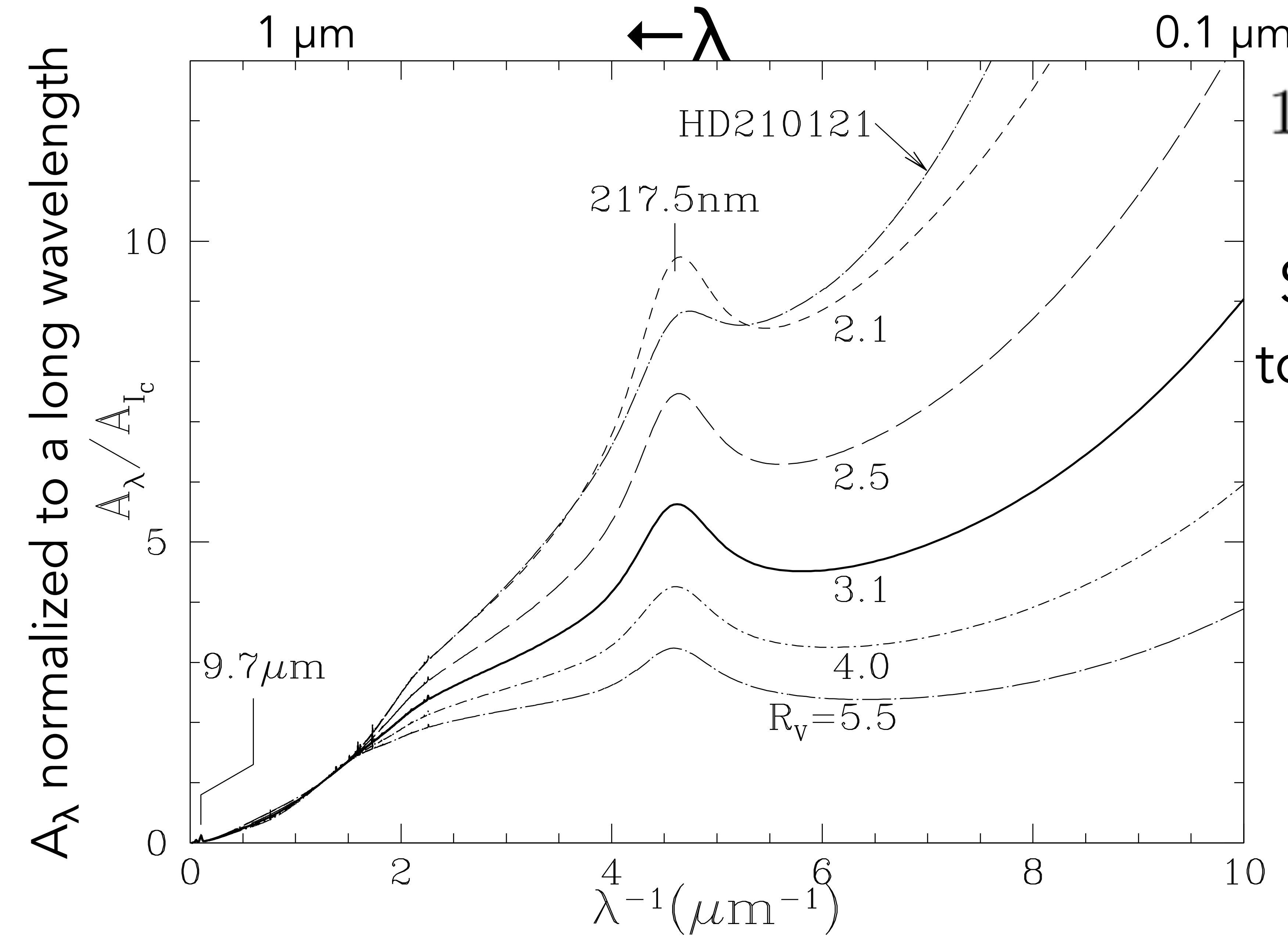
$$A(\lambda) = (2.5\pi \log e) \int d \ln a \, \frac{dN_{\mathrm{gr}}(a)}{da} \, a^3 Q_{\mathrm{ext}}(a, \lambda)$$

Extinction Curve



Continual rise to far-UV means there are more small grains than large grains.

Extinction Curve



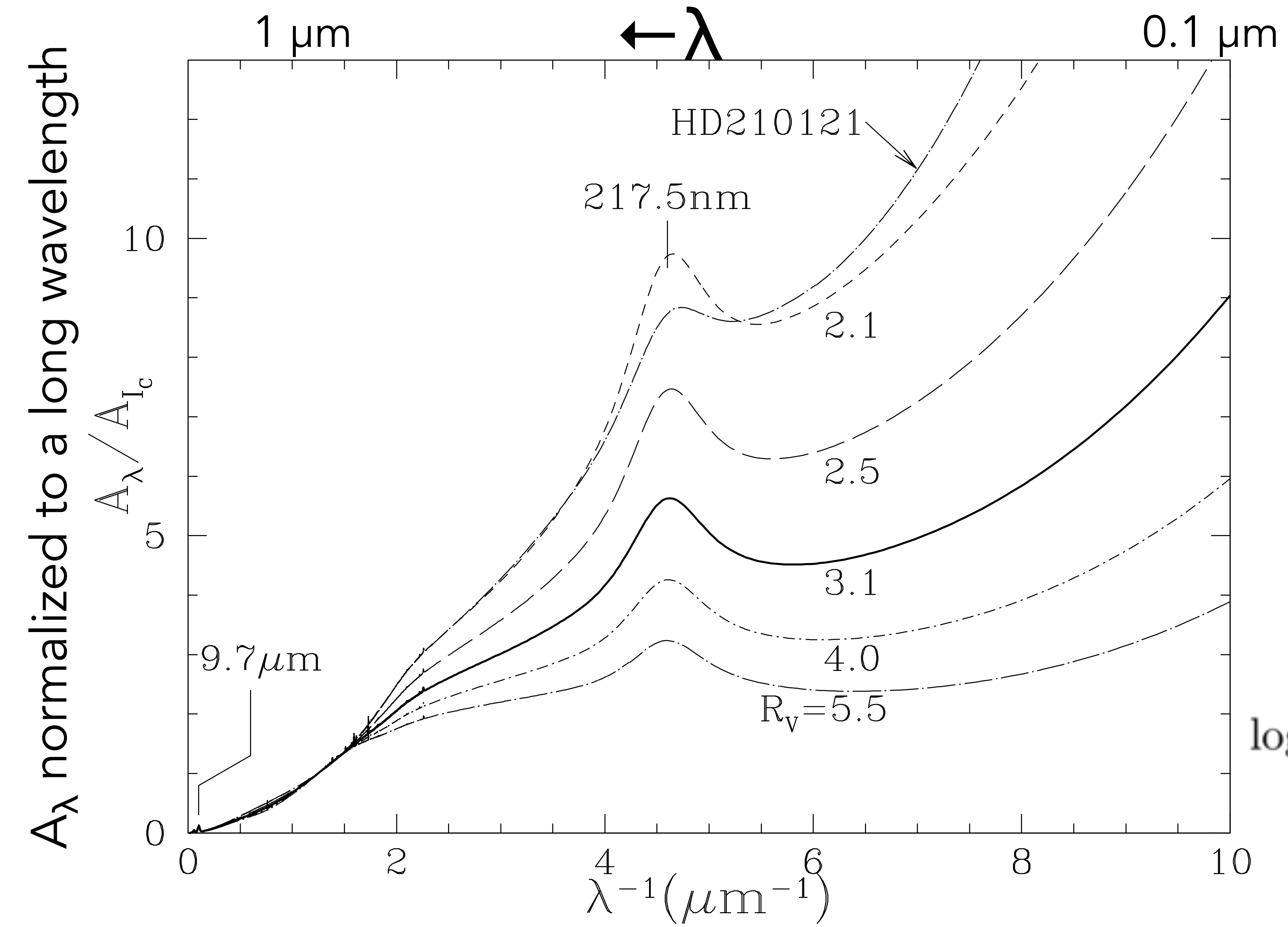
$$A_\lambda(a) = 1.086\ N_d(a)\ Q_{ext}(a)\ \pi a^2$$

Say grains only contribute to A_λ at their maximum Q_{ext}

$$\frac{A_{0.1\mu m}}{A_{0.5\mu m}} = \frac{N_{0.1\mu m}}{N_{0.5\mu m}} \left(\frac{0.1}{0.5}\right)^2$$

$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} \sim 150$$

Extinction Curve



$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} \sim \quad 150$$

Assume power-law slope to size distribution $N(a) \sim a^{-\beta}$

$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} = \left(\frac{0.1}{0.5}\right)^{-\beta}$$

$$\log_{10}\left(\frac{N_{0.1\mu m}}{N_{0.5\mu m}}\right) = -\beta \log_{10}\left(\frac{0.1}{0.5}\right)$$

$$\beta \sim 3$$

THE SIZE DISTRIBUTION OF INTERSTELLAR GRAINS

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ABSTRACT

The observed interstellar extinction over the wavelength range 0.11 μm < λ < 1 μm was fitted with a very general particle size distribution of uncoated graphite, enstatite, olivine, silicon carbide, iron, and magnetite. Combinations of these materials, up to three at a time, were considered. The cosmic abundances of the various constituents were taken into account as constraints on the possible distributions of particle sizes.

Excellent fits to the interstellar extinction, including the narrowness of the λ2160 feature, proved possible. Graphite was a necessary component of any good mixture, but it could be used with any of the other materials. The particle size distributions are roughly power law in nature, with an exponent of about -3.3 to -3.6. The size range for graphite is about 0.005 μm to about 1 μm. The size distribution for the other materials is also approximately power law in nature, with the same exponent, but there is a narrower range of sizes: about 0.025–0.25 μm, depending on the material. The number of large particles is not well determined, because they are gray. Similarly, the number of small particles is not well determined because they are in the Rayleigh limit. This power-law distribution is drastically different from an Oort–van de Hulst distribution, which is much more slowly varying for small particles but drops much faster for particles larger than average.

$$\frac{dn}{da} \propto a^{-3.5}$$

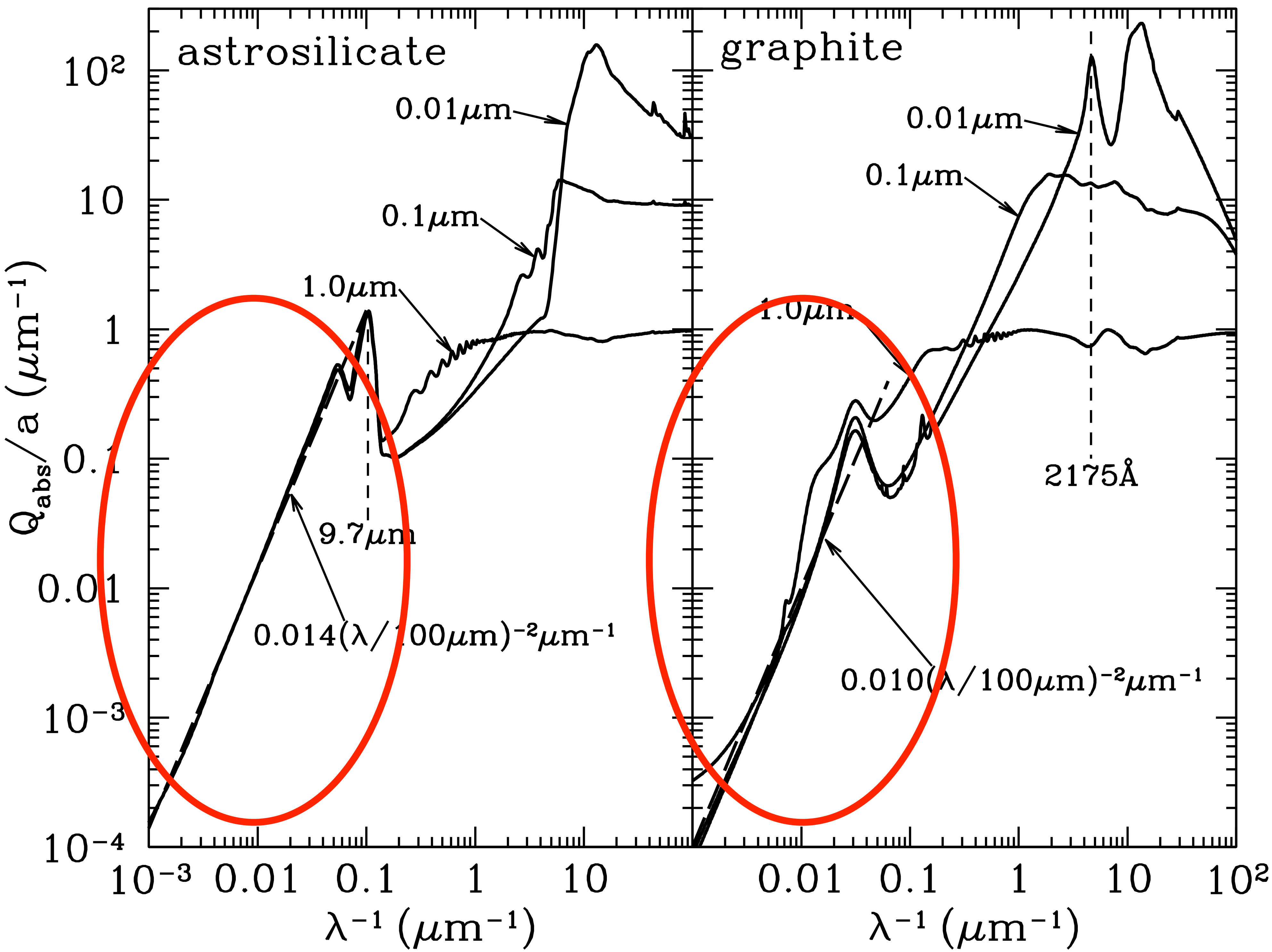
$$\mathrm{Mass}(a) \propto \int a^3 \frac{dn}{da} da \propto a^{0.5}$$

most mass in large grains

$$\mathrm{Area}(a) \propto \int a^2 \frac{dn}{da} da \propto a^{-0.5}$$

most area in small grains

←λ



At long wavelengths $Q_{abs}/a \propto \lambda^{-2}$

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int \underbrace{h\nu}_{\text{energy per absorbed photon}} \boxed{\frac{u_\nu}{h\nu}\, c\, Q_{\rm abs}(\nu)\pi a^2} d\nu$$

$n_{ph}\ v\ \sigma$

energy per absorbed photon

rate a dust grain of size a emits energy

$$\left(\frac{dE}{dt}\right)_{\rm emit} = \int 4\pi B_\nu(T)\ Q_{\rm em}(\nu)\ \pi a^2 d\nu$$

blackbody emitting over 4π str with efficiency Q_{em}

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \, \frac{u_\nu}{h\nu} \, c \, Q_{\rm abs}(\nu) \pi a^2 d\nu$$

<u>in LTE</u>

$$\boxed{\begin{aligned} u_\nu &= \frac{4\pi}{c} B_\nu(T) \\ \left(\frac{dE}{dt}\right)_{\rm abs} &= \left(\frac{dE}{dt}\right)_{\rm emit} \end{aligned}}$$

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\text{abs}} = \int h\nu \; \frac{u_\nu}{h\nu} \; c \; Q_{\text{abs}}(\nu)\pi a^2 d\nu$$

<u>in LTE</u>

$$\int \frac{4\pi}{c} B_\nu(T) \; c \; Q_{\text{abs}}(\nu)\pi a^2 d\nu = \int 4\pi B_\nu(T) \; Q_{\text{em}}(\nu)\pi a^2 d\nu$$

Therefore: $Q_{\text{abs}} = Q_{\text{em}}$

Dust Thermal Balance

rate a dust grain of size a absorbs energy

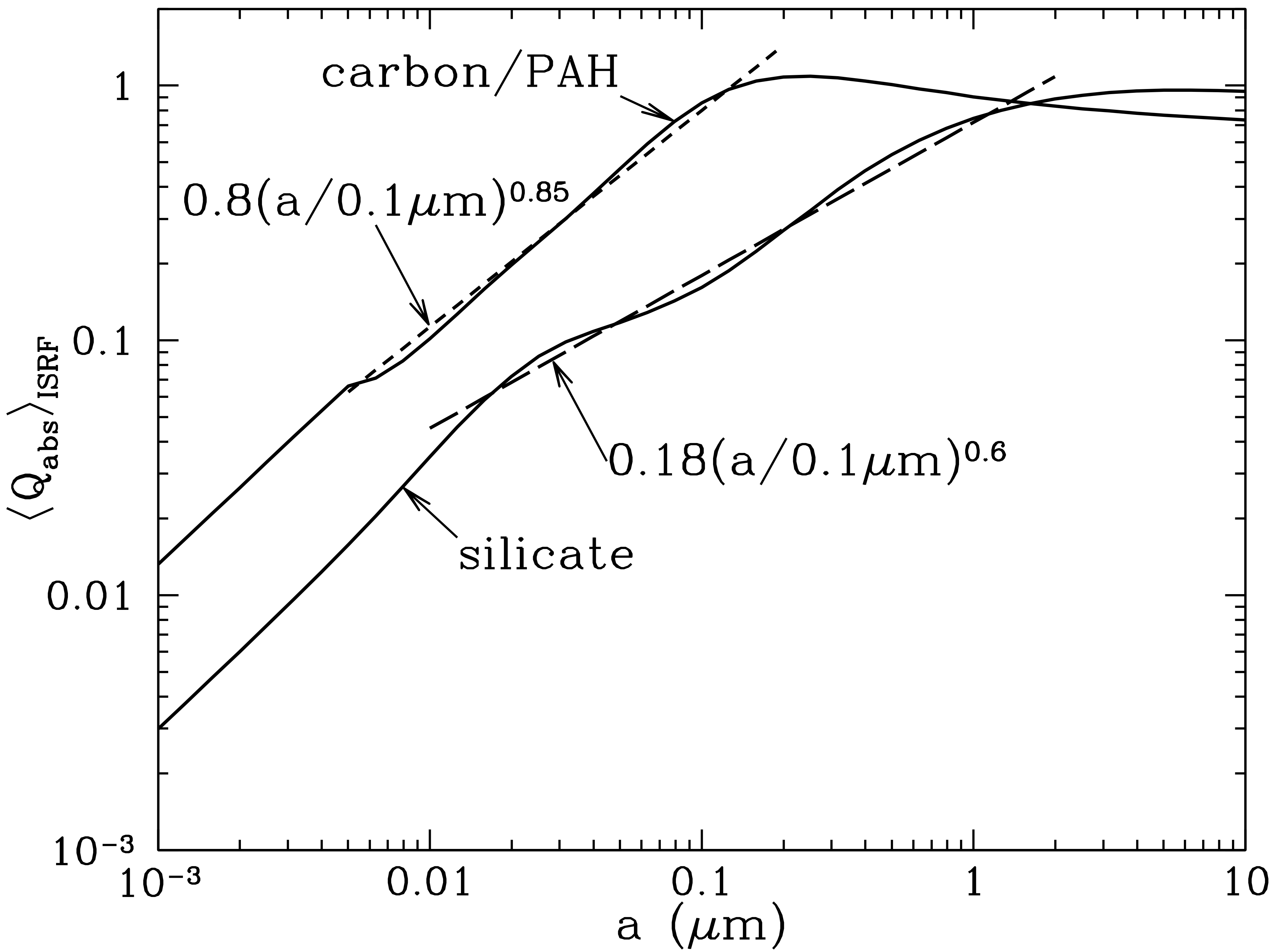
$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \; \frac{u_\nu}{h\nu} \; c \; Q_{\rm abs}(\nu) \pi a^2 d\nu$$

Define "spectrum averaged absorption cross section"

$$\langle Q_{\rm abs} \rangle_* \equiv \frac{\int u_{*\nu} Q_{\rm abs}(\nu) d\nu}{\int u_{*\nu} d\nu}$$

so that: $$\left(\frac{dE}{dt}\right)_{\rm abs} = \langle Q_{\rm abs} \rangle_* \pi a^2 \; u_* \; c$$

Dust Thermal Balance



$<Q_{abs}>_*$ for the average interstellar radiation field in the MW, and two astronomical dust analogs.

Dust Thermal Balance

rate a dust grain of size a emits energy

$$\left(\frac{dE}{dt}\right)_{\rm emit} = \int 4\pi B_\nu(T)\ Q_{\rm em}(\nu)\ \pi a^2 d\nu$$

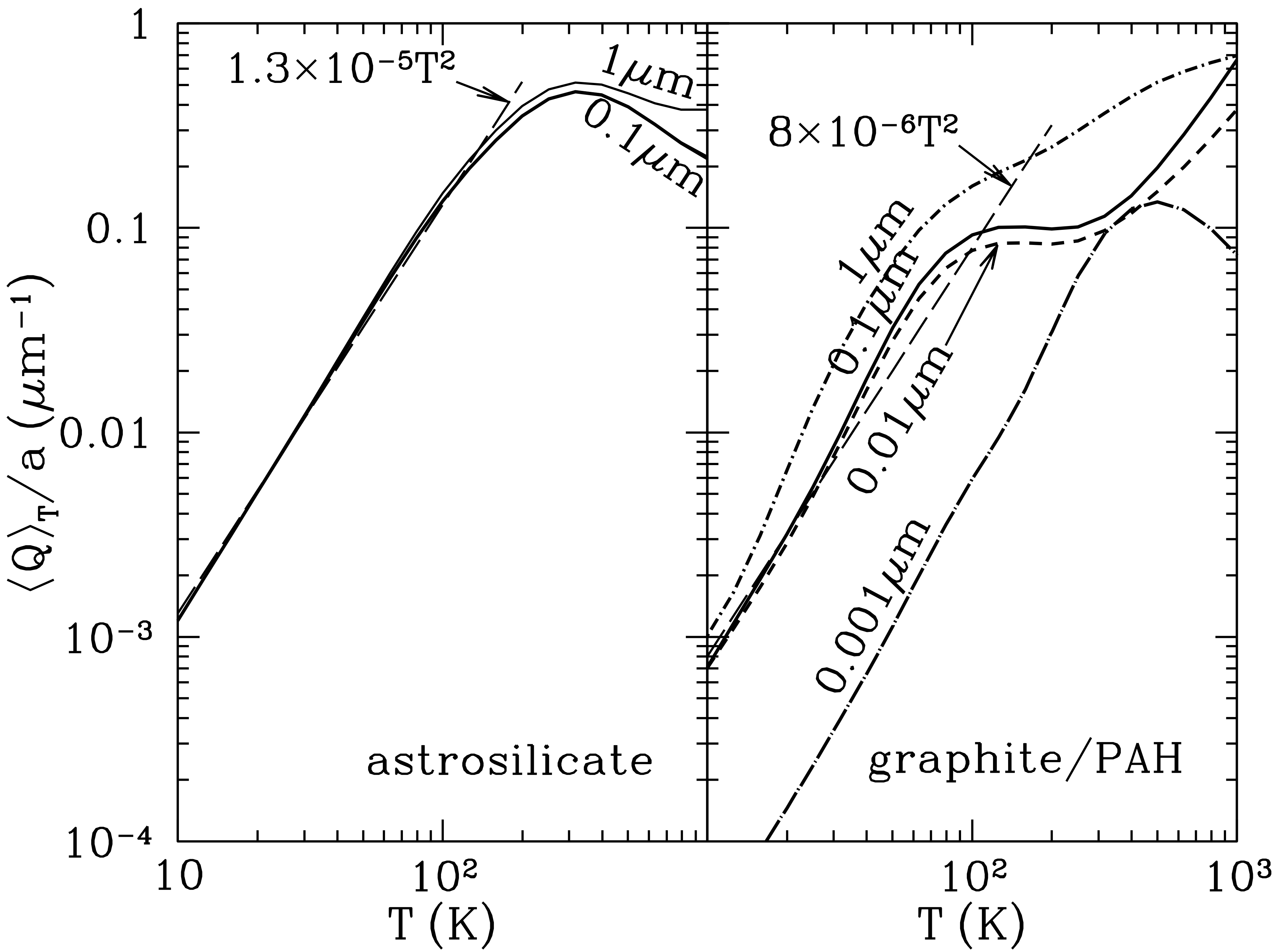
$$Q_{\rm abs} = Q_{\rm em}$$

Define "Planck averaged emission efficiency"

$$\langle Q_{\rm abs}\rangle_{\rm T} \equiv \frac{\int B_\nu(T) Q_{\rm abs} d\nu}{\int B_\nu(T) d\nu}$$

so that: $$\left(\frac{dE}{dt}\right)_{\rm em} = 4\pi a^2\ \langle Q_{\rm abs}\rangle_{\rm T}\, \sigma T^4$$

Dust Thermal Balance



$<Q_{abs}>_T/a$ for the a range of blackbody temperatures

Below ~100 K $<Q_{abs}>_T \sim T^2$

Dust Thermal Balance

In steady state, emission = absorption.

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \langle Q_{\rm abs}\rangle_* \, \pi a^2 \; u_* \; c$$

$$\left(\frac{dE}{dt}\right)_{\rm em} = 4\pi a^2 \, \langle Q_{\rm abs}\rangle_{\rm T} \, \sigma T^4$$

for MW interstellar radiation field and dust properties we found:

$\langle Q_{\rm abs}\rangle_* \sim 0.8(a/0.1\mu m)^{0.85}$ carbon

$\langle Q_{\rm abs}\rangle_* \sim 0.18(a/0.1\mu m)^{0.6}$ silicate

Dust Thermal Balance

In steady state, emission = absorption.

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \langle Q_{\rm abs}\rangle_* \, \pi a^2 \; u_* \; c$$

$$\left(\frac{dE}{dt}\right)_{\rm em} = 4\pi a^2 \, \langle Q_{\rm abs}\rangle_{\rm T} \, \sigma T^4$$

for MW interstellar radiation field and dust properties we found:

$\langle Q_{\rm abs}\rangle_T \sim 8 \times 10^{-6} T^2$ carbon

$\langle Q_{\rm abs}\rangle_T \sim 1.3 \times 10^{-5} T^2$ silicate

Dust Thermal Balance

Very weak dependence of equilibrium temperature on grain size.

$$T \approx 22.3(a/0.1\mu m)^{-1/40} U^{1/6} K$$

carbon

$$T \approx 16.4(a/0.1\mu m)^{1/15} U^{1/6} K$$

silicate