Physics 224
The Interstellar Medium

Lecture #7: Ionization & Recombination

Radiation Field Definitions

$$S_\nu = \int I_\nu d\Omega$$

Flux Density
units [erg/s/cm^2/Hz]

$$L_\nu = 4\pi d^2 S_\nu$$

distance to source

Spectral Luminosity
units [erg/s/Hz]

$$L_{bol} = \int L_\nu d\nu$$

Bolometric Luminosity
units [erg/s]

Radiation Field Definitions

$$u_\nu(\Omega) = \frac{1}{c} I_\nu$$

Energy density per solid angle
units [erg/cm^3/Hz/sr]

$$u_\nu = \frac{1}{c} \int I_\nu d\Omega$$

Energy density
units [erg/cm^3/Hz]

- Part I: Absorption Lines (continued from last time)
- Part II: Ionization Processes
- Part III: Recombination Processes
- Part IV: HII Regions

Part I: Absorption Lines

Absorption Lines

$$\kappa_\nu = \frac{h\nu}{4\pi} n_l B_{lu} \phi_\nu \left(1 - e^{-E_{ul}/kT_{exc}}\right)$$

For most optical absorption lines $E_{ul} \gg kT_{exc}$

This means that upper level is generally not populated, so stimulated emission is negligible!

Absorption Lines

In that case, we can integrate κ_ν over the path length (s) to get optical depth and show:

$$\tau_\nu = \frac{\pi e^2}{m_e c} f_{lu} \; N_l \; \phi_\nu$$

"oscillator strength" related to Einstein coeff

column density of absorbers

$$A_{ul} = \frac{8\pi^2 e^2 \nu_{lu}^2}{m_e c^3} \frac{g_l}{g_u} f_{lu}$$

Absorption Lines

General line w/o stimulated emission:

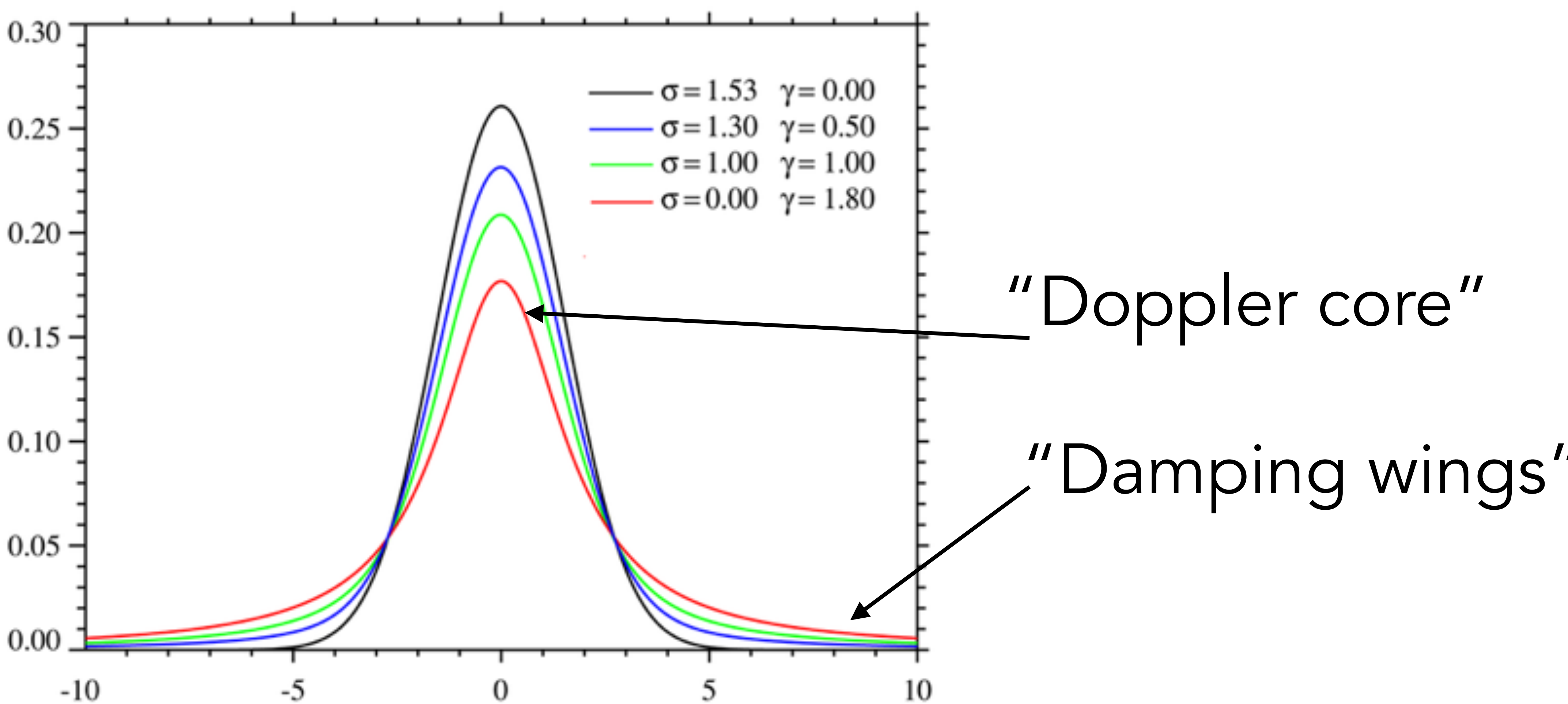
$$\tau_\nu = \text{const.}\ N_l\ \phi_\nu$$

line profile

Absorption Lines

Voigt Profile: convolution of Lorentz & Gaussian

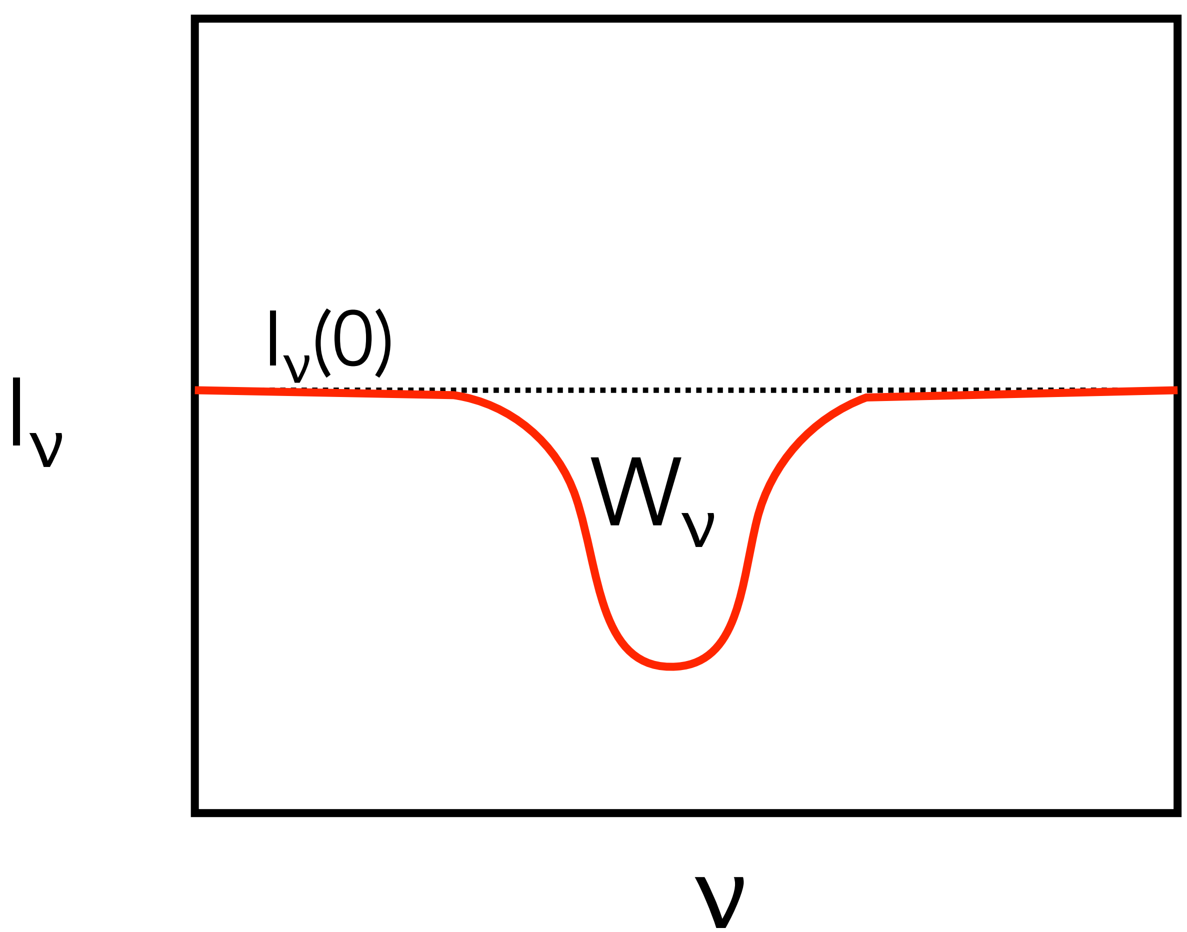
$$\phi_\nu = \frac{1}{\sqrt{2\pi\sigma_v^2}} \int_{-\infty}^{\infty} e^{-v^2/2\sigma_v^2} \frac{4\gamma_{u\ell}}{16\pi^2(\nu - (1 - v/c)\nu_{u\ell})^2 + \gamma_{u\ell}^2}\, dv$$



Absorption Lines

Define "equivalent width" of a line:

$$W_\nu \equiv \int_{-\infty}^{\infty} \frac{I_\nu(0) - I_\nu}{I_\nu(0)} d\nu = \int_{-\infty}^{\infty} \left(1 - e^{-\tau_\nu}\right) d\nu$$



Why is this useful?
if we know what $I_\nu(0)$ is and absorption is happening in a narrow freq range,
we can relate EW to τ_ν

Absorption Lines

Online demo…

Absorption Lines

When $\tau_\nu \ll 1$, Taylor expansion of $1 - e^{-\tau_\nu}$

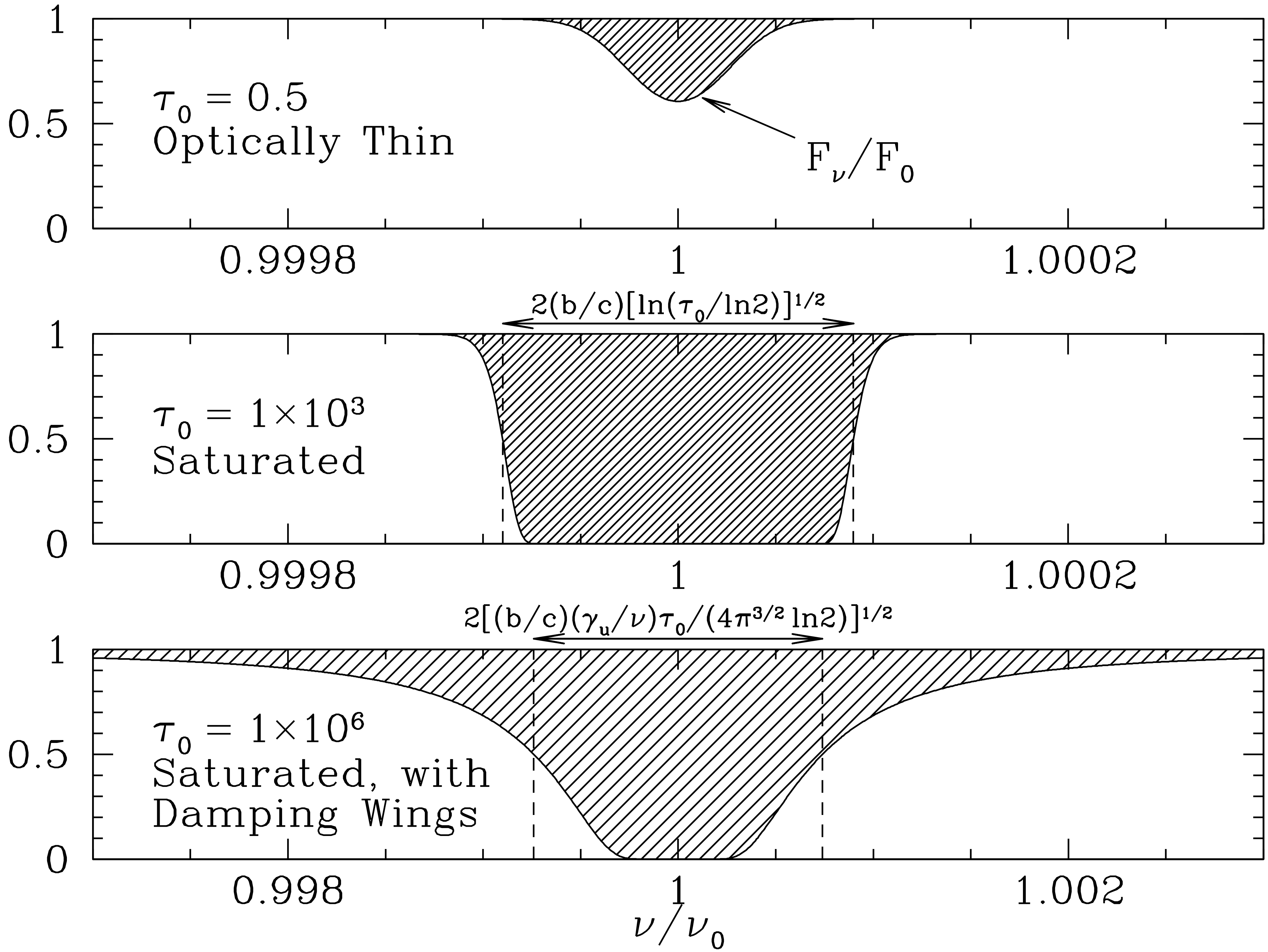
$$W_\nu = \int 1 - (1 - \tau_\nu) d\nu = \int \tau_\nu d\nu$$

$$= \sqrt{\pi}\frac{b}{c}\tau_0 = \pi \frac{e^2}{m_e c^2} f_{\ell u} \lambda_{u\ell} N_\ell$$

Doppler broadening parameter $b = 2^{1/2}\sigma_v$

optical depth at line center

Absorption Lines



Absorption Lines

"Linear"	$W \propto N$	$\tau_o \ll 1$	$\tau_o = \frac{\pi e^2}{mc^2} \lambda^2 N f$
"Flat"	$W \propto b\sqrt{\ln(N/b)}$	$10 \leq \tau_o \leq 10^3$	$\tau_o = \frac{\pi^{1/2} e^2}{mc} \frac{\lambda}{b} N f$
"Damped"	$W \propto \sqrt{N}$	$\tau_o \geq 10^4$	$\tau_o = \frac{1}{4} \frac{e^2 \Gamma}{mc^3} \lambda^4 N f$

Absorption Lines

"Curve of Growth"

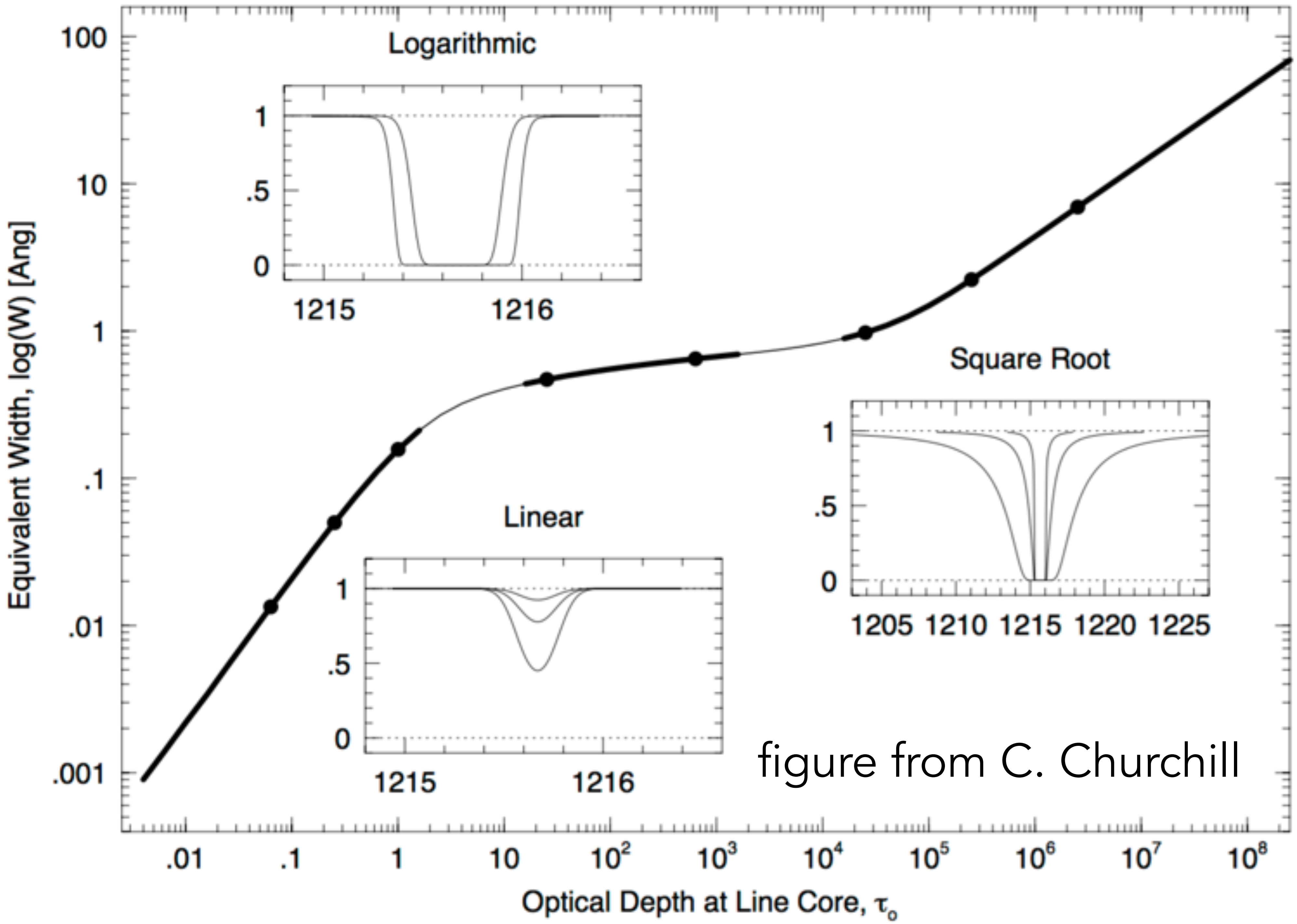


figure from C. Churchill

Part II: Ionization Processes

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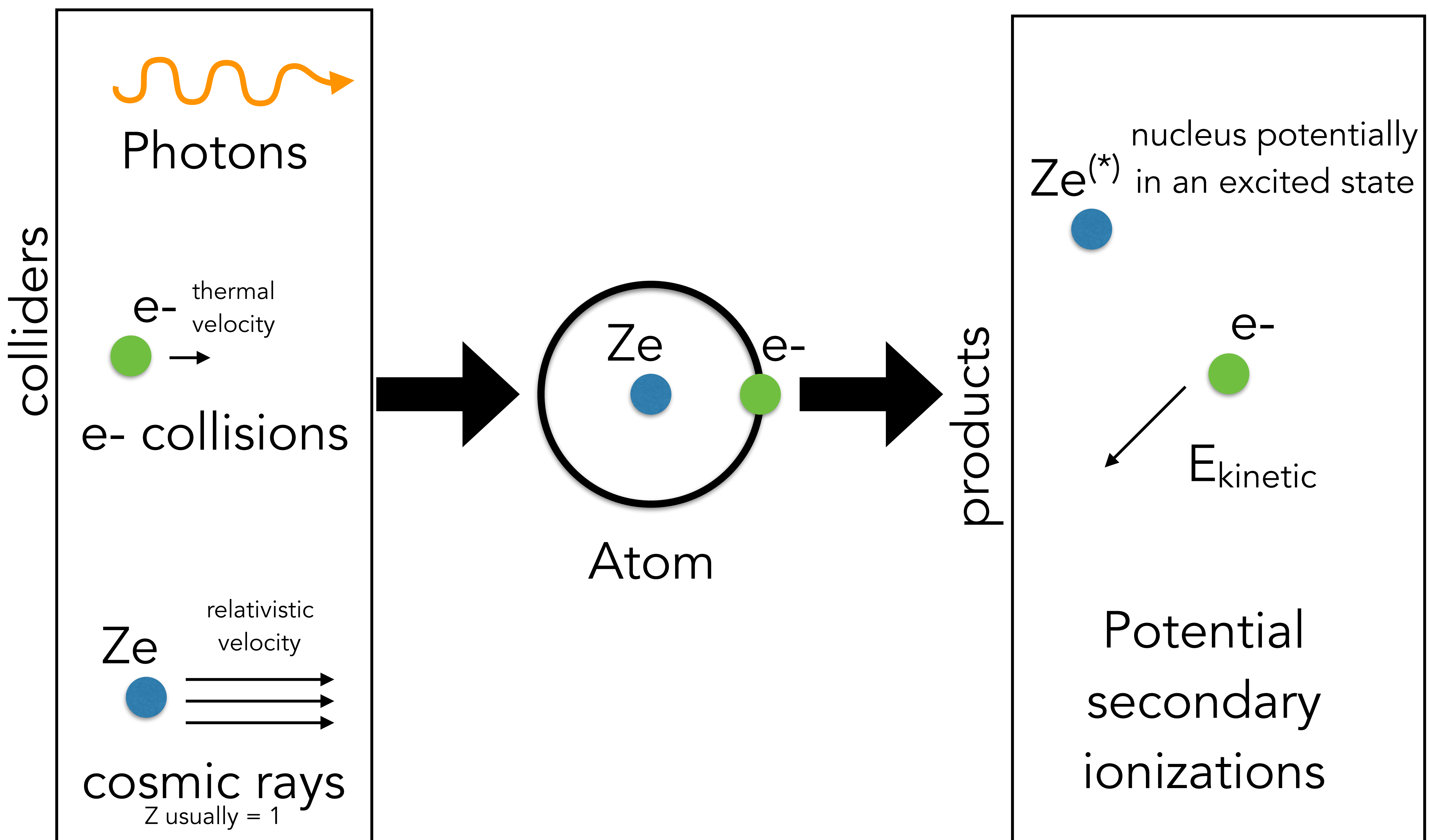
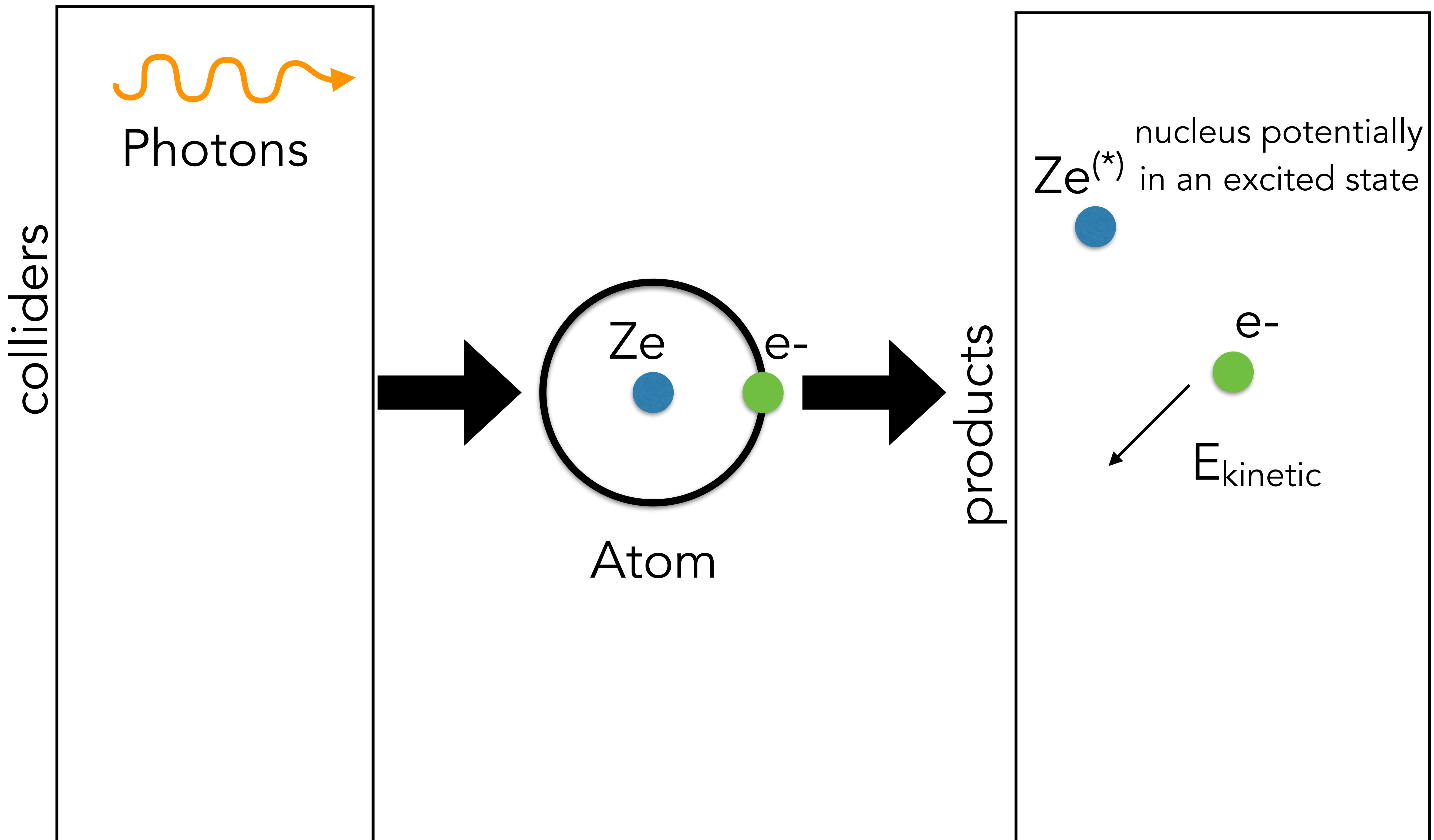


Part II: Ionization Processes



Photoionization

Cross section can be determined analytically for Hydrogen (and "hydrogenic" ions - those with 1 e- remaining)

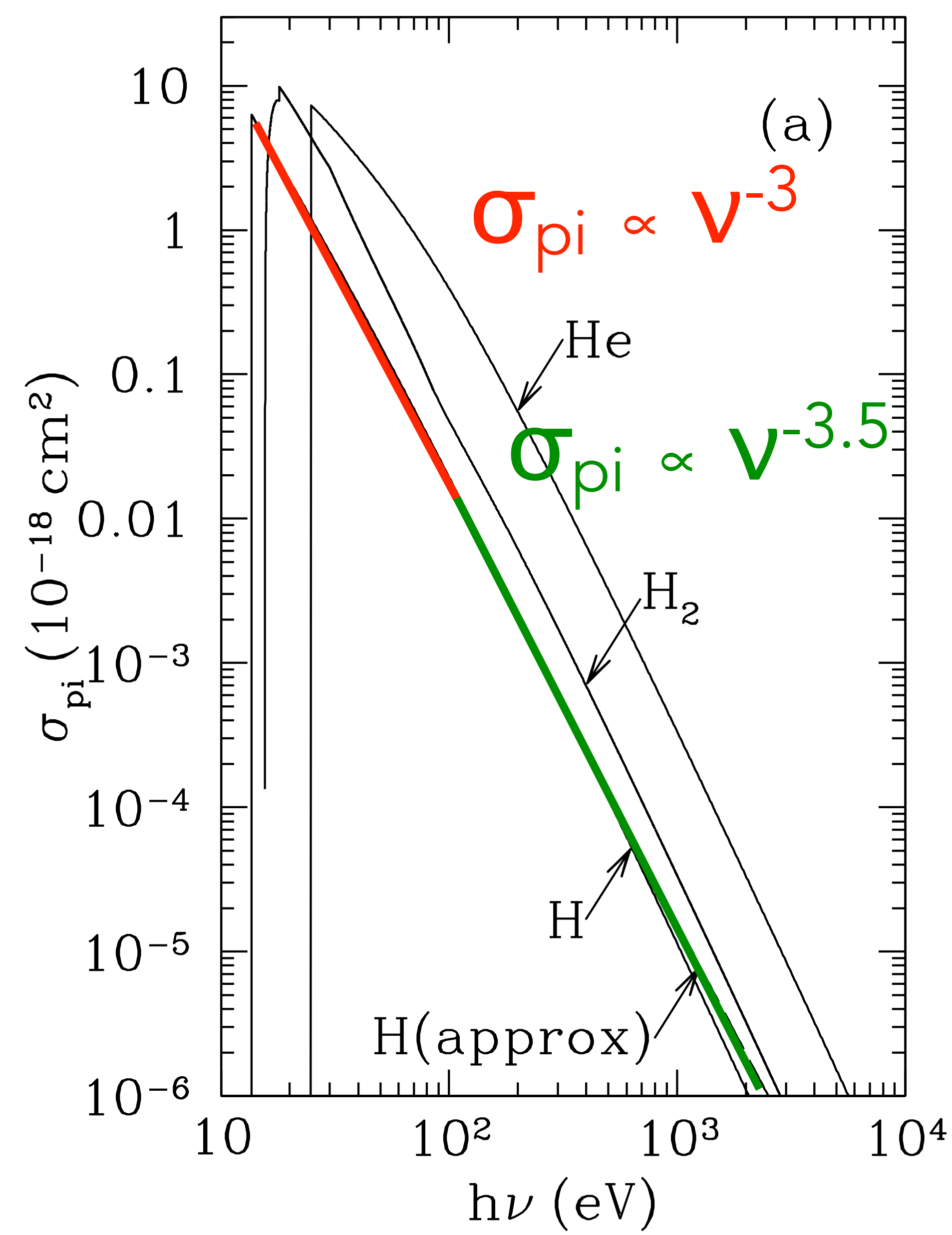
when hν > 13.6 Z^2 eV

$$\sigma_{\rm pi}(\nu) = \sigma_0 \left(\frac{Z^2 I_{\rm H}}{h\nu}\right)^4 \frac{e^{4-(4\tan^{-1}x)/x}}{1-e^{-2\pi/x}}$$

where: $x = \sqrt{\frac{h\nu}{Z^2 I_{\rm H}} - 1}$

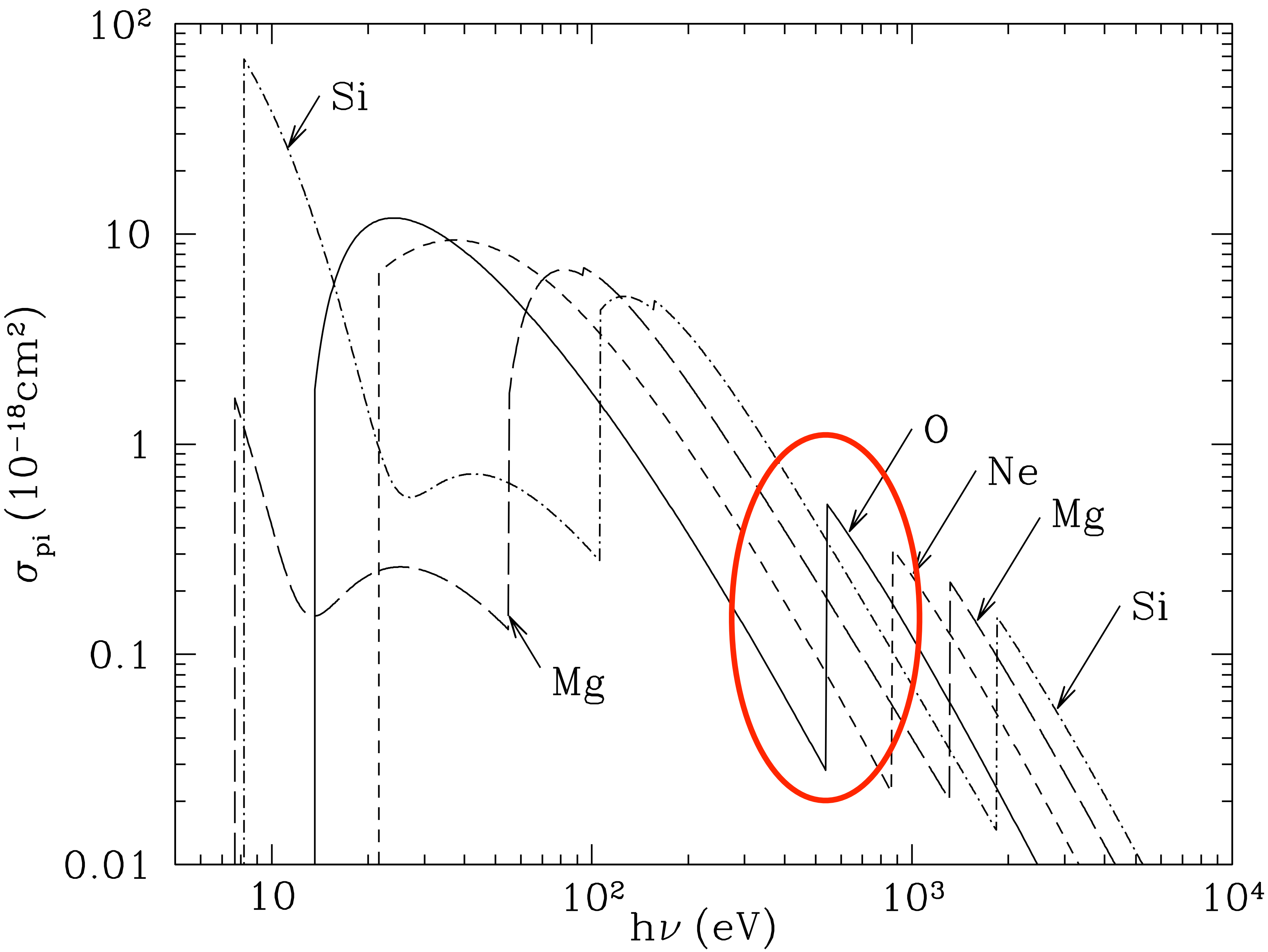
and "cross section at threshold" is

$$\sigma_0 = \frac{2^9\pi}{3e^4} Z^{-2}\alpha\pi a_0^2 = 6.304 \times 10^{-18} Z^{-2}\ {\rm cm}^{-2}$$



Photoionization

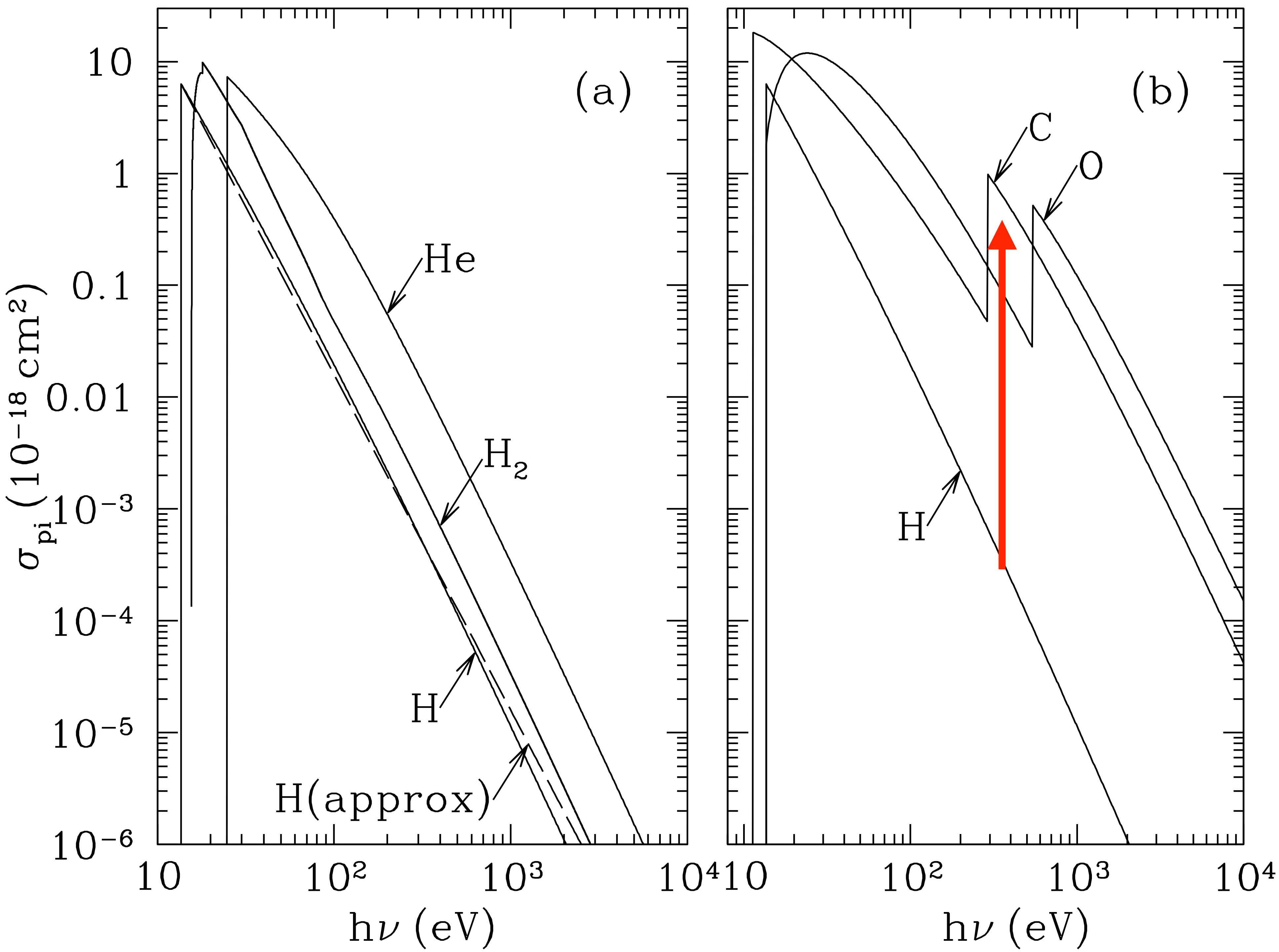
Cross section complexity increases with multiple electrons.



"absorption edge" due to K shell (the 1s shell)

above binding energy of K shell cross section increases sharply

Photoionization



<u>Note:</u>

cross section of C and O and other metals far exceeds H at high energy

Even though they are less abundant, <u>metals dominate PI rate of gas at high energies.</u>

Photoionization

rate per volume ~ n_{atom} $\boxed{n_{collider}\ \sigma\ c}$

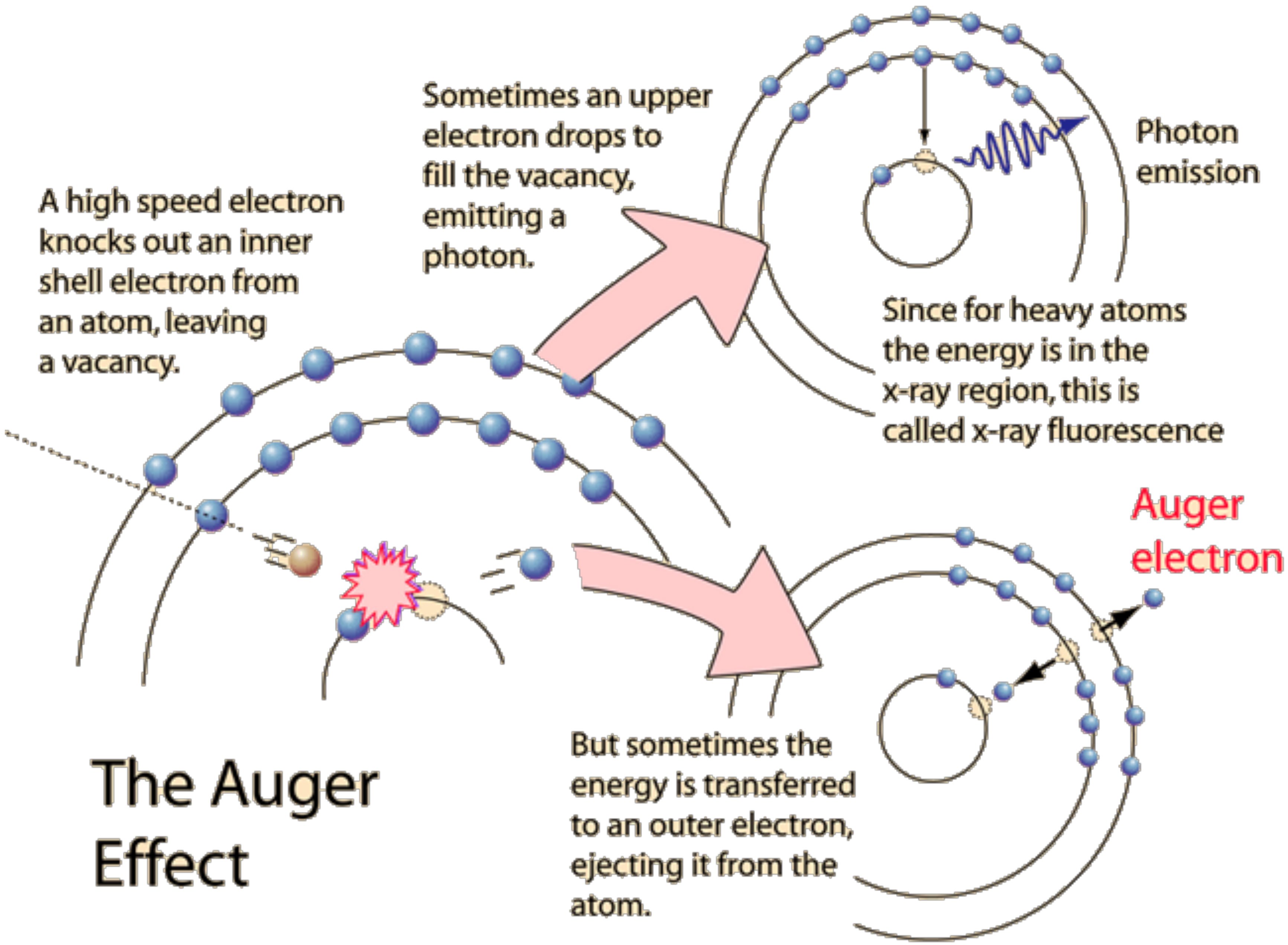
ζ_{pi} =
photoionization rate

$$\zeta_{pi} = \int_{\nu_1}^{\infty} \sigma_{pi}(\nu)\ c\ \frac{u_\nu}{h\nu}\ d\nu$$

minimum energy for ionization

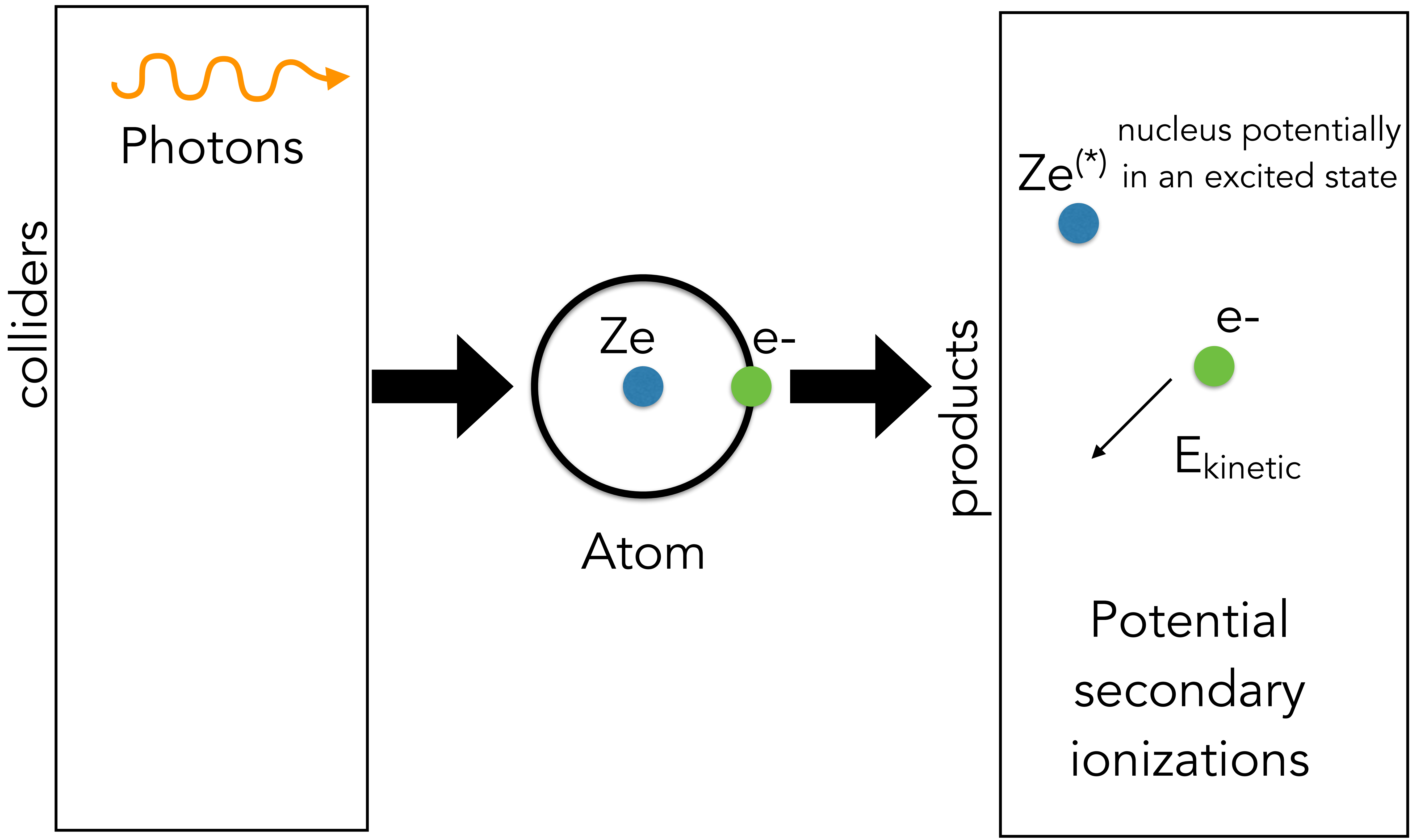
number density of photons

Photoionization



http://hyperphysics.phy-astr.gsu.edu/hbase/atomic/auger.html

Ionization Processes



Secondary Ionizations

$$E_{pe} = h\nu - I_s$$

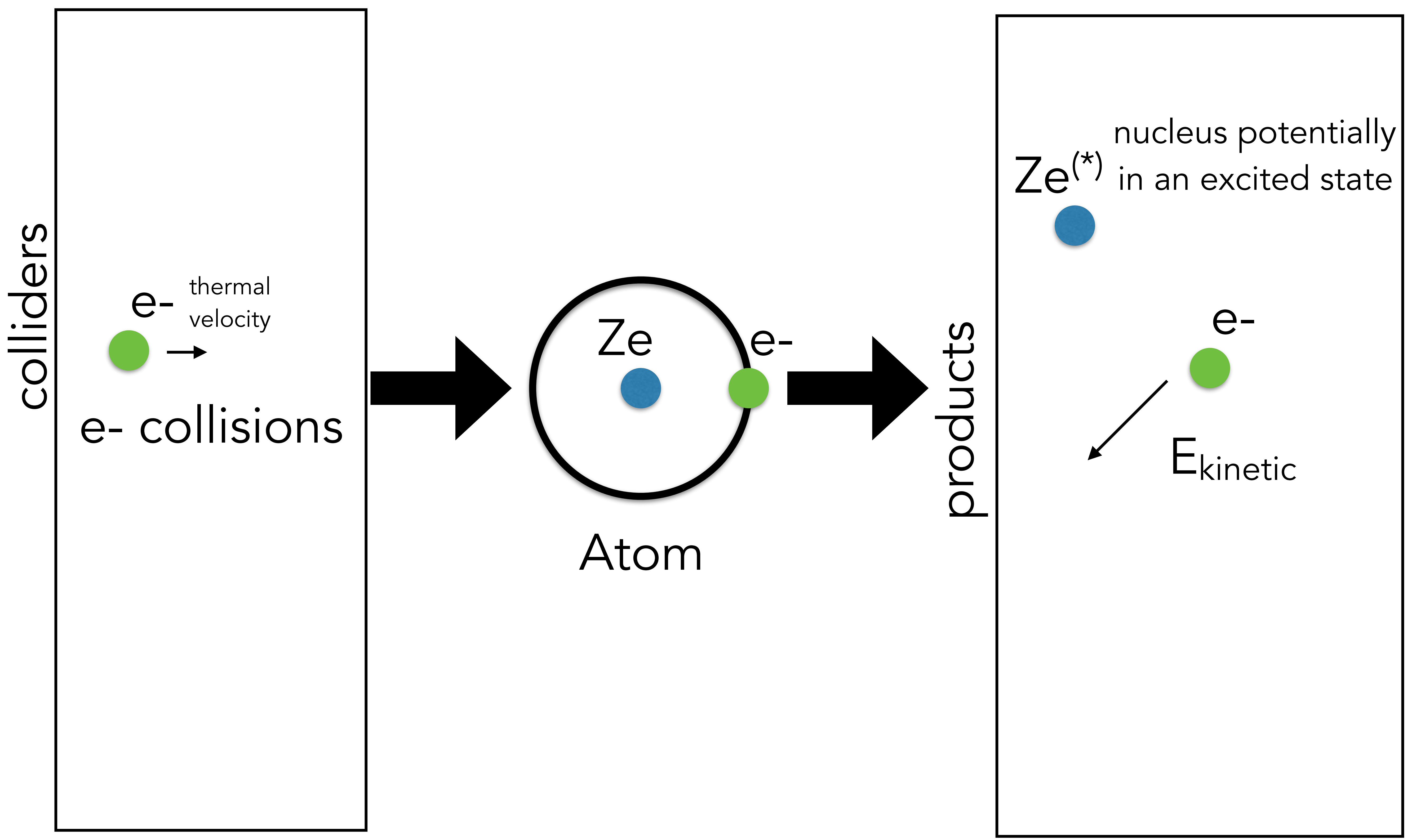
Energy of ejected photoelectron

difference between photon energy and ionization potential

For x-ray ionization E_{pe} can be big!
May go on to ionize other atoms/ions in the gas.

Secondary ionization rate depends on E_{pe} and ionization state of the gas.

Part II: Ionization Processes



Collisional Ionization

$$\zeta_{\mathrm{ci}} = k_{\mathrm{ci}} n_e n_I$$

rate coefficient

collisional
ionization rate

$$k_{ci} = \int_I^\infty \sigma_{ci}(E)\ v\ f(E) dE$$

integral of cross section over
Maxwellian velocity distribution

Collisional Ionization

A pretty good estimate of collisional ionization cross sec when E>I:

$$\sigma_{ci}(E) = C\pi a_0^2(1 - I/E)$$

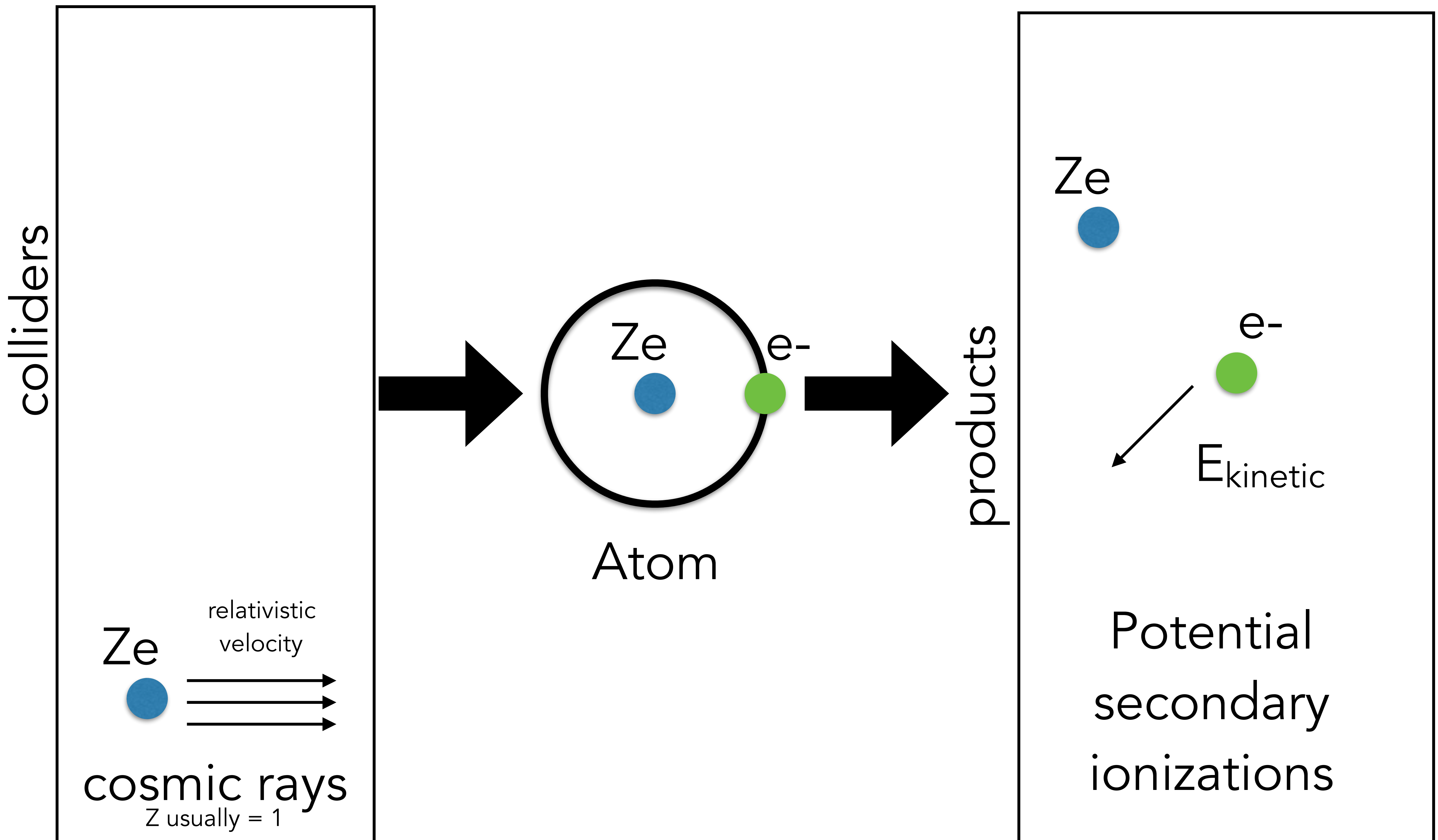
constant of order unity

Bohr radius cross section

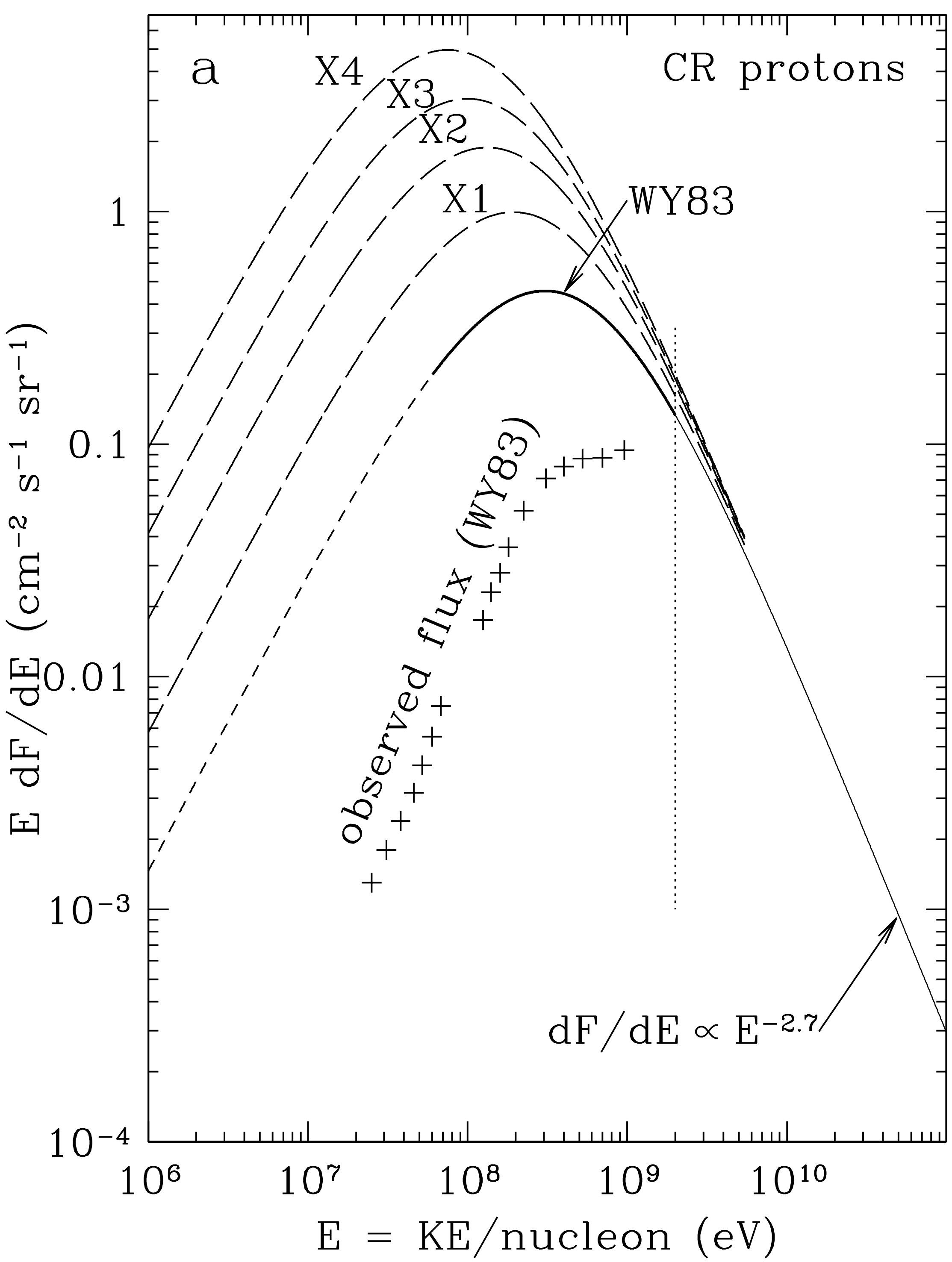
cross sec goes to zero when E=I

At higher E, cross section ~ 1/E
(can show this from the impact approx from Lecture 2)

Part II: Ionization Processes



Cosmic Ray Ionization



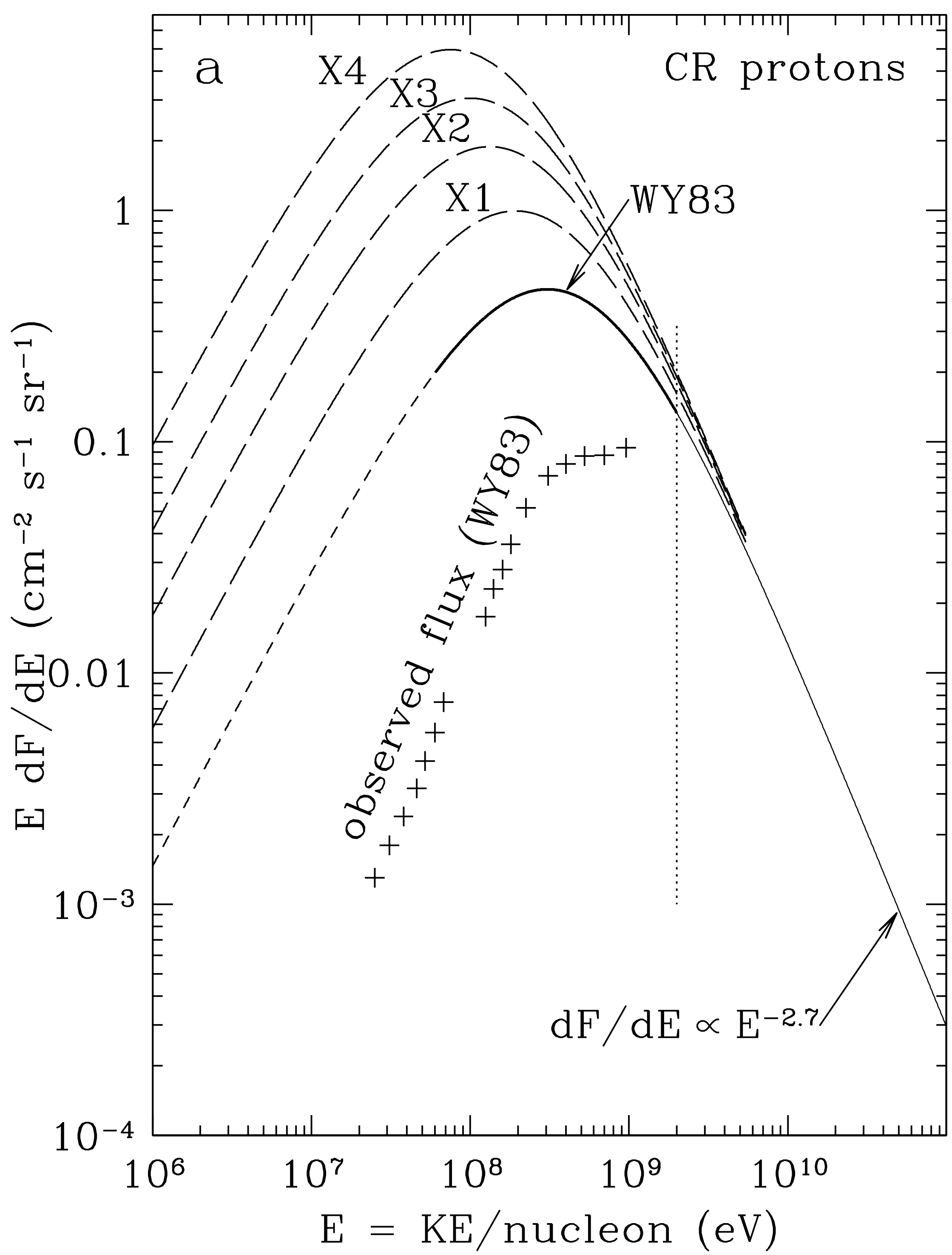
Cosmic ray energy flux is dominated by protons.

$$\zeta_{\rm CR} = 4\pi \int_{E_{\rm min}}^{\infty} \sigma_{\rm ci}(E) E \frac{dF}{dE} \cdot \frac{dE}{E}$$

Similar to before but velocity distribution is <u>not Maxwellian</u>

Big uncertainties in CR flux at low energies due to solar wind.

Cosmic Ray Ionization



CR ionization is very important in dense gas, where extinction by dust and other absorption has blocked most photons.

Will come back to this in discussing molecular clouds!

Part III: Recombination Processes

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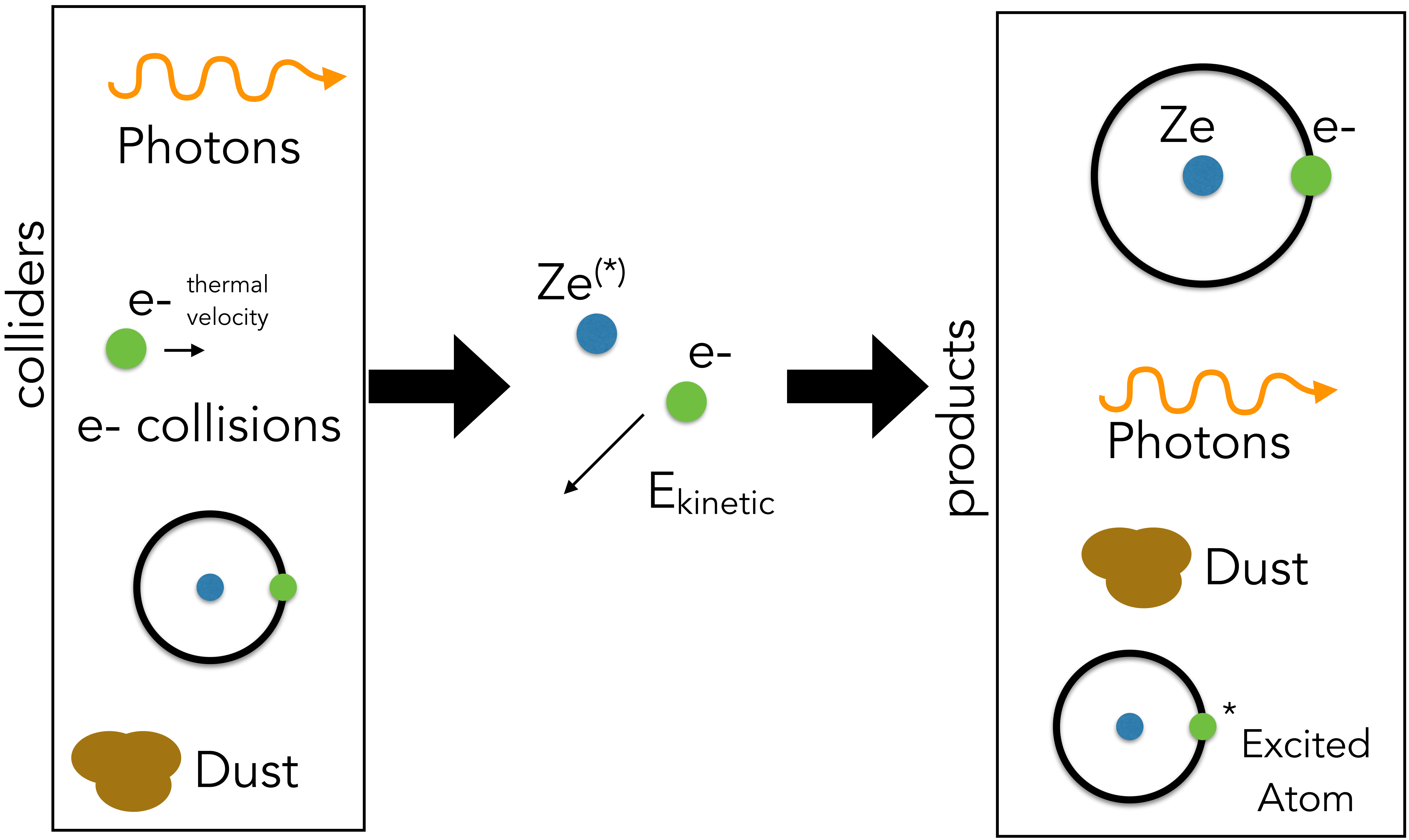
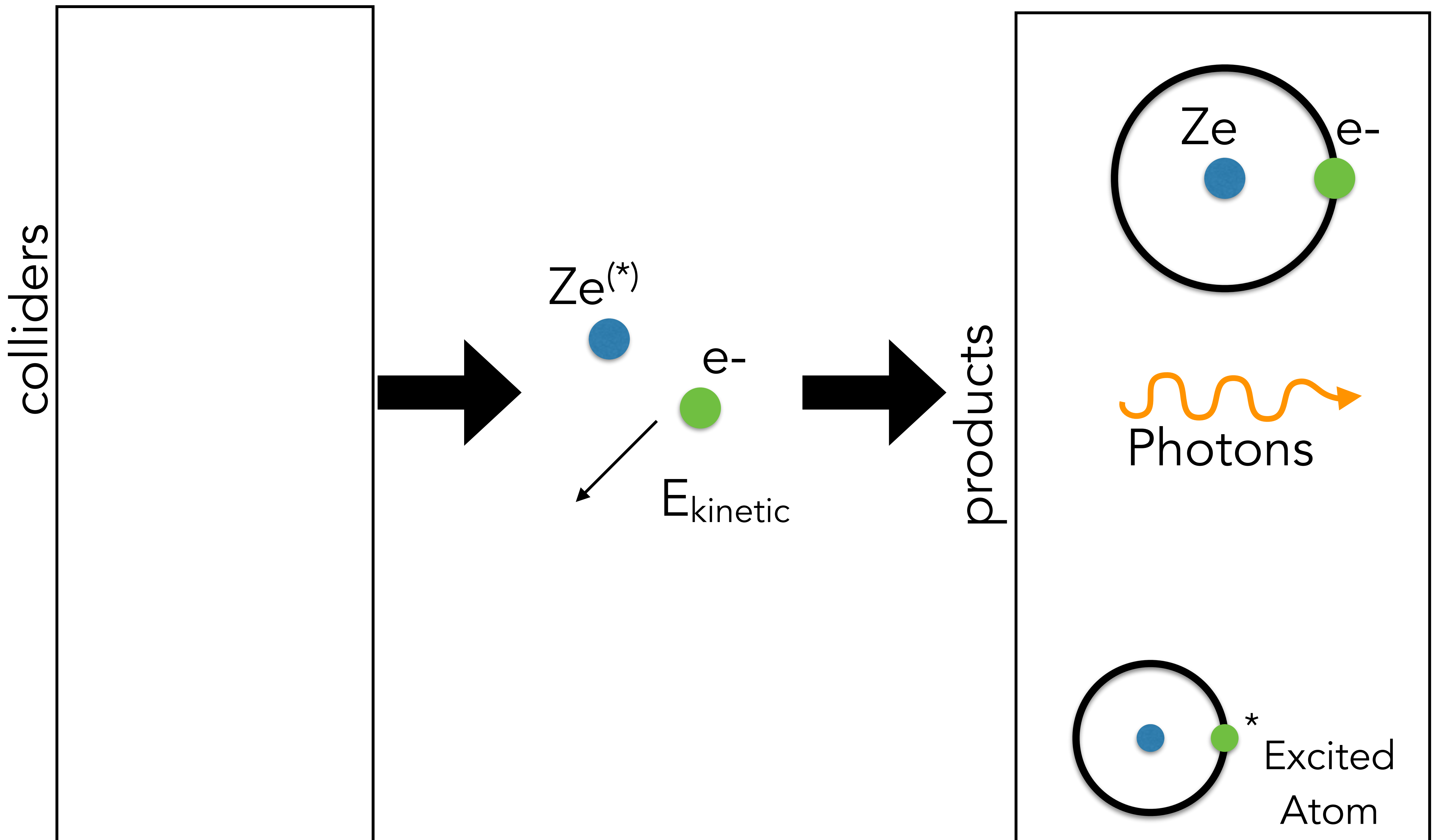
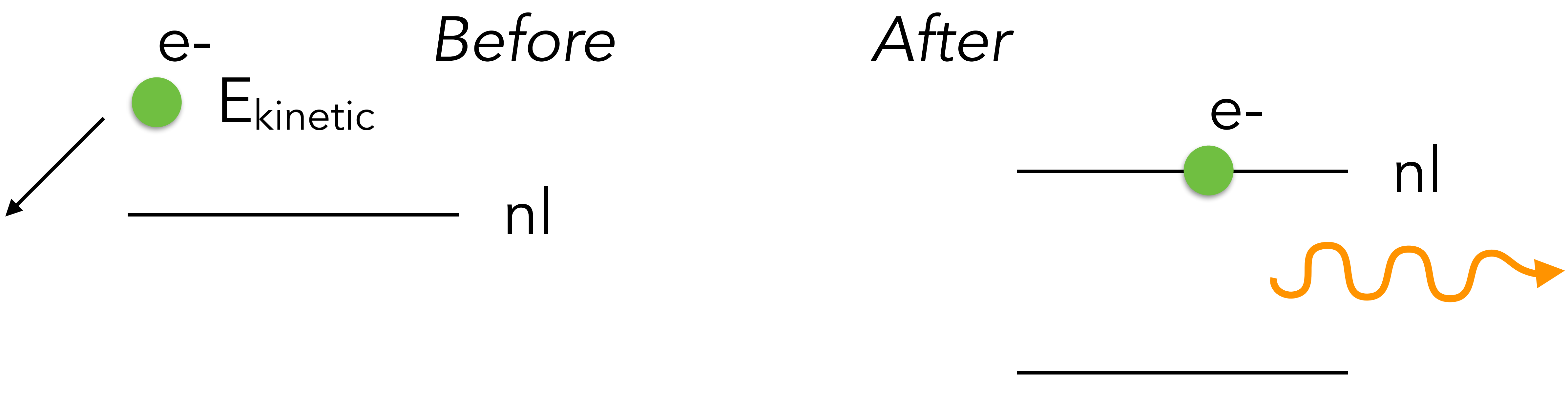


Part III: Recombination Processes



Radiative Recombination

$$X^+ + e^- \rightarrow X + h\nu,$$



$E_{photon} = I_{nl} + E_{kinetic}$

I_{nl} = ionization potential from nl

Radiative Recombination

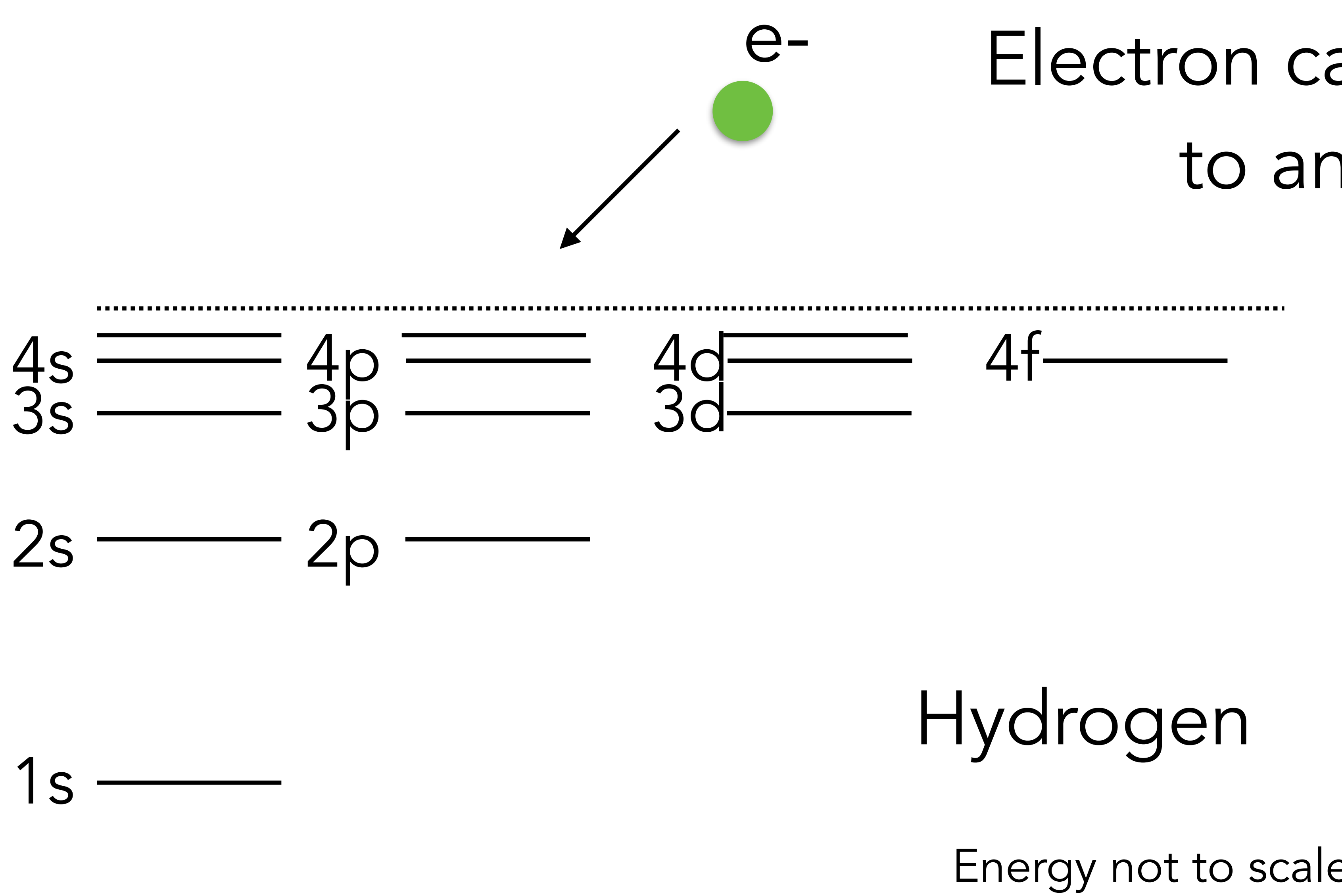
Given photoionization cross section from before, we can use detailed balance to work out radiative recombination cross section.

<u>Milne Relation:</u>

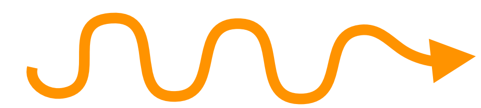
$$\sigma_{\rm rr}(E) = \frac{g_\ell}{g_u}\frac{(I_{X,u\ell}+E)^2}{E m_e c^2}\sigma_{\rm pi}(h\nu = I_{X,u\ell}+E)$$

Radiative Recombination

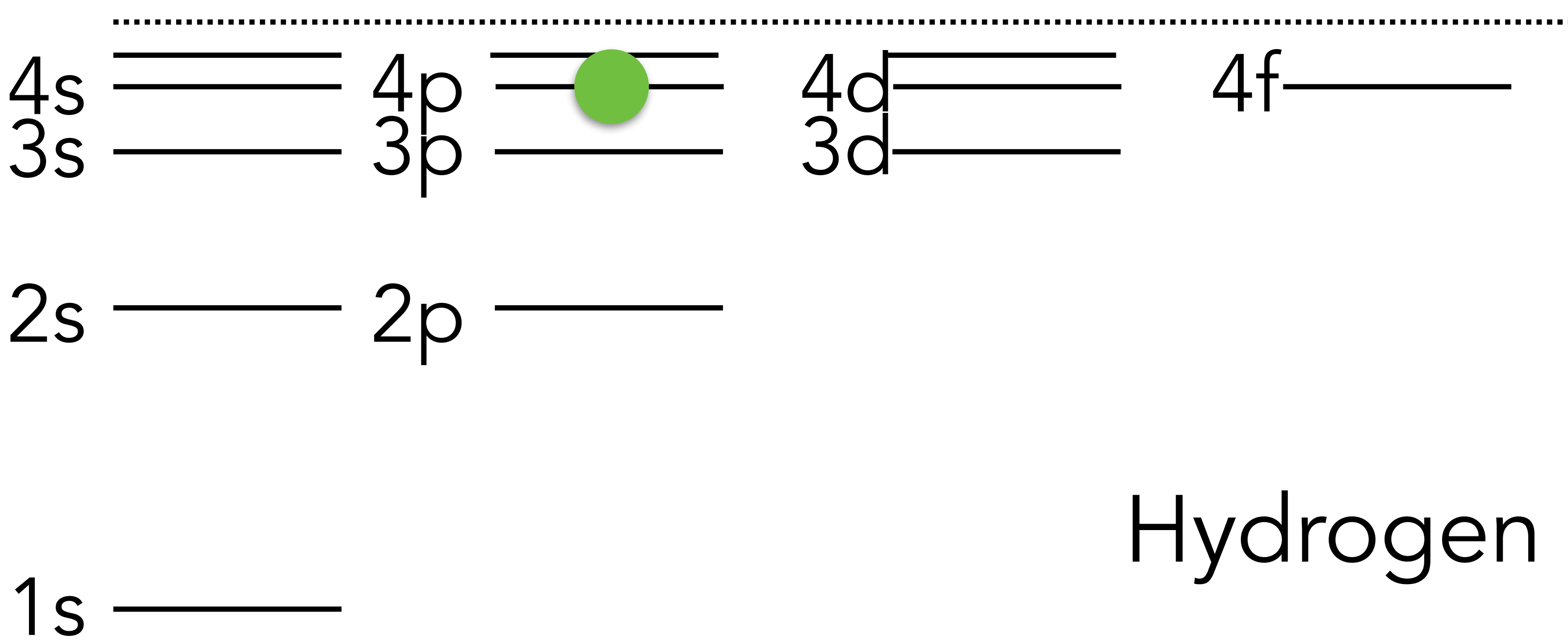
Electron can recombine to any level.



Radiative Recombination



$$E_{photon} = I_{nl} + E_{kinetic}$$



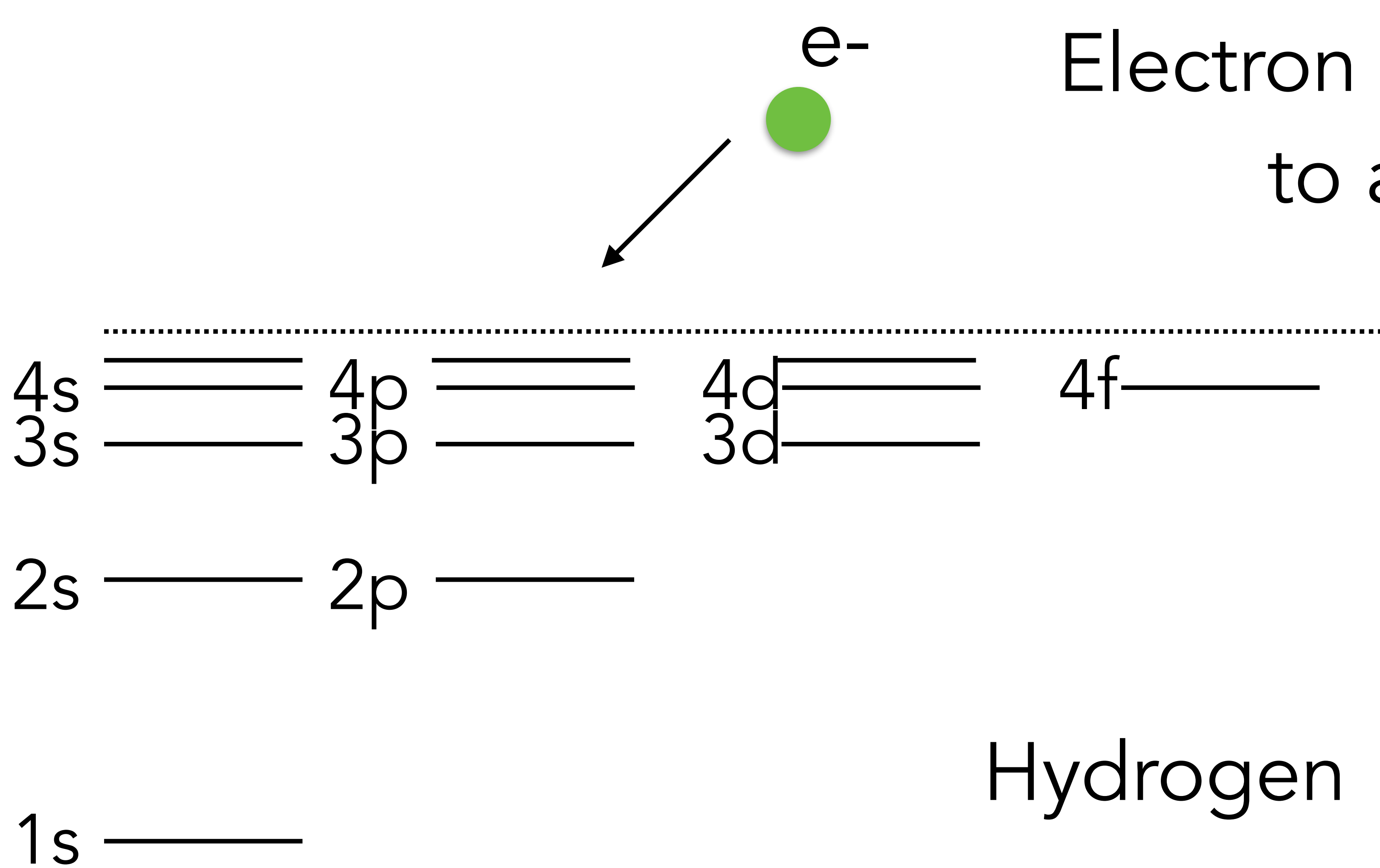
$$I_{nl} = 13.6 \text{ eV}/n^2$$

Hydrogen

Energy not to scale

Radiative Recombination

Electron can recombine to any level.

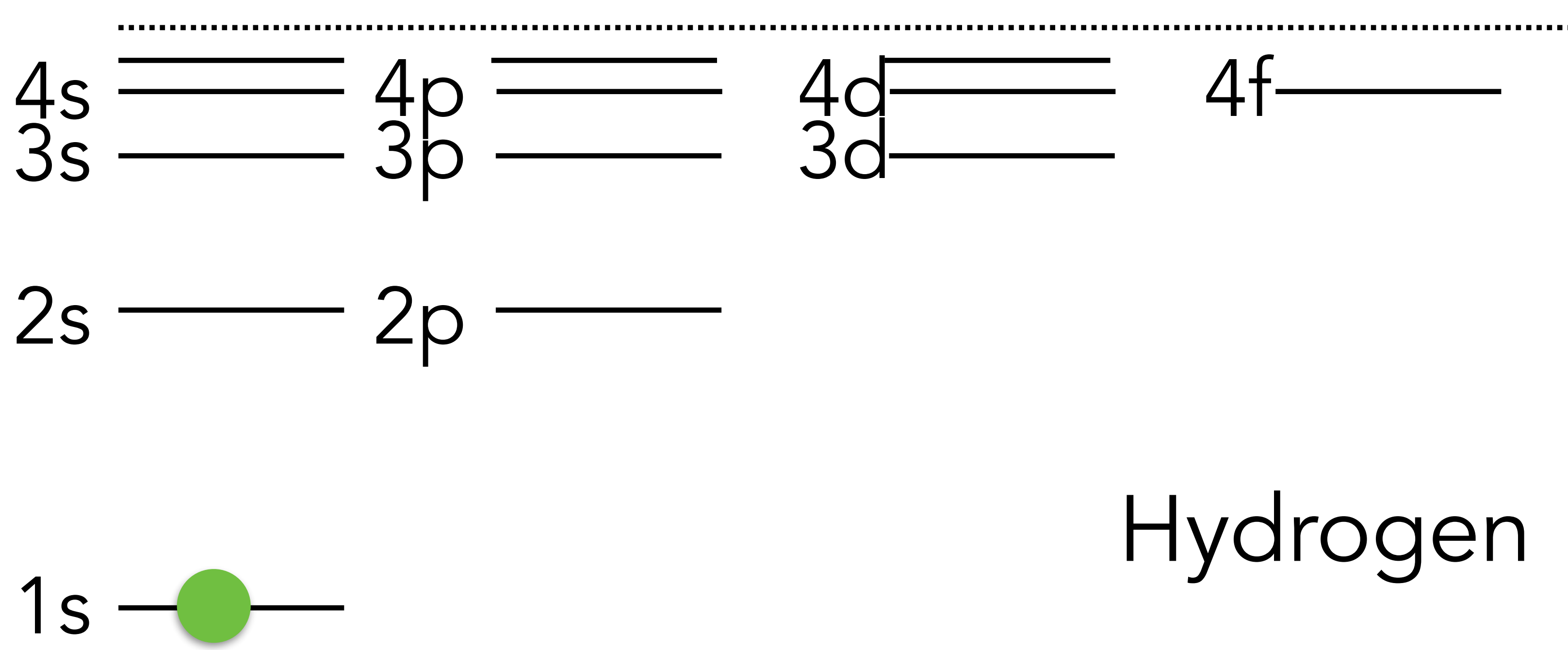


Hydrogen

Energy not to scale

Radiative Recombination

$E_{photon} = I_{nl} + E_{kinetic}$



$E_{photon} > 13.6$ eV

Hydrogen

Energy not to scale

Photon can ionize another H atom immediately if there is enough H around!

Radiative Recombination

"Case A": optically thin to ionizing radiation,
every ionizing photon from a recombination can escape

good approx for hot, collisionally ionized gas

"Case B": Optically thick to ionizing radiation,
recombinations to n=1 do not reduce ionization state of gas

good approx for "HII regions" =
photoionized nebulae around young, massive stars

Radiative Recombination

"Case A": optically thin to ionizing radiation, every ionizing photon from a recombination can escape

$$\alpha_A(T) = \sum_{n=1}^{\infty} \sum_{\ell=0}^{n-1} \alpha_{n\ell}(T)$$

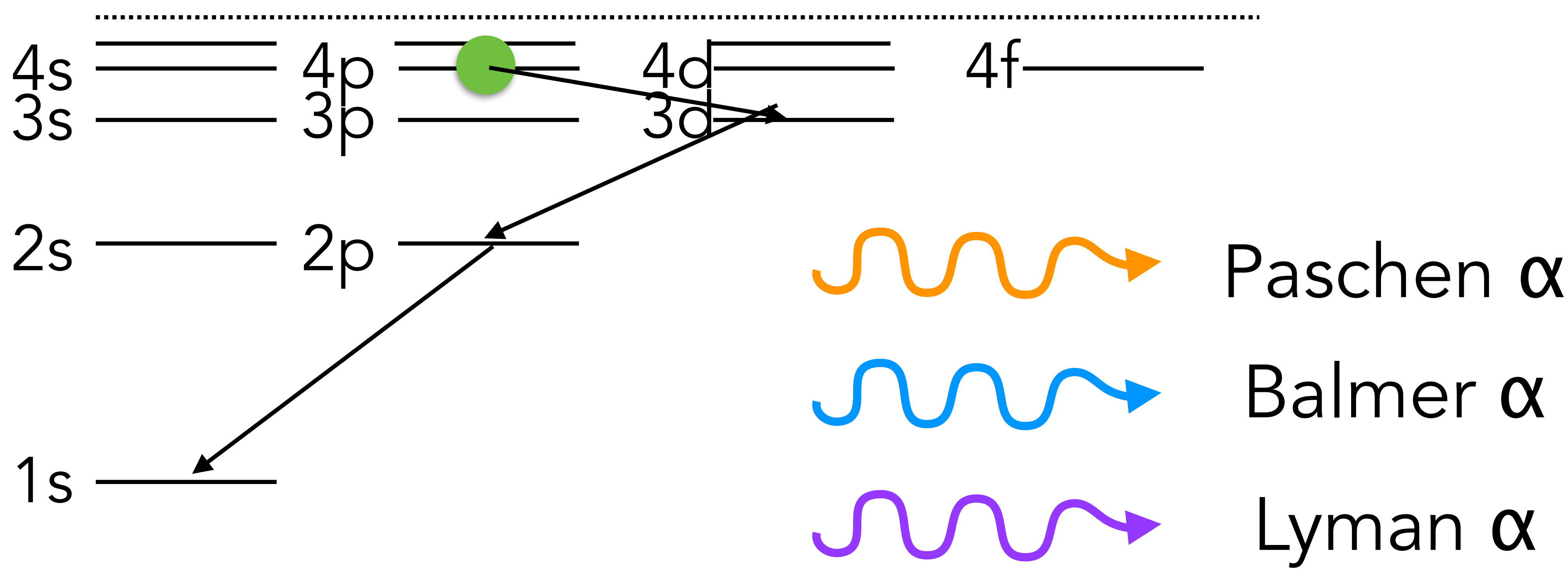
total recombination rate = sum of recombination rates to all levels

"Case B": Optically thick to ionizing radiation, recombinations to n=1 do not reduce ionization state of gas

$$\alpha_B(T) = \sum_{n=2}^{\infty} \sum_{\ell=0}^{n-1} \alpha_{n\ell}(T) = \alpha_A(T) - \alpha_{1s}(T)$$

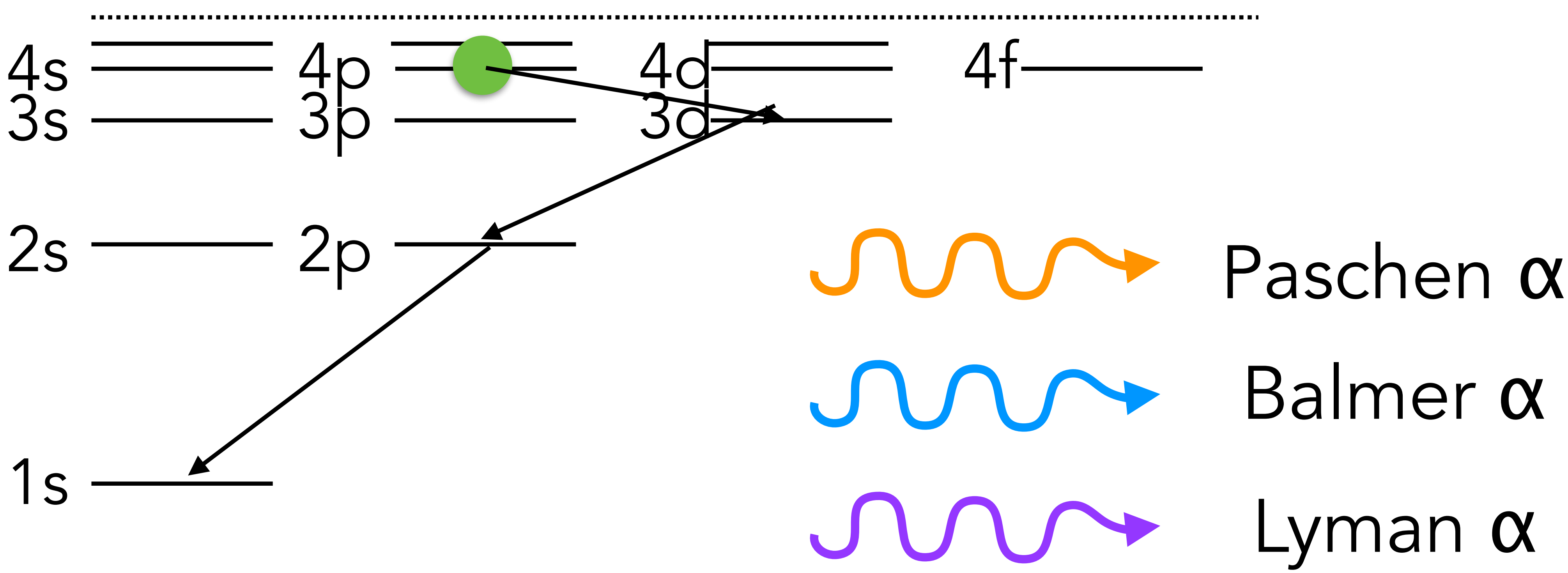
same but 1s rate is omitted

Radiative Recombination



For all but the highest n levels, collisions are much slower than radiative transitions -> recombination produces a characteristic spectrum of Hydrogen emission lines.

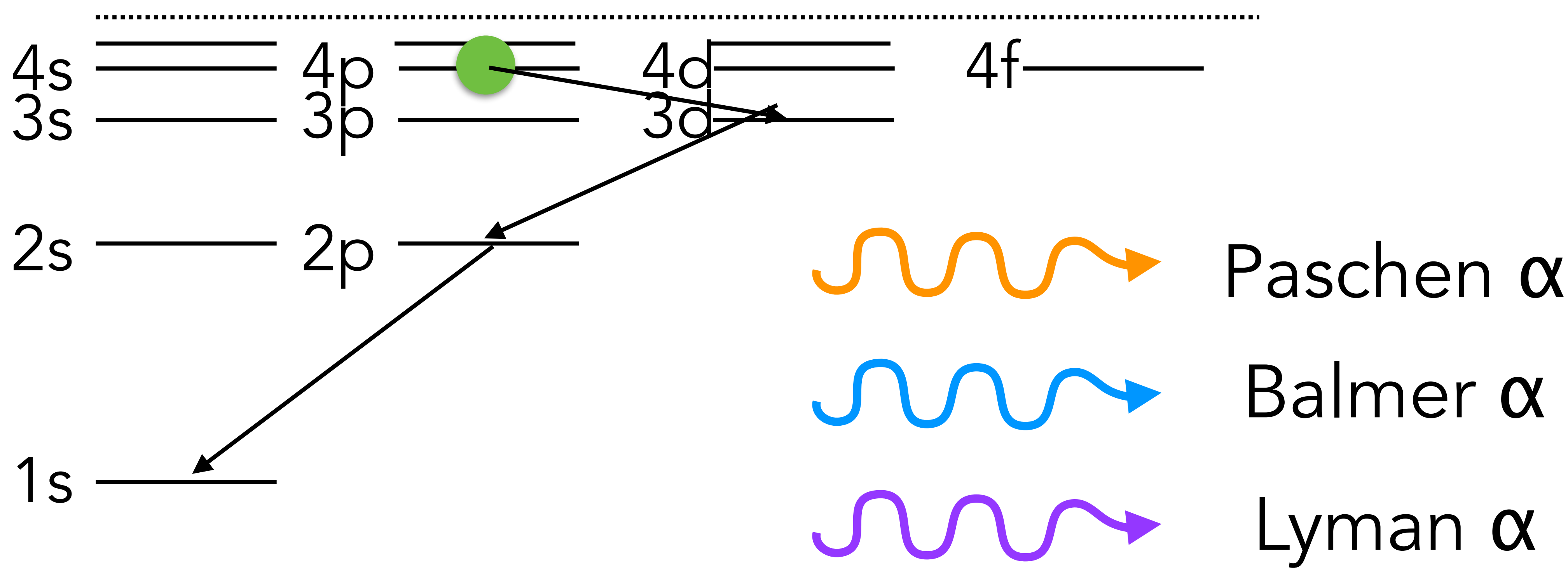
Radiative Recombination



allowed radiative decays for: $n>n'$ and l-$l'=\pm1$

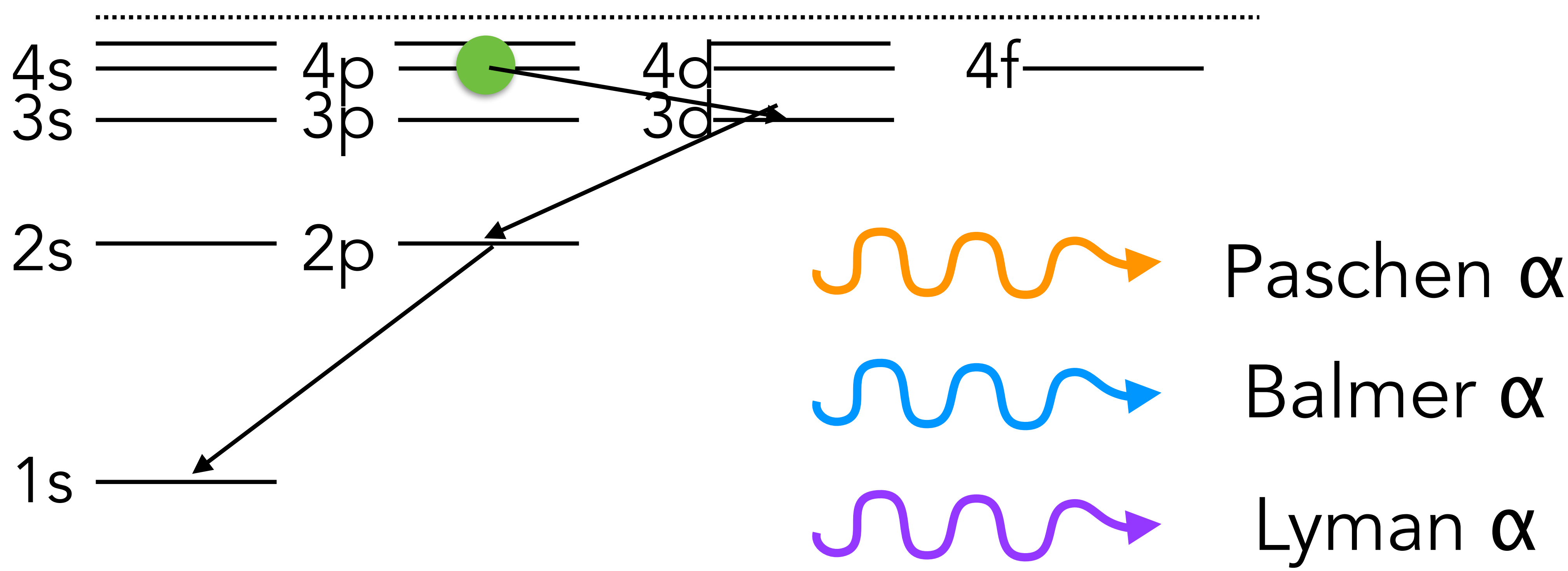
Einstein A coefficients + selection rules -> “branching ratios”

Radiative Recombination



For Case A this is straightforward.

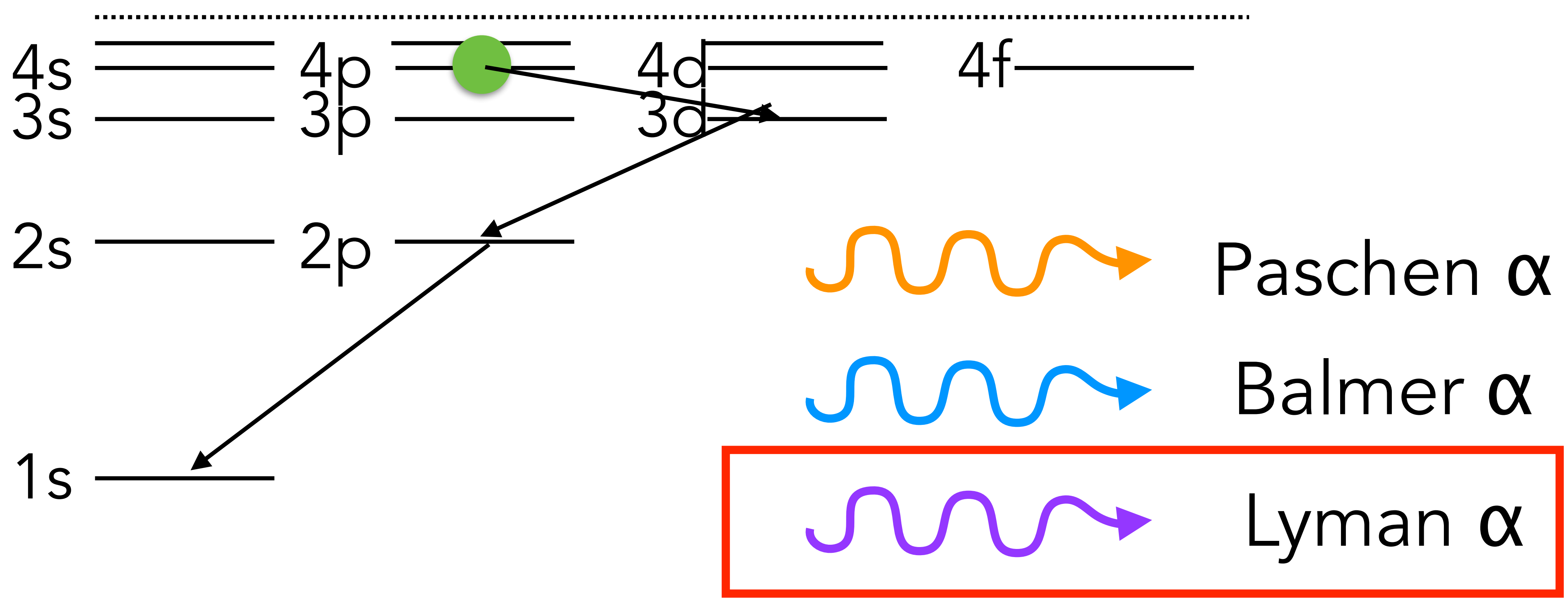
Radiative Recombination



For Case B, need to recognize that cross section for Lyman transitions is big, bigger than even photoionization cross section.

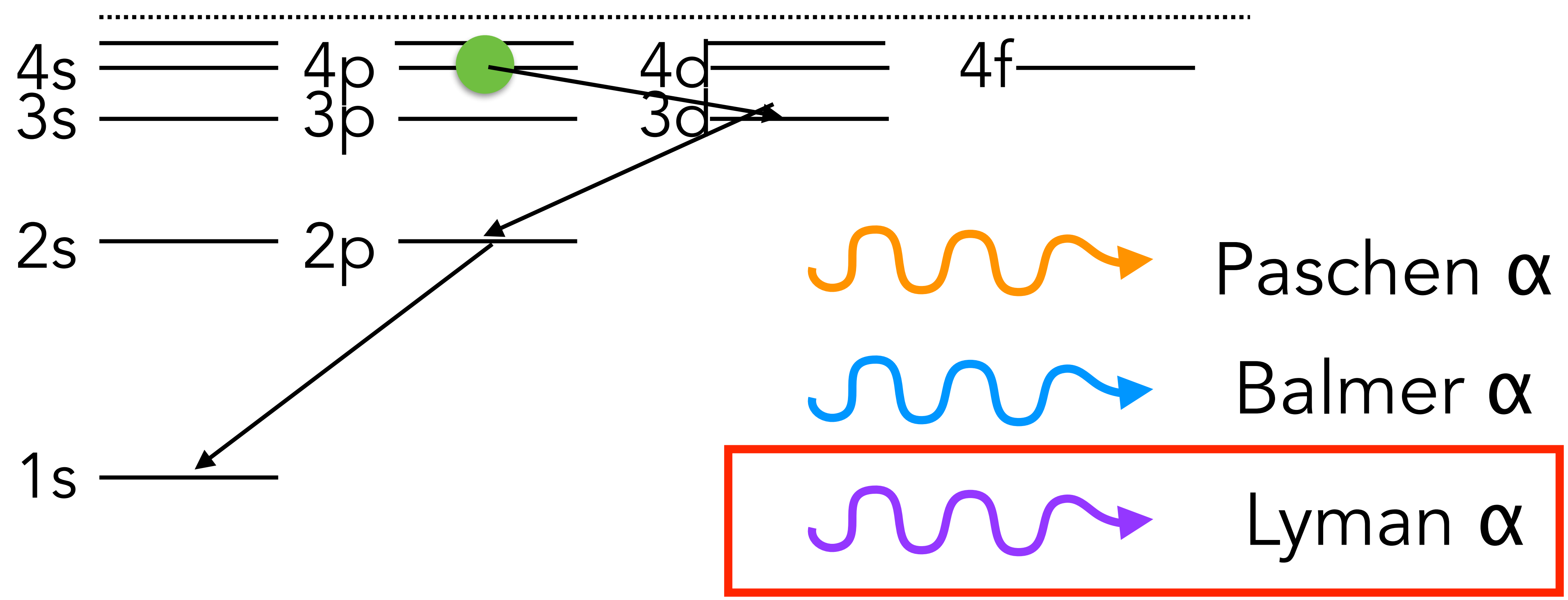
for example: $$\tau_{\mathrm{Ly}\alpha} = 8.0 \times 10^4 \left(\frac{15 \text{ km s}^{-1}}{b} \right) \tau_{\mathrm{LyC}}$$

Radiative Recombination



Lyman photons will be absorbed immediately.
"resonantly scattered" with small changes in freq
until a non-Lyman transition occurs

Radiative Recombination



Case B: rates for Lyman transitions -> 0
distributed instead among other transitions

Other Recombination Processes

- Dielectronic: capture of incoming electron excites one of the other bound electrons -> 2 excited e-
- Dissociative: molecular ion captures e-, dissociates
- Charge exchange: one important reaction is O^+ + H <-> O + H^+
- Neutralization by dust grains