Ionized Bubbles in the ISM

The Physical State of Interstellar Hydrogen

Authored by: Bengt Strömgren

Presentation by: Nathan Butcher

Motivation

- In 1934, a paper by Arthur Eddington about the density of metals states the hydrogen in the ISM almost entirely neutral (even molecular)
- Only very close to a star will hydrogen get ionized.
- The photons with $E \geq 13.6$ eV will be absorbed and “transmuted” to lower energies.
- Bengt Strömgren performed a theoretical analysis of the ionized region

Saha Equation

- Equlibrium ratio of ionized and neutral hydrogen can be calculated with the Saha equation (Draine eq. 3.16)

$$\frac{N''N_e}{N'} = \frac{(2\pi m_e)^{3/2}}{h^3} \frac{2q''}{q'} (kT)^{3/2} e^{-I/kT} \cdot \sqrt{\frac{T_{\mathrm{el}}}{T}} \cdot w\, e^{-\tau_u}$$

$$w = \frac{R^2}{4s^2}$$

Is called the dilution factor

Eq. From Stromgren, 1939

Assumptions

- Assume all electrons come from hydrogen (N_e = N’’). Greatly simplifies the problem. Validity will be discussed later.
- Assume the absorption coefficient per hydrogen atom (a_u) to be independent of wavelength. This is valid because the cross section approximately drops off as ν^{-3} (Draine 2010, eq. 13.3) so only a small portion of the spectrum will cause ionizations.

Eq. from Stromgren, 1939

Degree of Ionization

- Introduce the degree of ionization, x, as a new variable. This allows for quantities to be rewritten

$$\left.\begin{aligned} N &= N' + N'' , \\ N'' &= xN , \\ N' &= (1 - x)N , \\ N_e &= xN , \end{aligned}\right\}$$

Eq. From Stromgren, 1939

Rewriting the Saha Equation

- The Saha equation can now be rewritten as

$$\frac{x^2}{1 - x} N = C_1 \cdot \frac{1}{s^2} \cdot e^{-\tau u}$$

$$C_1 = 10^{-0.51 - \theta I} \cdot \frac{2q''}{q'} \sqrt{\frac{T_{\text{el}}}{T}}\, T^{3/2} \cdot R^2$$

$$\theta = \frac{5040^\circ}{T}$$

Eq. From Stromgren, 1939

Method

$$d\tau_u = (1 - x) N a_u \cdot 3.08 \cdot 10^{18} ds$$

Solve for (1-x) and plug into the rewritten Saha equation on the previous slide to get

$$e^{-\tau_u} d\tau_u = \frac{N^2}{C_1} x^2 s^2 \cdot 3.08 \cdot 10^{18} a_u ds$$

Eq. From Stromgren, 1939

Method

$$y = e^{-\tau_u} \qquad (1 \geqq y > 0)$$

$$dz = \frac{N^2}{C_1} \cdot 3.08 \cdot 10^{18} a_u \cdot s^2 ds$$

Set z = 0 for s = 0 to write

$$s = \left(\frac{3C_1}{N^2 \cdot 3.08 \cdot 10^{18} a_u} \right)^{1/3} \cdot z^{1/3}$$

Note that this implies y = 1 when z = 0.

Eq. From Stromgren, 1939

Method

$$\frac{dy}{dz} = -x^2$$

$$\frac{1 - x}{x^2} = a \frac{1}{y} z^{2/3}$$

$$a = \left(\frac{9}{NC_1 \cdot (3.08 \cdot 10^{18} a_u)^2} \right)^{1/3}$$

When z is small, if a is small and y is not much less than 1 then x is nearly 1.

As z goes to 1, y becomes small compared to 1. This causes x to quickly go to zero.

Eq. From Stromgren, 1939

Strömgren Sphere

- There is a radius, s_0, within which hydrogen is almost fully ionized. Outside of s_0, hydrogen is almost fully neutral. This sphere of ionized hydrogen is not called a "Strömgren sphere."
- As soon as neutral hydrogen is present, the distance ionizing radiation can travel decreases rapidly, which leads to the sharp boundary.

$$s_0 = \left(\frac{3C_1}{N^2 \cdot 3.08 \cdot 10^{18} a_u} \right)^{1/3}$$

Eq. From Stromgren, 1939

Electron Number Density

- If the HII region has a high density of helium, it will only be ionized in an even smaller radius than the radius of hydrogen. Close to the edge of the HII region, helium is neutral.
- If the HII region has a low density of helium, the ionized region will be of a similar size to that of hydrogen. However, the contribution of electrons by helium is small due to the low relative density

Dependence on a

x^2	s/s_0		
	$a=0.001$	$a=0.01$	$a=0.1$
1.0	0.000	0.000	0.00
0.8	1.000	0.988	0.82
0.6	1.002	1.009	0.97
0.4	1.003	1.020	1.05
0.2	1.004	1.028	1.12

The quantity (as_0) is independent of C_1, so the width of the transition does not depend on any characteristics of the star, just on a.

Table and Eq. From Stromgren, 1939

s0 for Different Stars

Sp.	T	M_{vis}	s_0
O5	79,000°	$-4^m.2$	140 parsecs $\times N^{-2/3}$
O6	63,000	-4.1	110
O7	50,000	-4.0	87
O8	40,000	-3.9	66
O9	32,000	-3.6	46
B0	25,000	-3.1	26
B1	23,000	-2.5	17
B2	20,000	-1.8	11
B3	18,600	-1.2	7.2
B4	17,000	-1.0	5.2
B5	15,500	-0.8	3.7
A0	10,700	$+0.9$	0.5

Table From Stromgren, 1939

Recent Work

- Problems with analytic solutions make excellent tests for numerical codes.
- The radius of the Strömgren sphere generated by a single frequency source has an analytic solution.
- Numerical radiative transfer methods should be able to reproduce the analytic result accurately. This was tested in Iliev, et al. (2006)

Code Test

- Hydrogen only, kept a fixed temperature of 10^4 K.
- Density of 10^{-3} cm^{-3}.
- Source emits only 13.6 eV photons at a rate of
  5×10^{48} s^{-1}.
- Case B recombination coefficient used.

Analytic Expressions

The position of the ionization front at time t is given by the expression

$$r_I = r_s(1 - e^{-t/t_{rec}})^{1/3}$$

r_s is the Strömgren radius

$$r_s = \left(\frac{3\dot{N}_\gamma}{4\pi\alpha_\beta(T)n_H^2} \right)^{1/3}$$

t_{rec} is the recombination time (around 122.4 Myr)

$$t_{rec} = (\alpha_\beta n_H)^{-1}$$

Eq. from Iliev, et al. 2006

Results of Recent Work

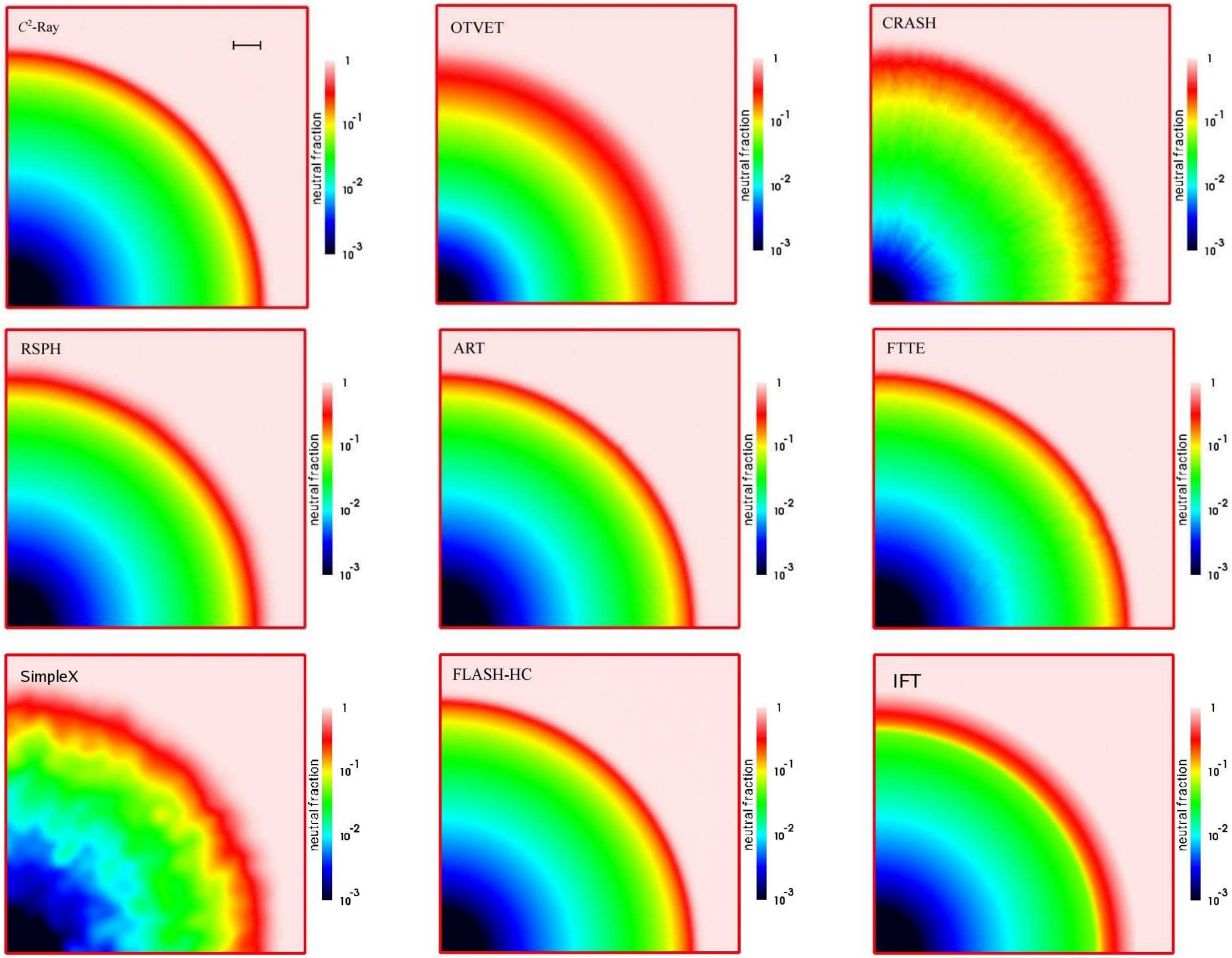

Figure from Iliev, et al. 2006

Results of Recent Work

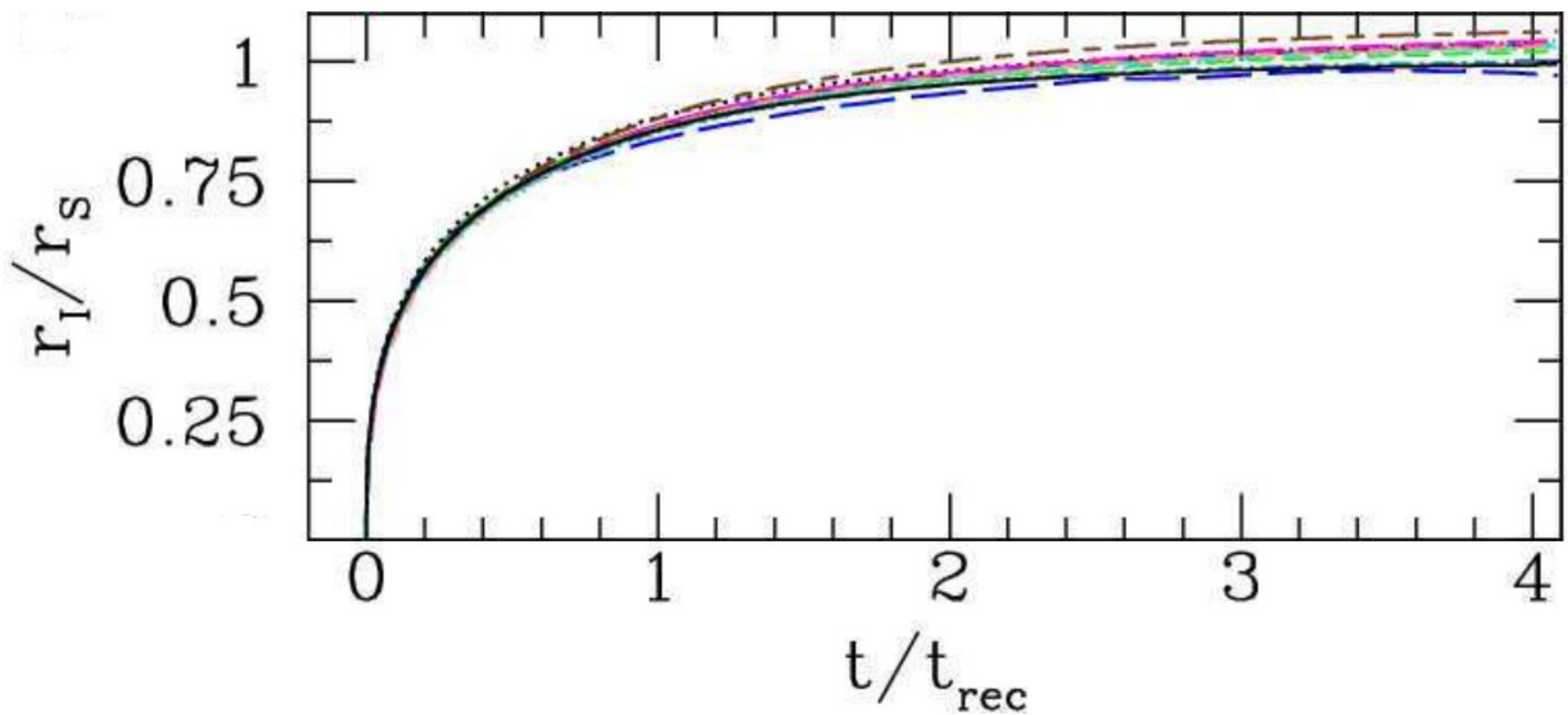

Figure from Iliev, et al. 2006

Citations

- Strömgren, Bengt. "The Physical State of Interstellar Hydrogen." The Astrophysical Journal 89 (1939): 526.
- Eddington, Arthur Stanley. "The density of interstellar calcium and sodium." Monthly Notices of the Royal Astronomical Society 95 (1934): 2.
- Iliev, Ilian T., et al. "Cosmological radiative transfer codes comparison project–I. The static density field tests." Monthly Notices of the Royal Astronomical Society 371.3 (2006): 1057-1086.
- Draine, Bruce T. Physics of the interstellar and intergalactic medium. Princeton University Press, 2010.