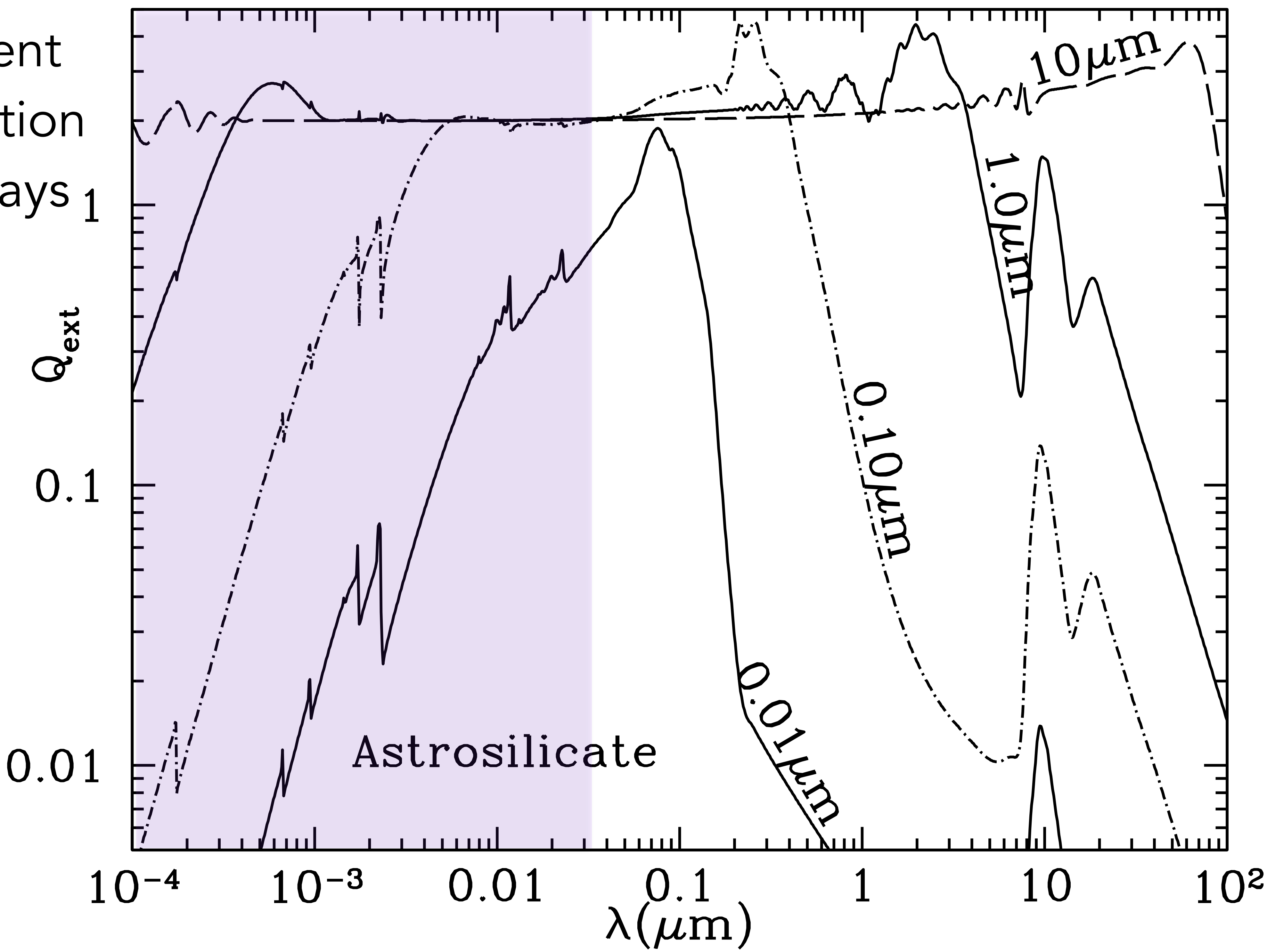
Physics 224
The Interstellar Medium

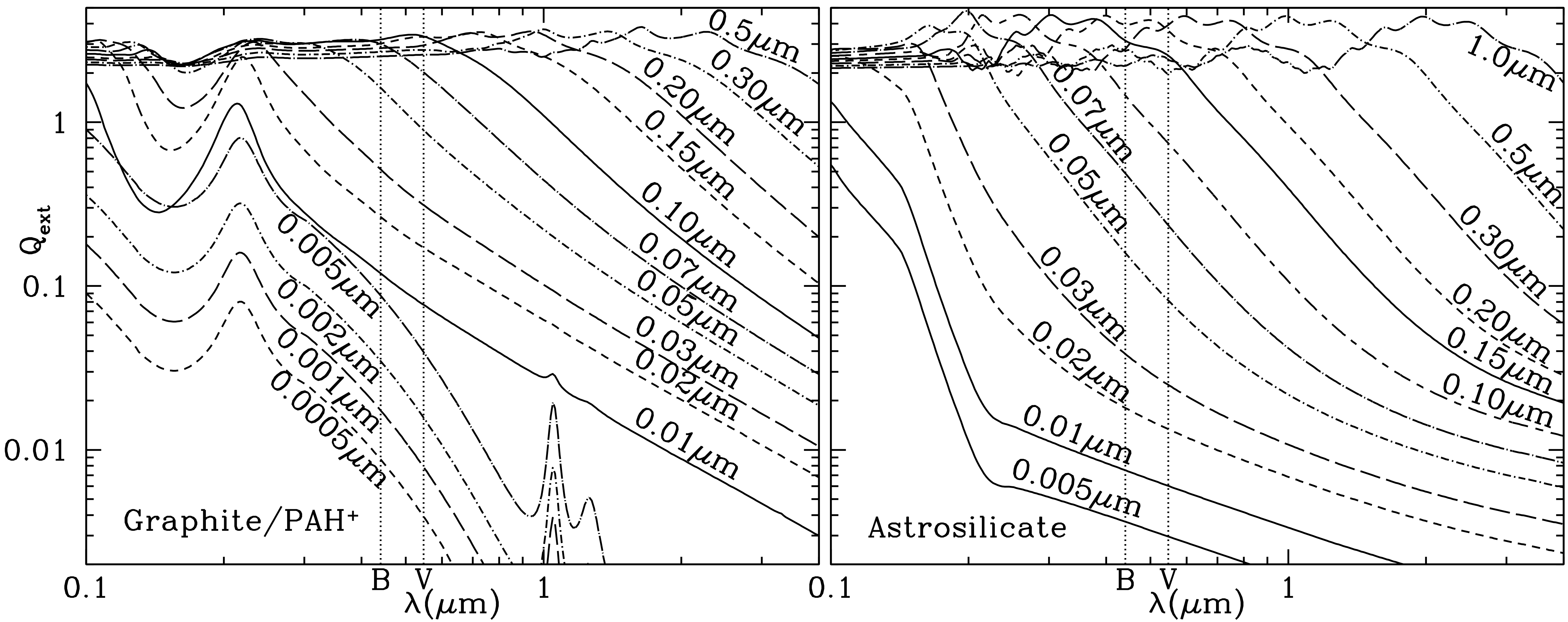
Lecture #12

Astronomical Dust

Need different calculation for x-rays

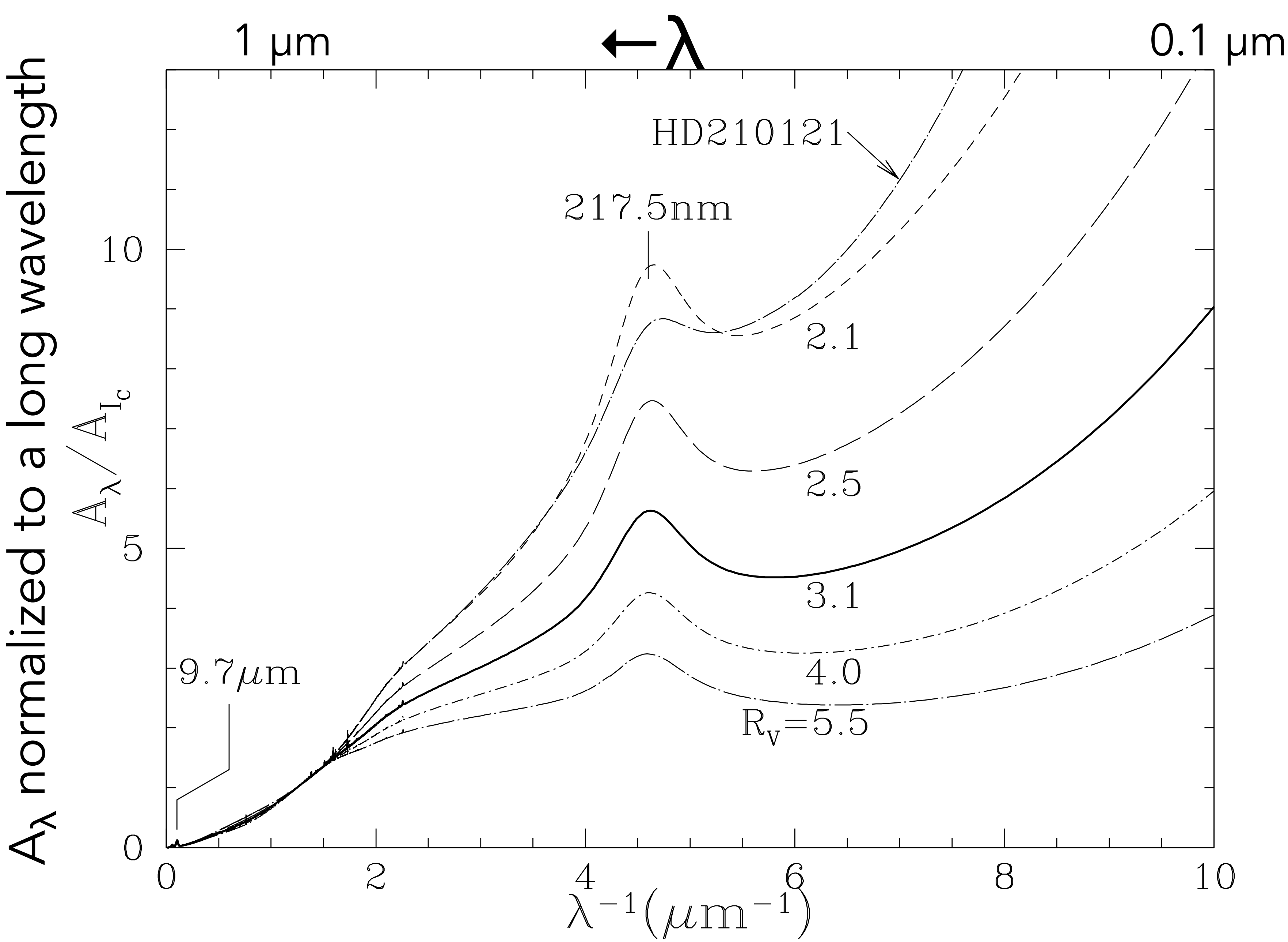


Astronomical Dust



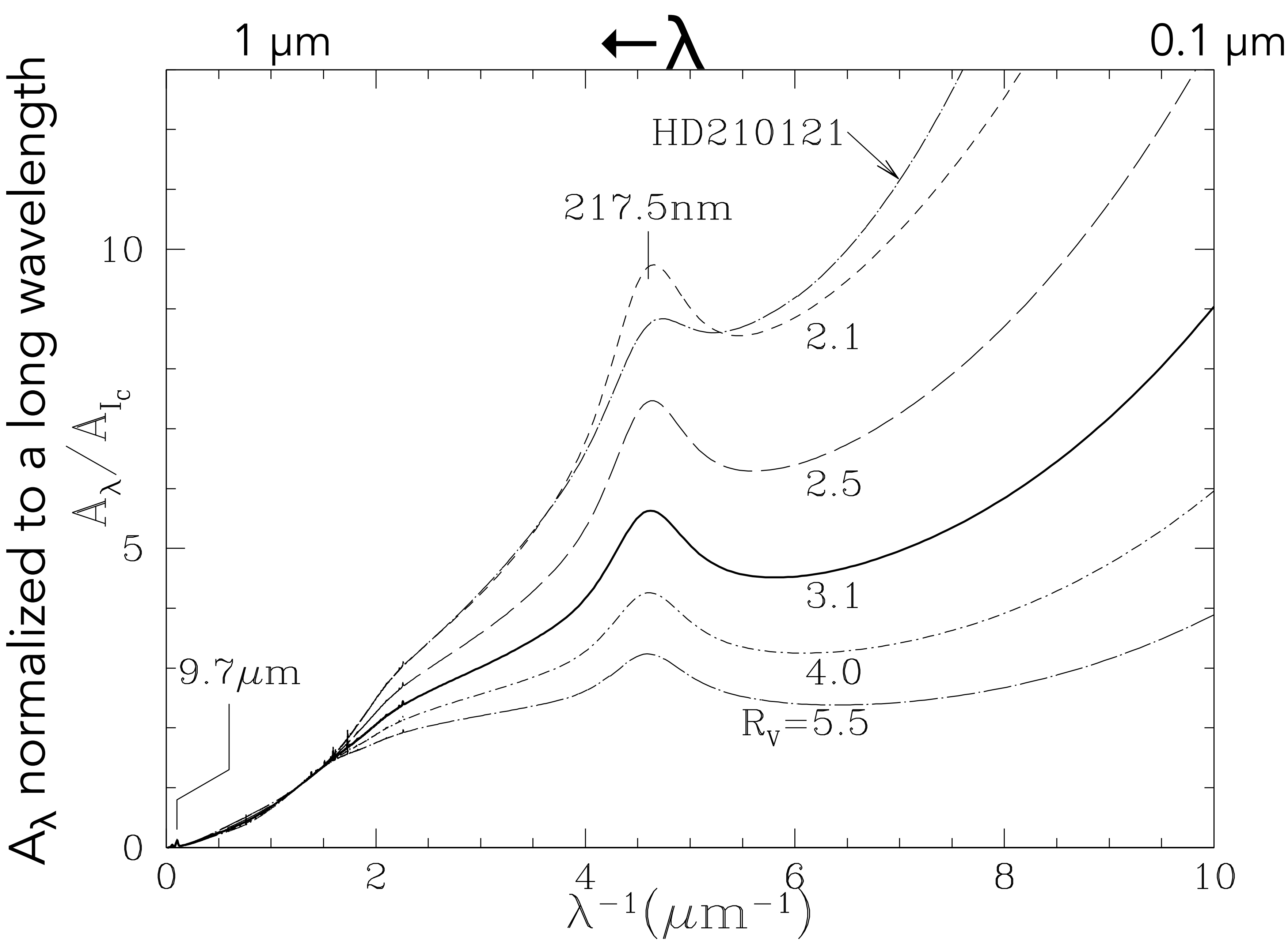
Q_{ext} for astronomical dust analogs

Extinction Curve



This does not look like the Q_{ext} plots from before - why?

Extinction Curve



This does not look like the Q_{ext} plots from before - why?

There is a range of grain sizes!

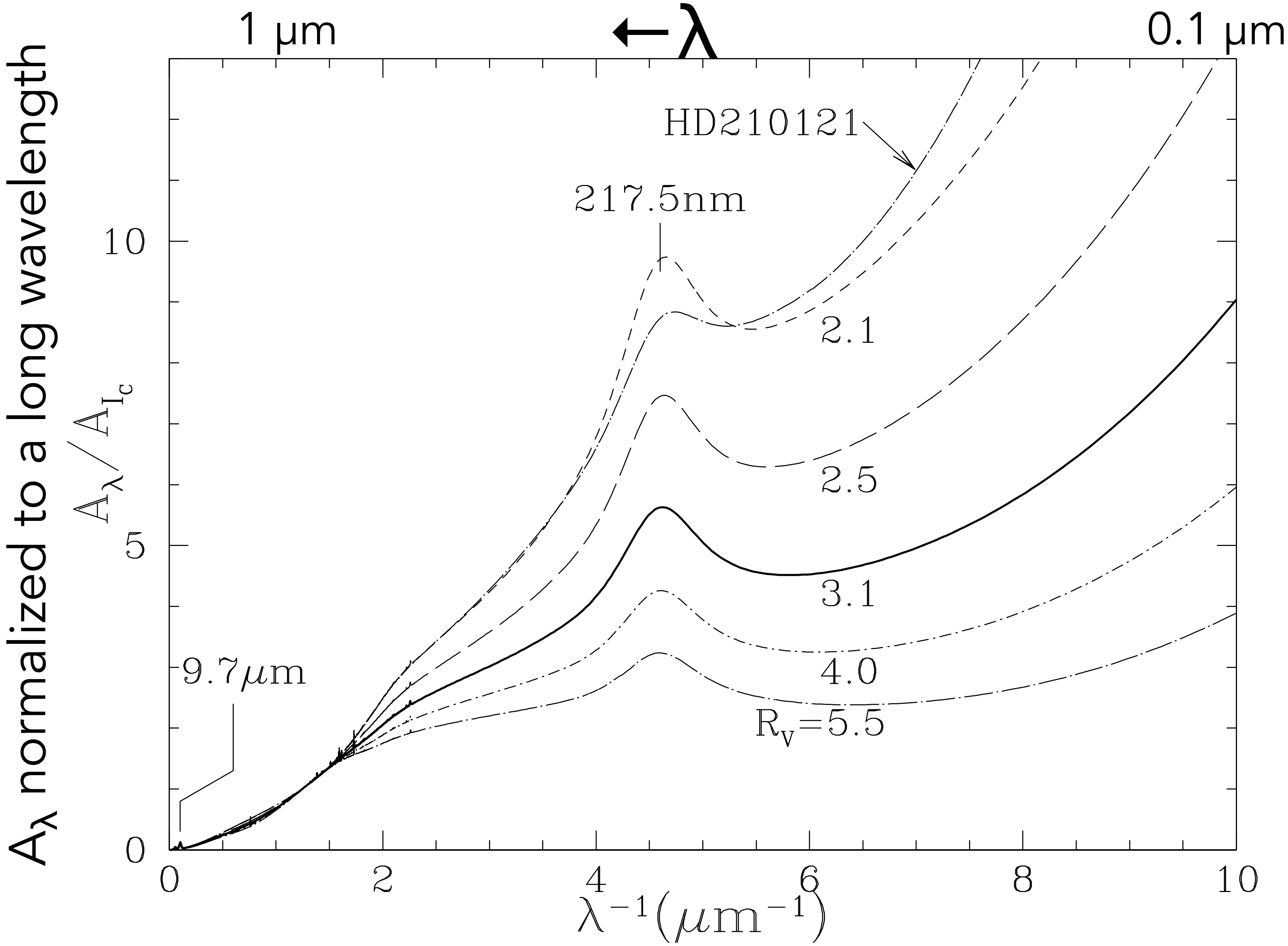
Extinction Curve

$$\frac{A_\lambda}{\mathrm{mag}} = 2.5 \log\ [e^{\tau_\lambda}] = 1.086 \tau_\lambda$$

For a given grain size:

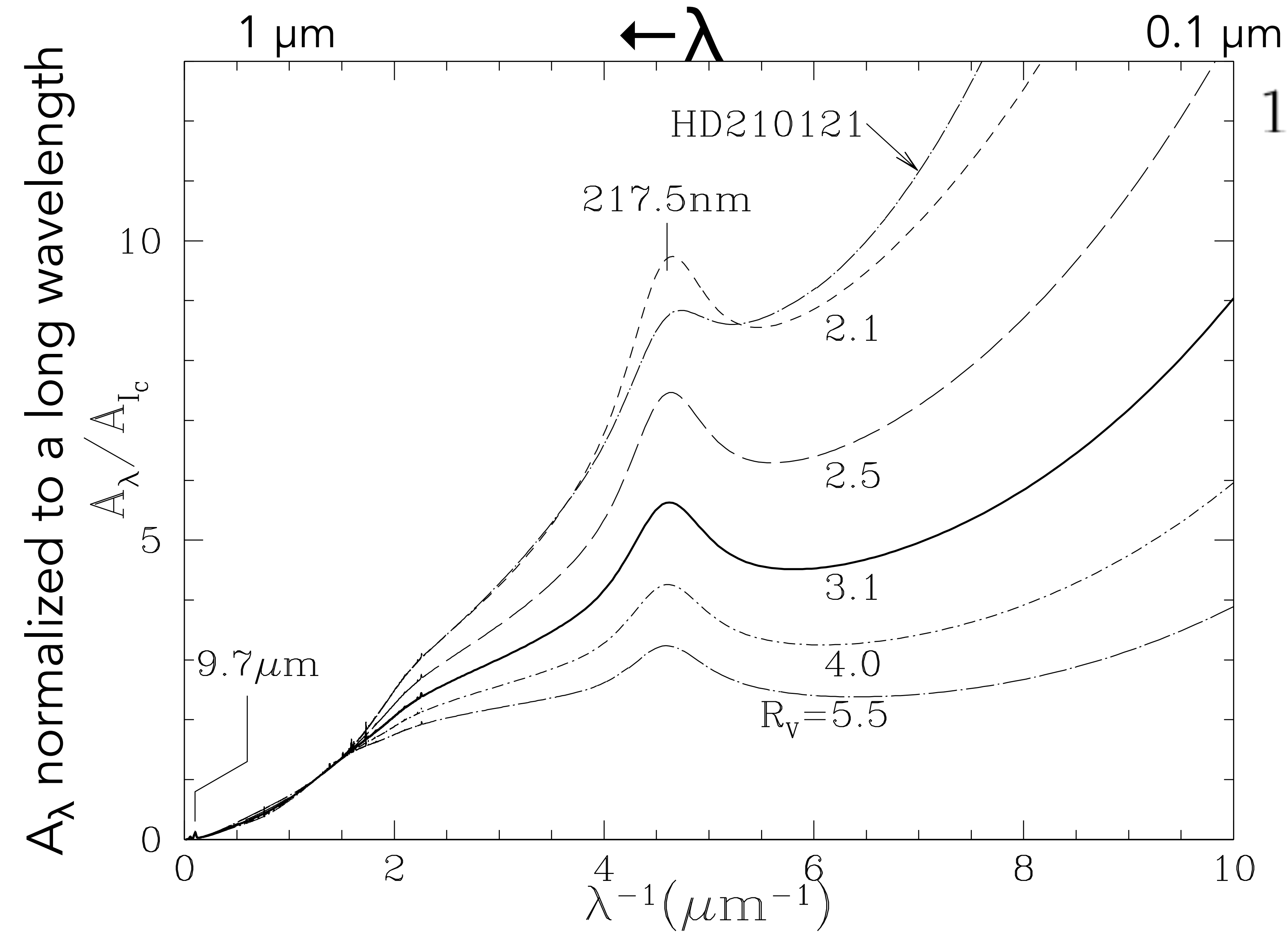
$$\tau_\nu(a) = N_d(a)\ Q_{ext}(a)\ \pi a^2$$

Extinction Curve



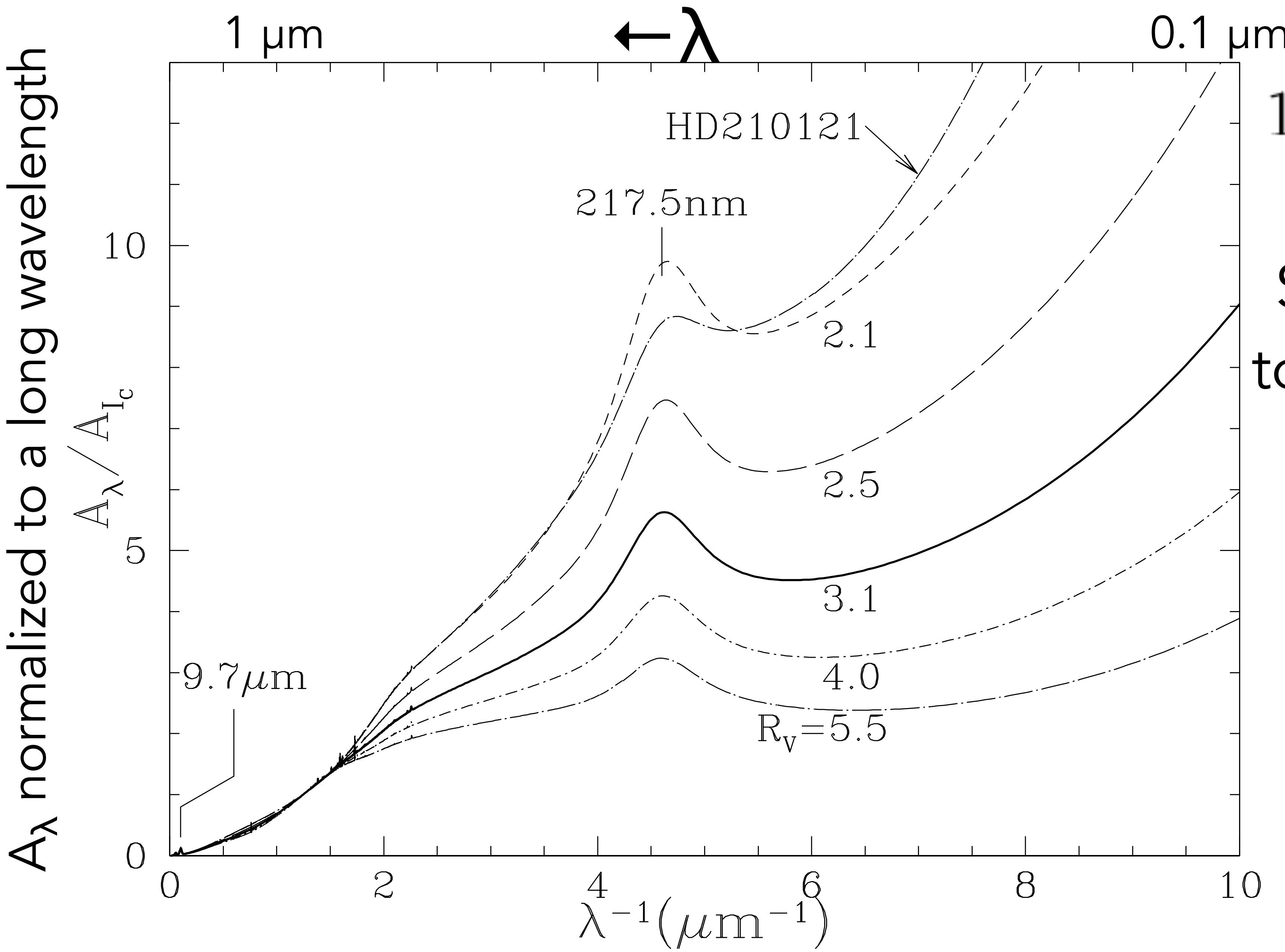
Continual rise to far-UV means there are more small grains than large grains.

Extinction Curve



$$A_\lambda(a) = 1.086\ N_d(a)\ Q_{ext}(a)\ \pi a^2$$

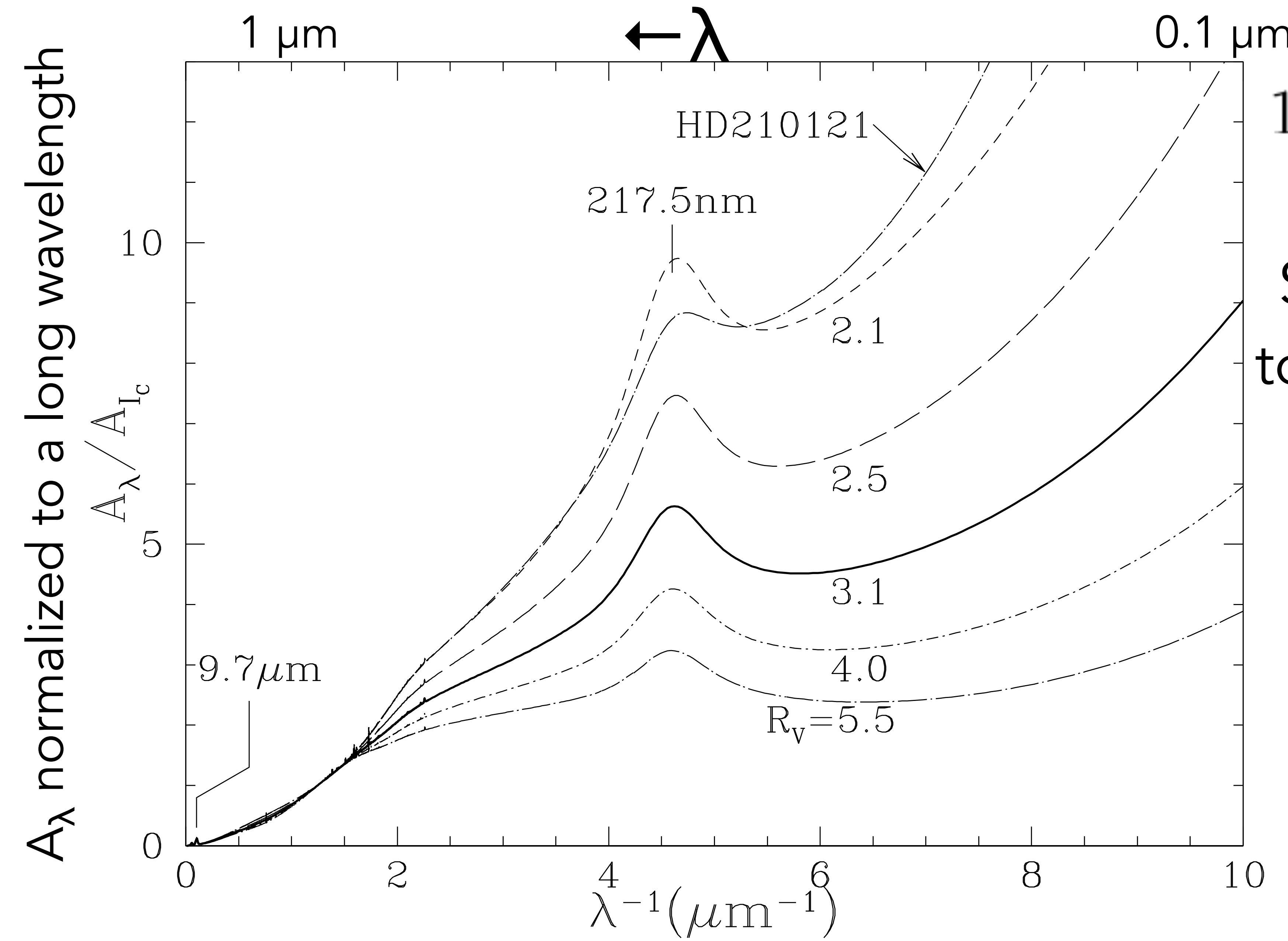
Extinction Curve



$$A_\lambda(a) = 1.086\ N_d(a)\ Q_{ext}(a)\ \pi a^2$$

Say grains only contribute to A_λ at their maximum Q_{ext}

Extinction Curve

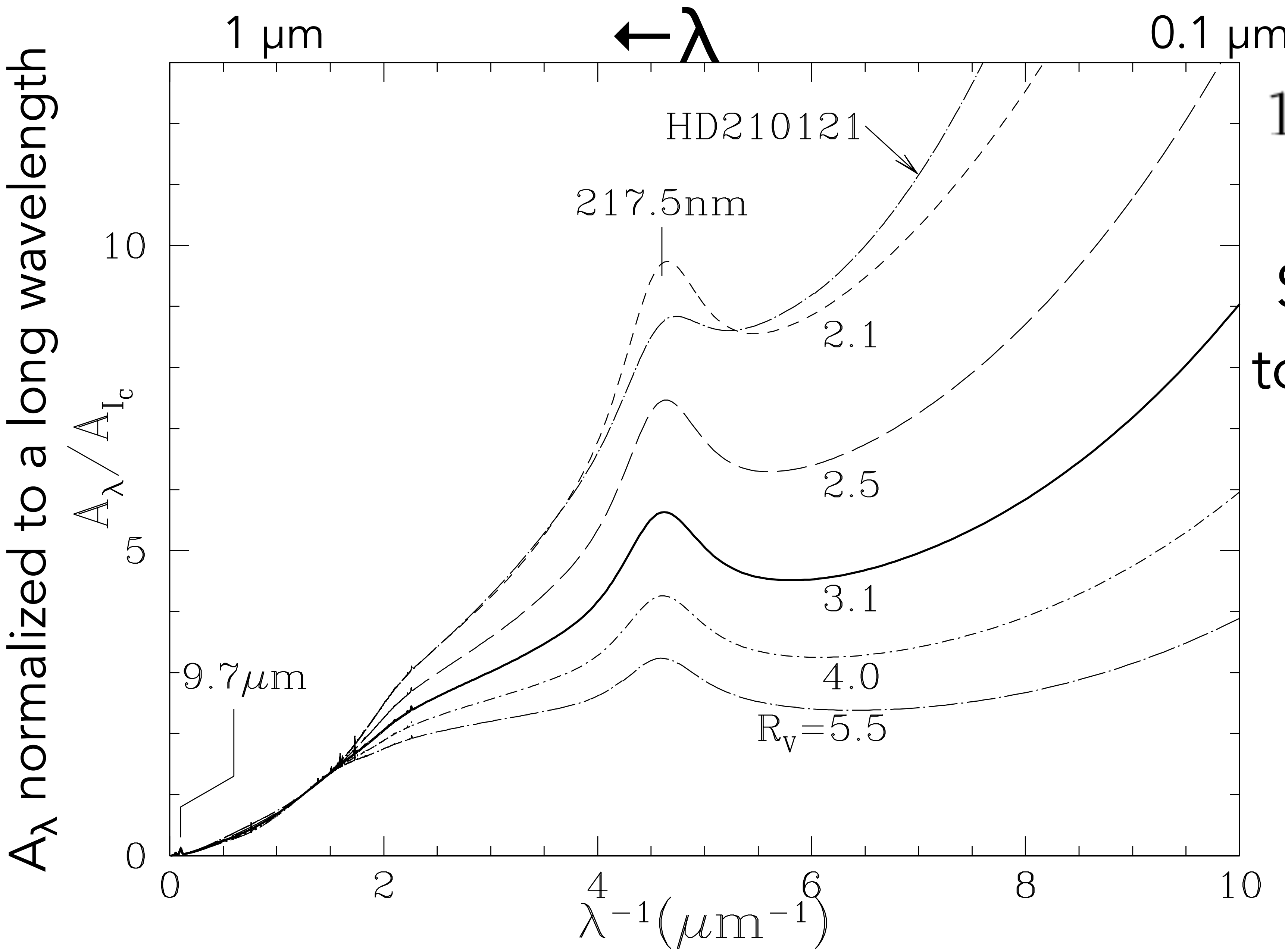


$$A_\lambda(a) = 1.086\ N_d(a)\ Q_{ext}(a)\ \pi a^2$$

Say grains only contribute to A_λ at their maximum Q_{ext}

$$\frac{A_{0.1\mu m}}{A_{0.5\mu m}} = \frac{N_{0.1\mu m}}{N_{0.5\mu m}} \left(\frac{0.1}{0.5}\right)^2$$

Extinction Curve



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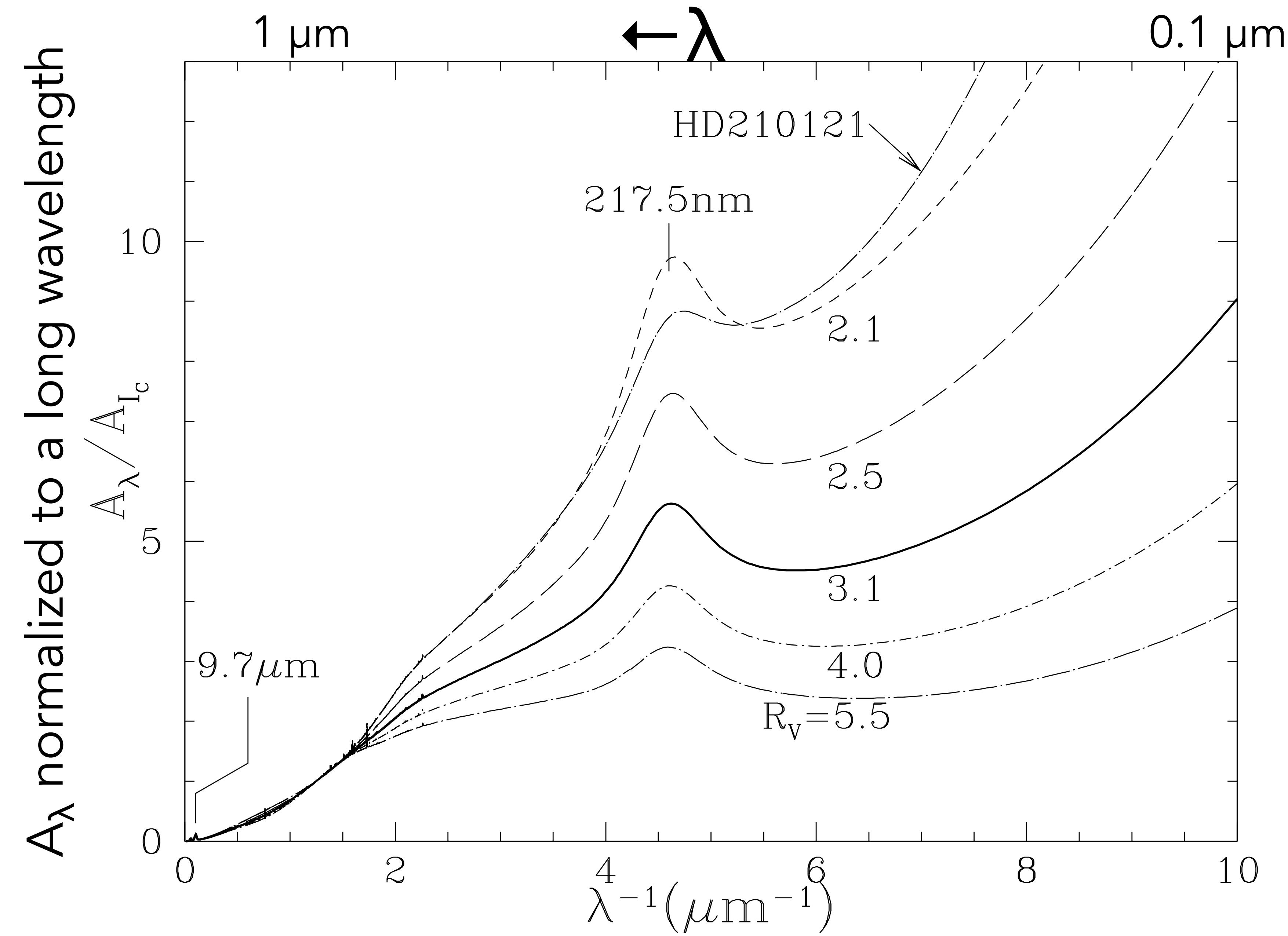
Say grains only contribute to A_λ at their maximum Q_{ext}

$$\frac{A_{0.1\mu m}}{A_{0.5\mu m}} = \frac{N_{0.1\mu m}}{N_{0.5\mu m}} \left(\frac{0.1}{0.5}\right)^2$$

$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} \sim 125$$

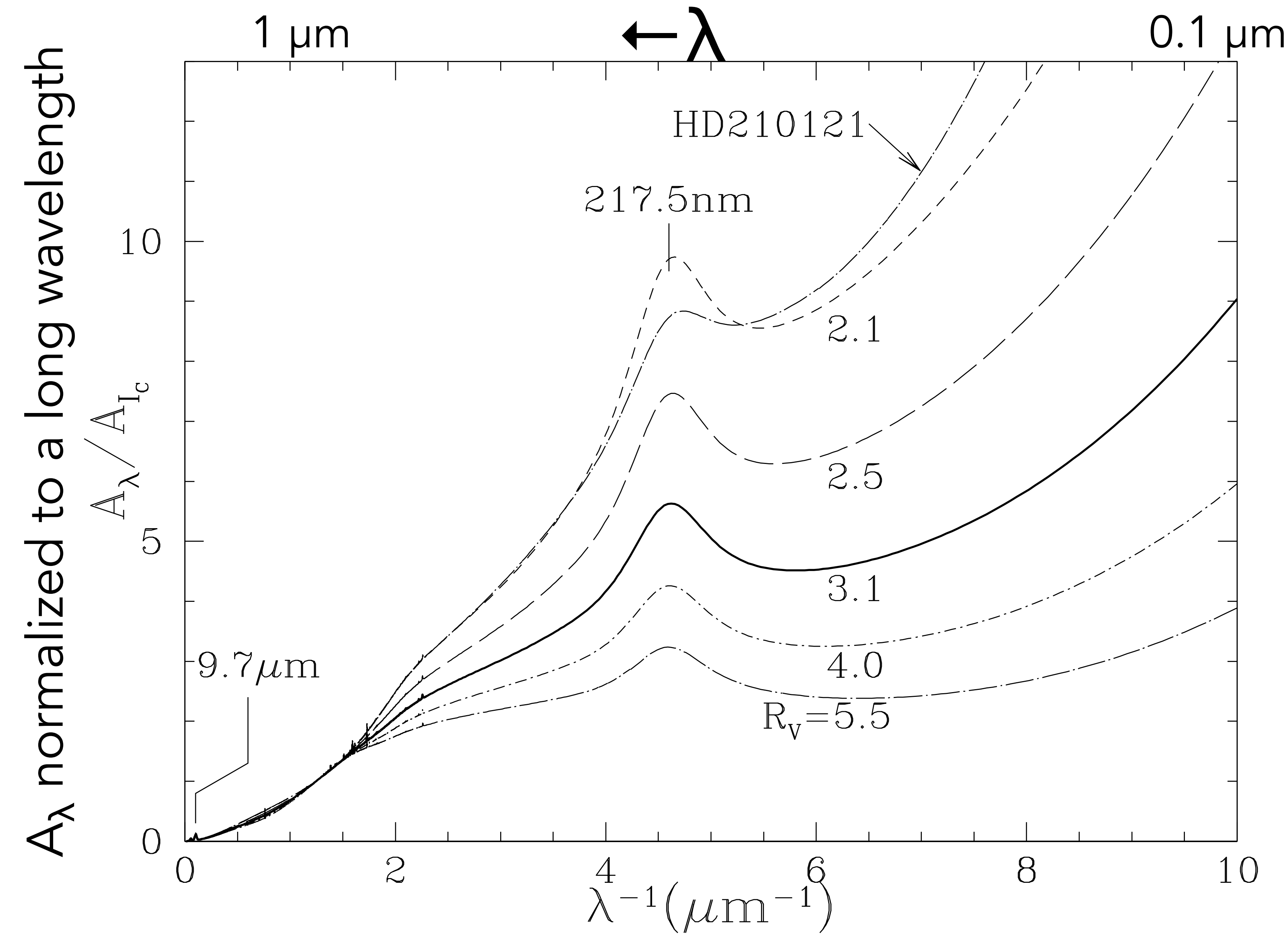
Extinction Curve



$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} \sim \quad 125$$

Assume power-law slope to size distribution $N(a) \sim a^{-\beta}$

Extinction Curve

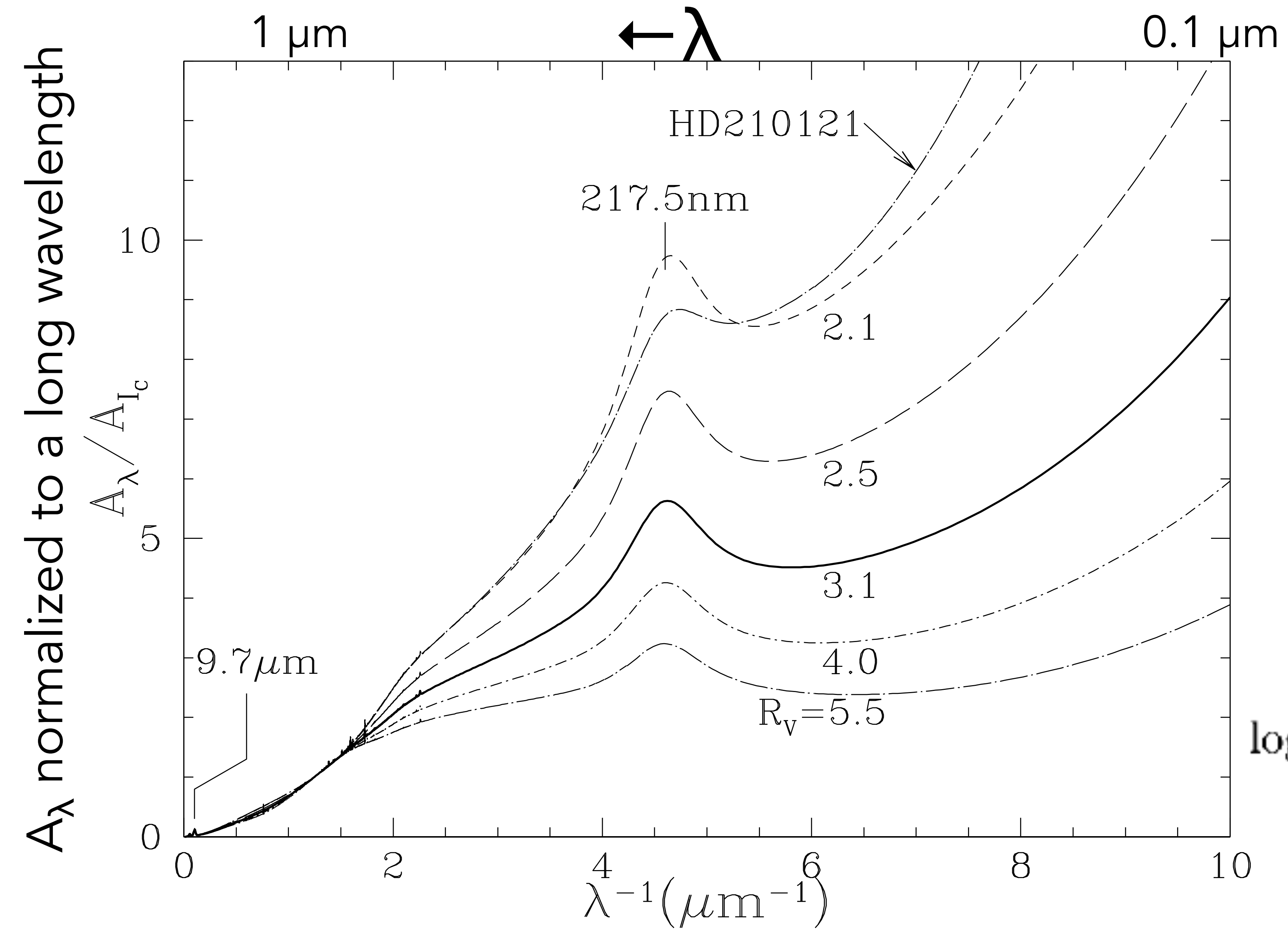


$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} \sim \quad 125$$

Assume power-law slope to size distribution $N(a) \sim a^{-\beta}$

$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} = \left(\frac{0.1}{0.5}\right)^{-\beta}$$

Extinction Curve



$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} \sim \quad 125$$

Assume power-law slope to size distribution
N(a) ~ a$^{-\beta}$

$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} = \left(\frac{0.1}{0.5}\right)^{-\beta}$$

$$\log_{10}\left(\frac{N_{0.1\mu m}}{N_{0.5\mu m}}\right) = -\beta \log_{10}\left(\frac{0.1}{0.5}\right)$$

β ~ 3

THE SIZE DISTRIBUTION OF INTERSTELLAR GRAINS

JOHN S. MATHIS, WILLIAM RUMPL, AND KENNETH H. NORDSIECK
Washburn Observatory, University of Wisconsin–Madison
Received 1977 January 24; accepted 1977 April 11

ABSTRACT

The observed interstellar extinction over the wavelength range $0.11\ \mu m < \lambda < 1\ \mu m$ was fitted with a very general particle size distribution of uncoated graphite, enstatite, olivine, silicon carbide, iron, and magnetite. Combinations of these materials, up to three at a time, were considered. The cosmic abundances of the various constituents were taken into account as constraints on the possible distributions of particle sizes.

Excellent fits to the interstellar extinction, including the narrowness of the λ2160 feature, proved possible. Graphite was a necessary component of any good mixture, but it could be used with any of the other materials. The particle size distributions are roughly power law in nature, with an exponent of about -3.3 to -3.6. The size range for graphite is about 0.005 μm to about 1 μm. The size distribution for the other materials is also approximately power law in nature, with the same exponent, but there is a narrower range of sizes: about 0.025–0.25 μm, depending on the material. The number of large particles is not well determined, because they are gray. Similarly, the number of small particles is not well determined because they are in the Rayleigh limit. This power-law distribution is drastically different from an Oort–van de Hulst distribution, which is much more slowly varying for small particles but drops much faster for particles larger than average.

$$\frac{dn}{da} \propto a^{-3.5}$$

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$$\frac{dn}{da} \propto a^{-3.5}$$

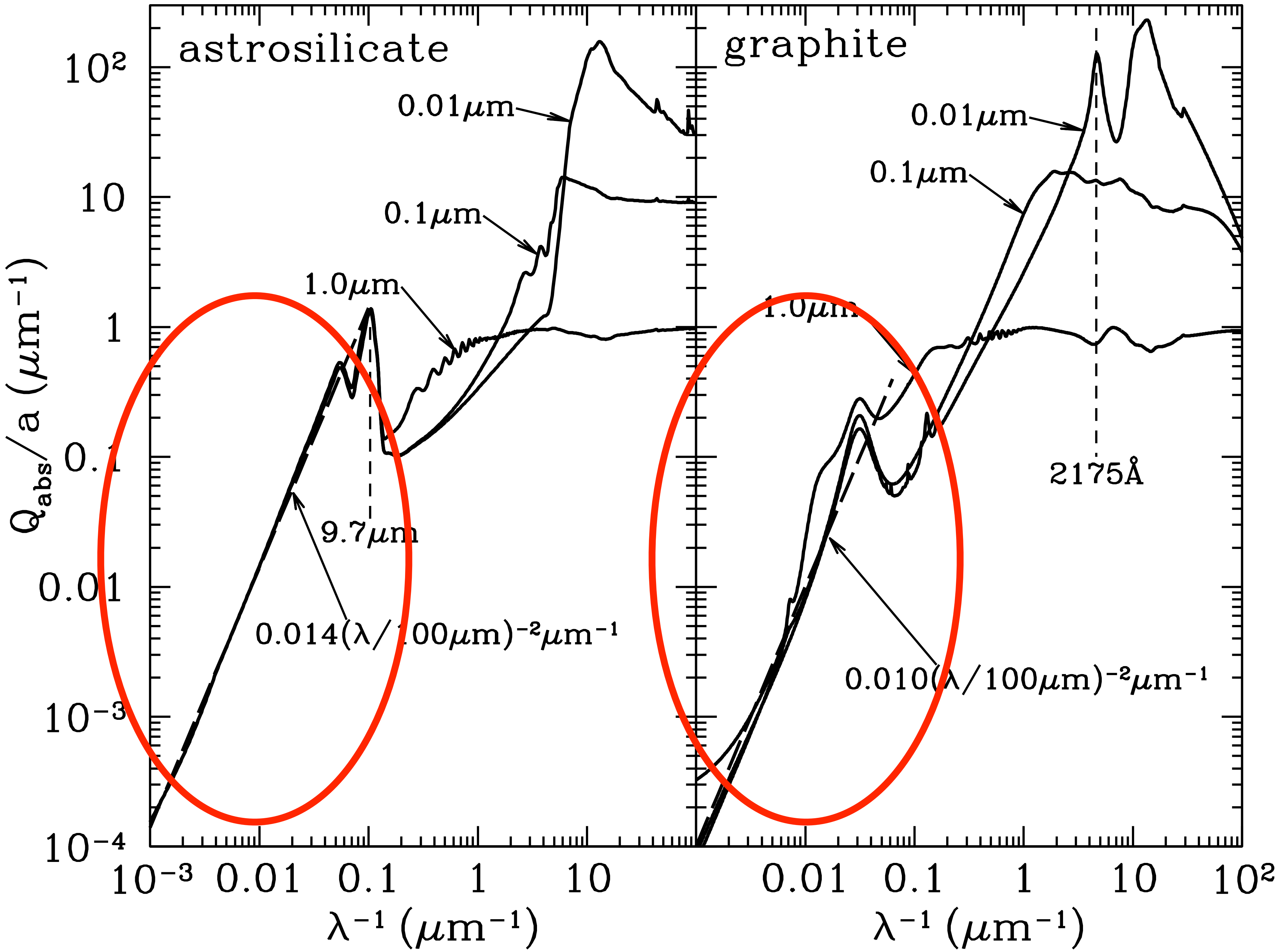
$$\text{Mass}(a) \propto \int a^3 \frac{dn}{da} da \propto a^{0.5}$$

most mass in large grains

$$\text{Area}(a) \propto \int a^2 \frac{dn}{da} da \propto a^{-0.5}$$

most area in small grains

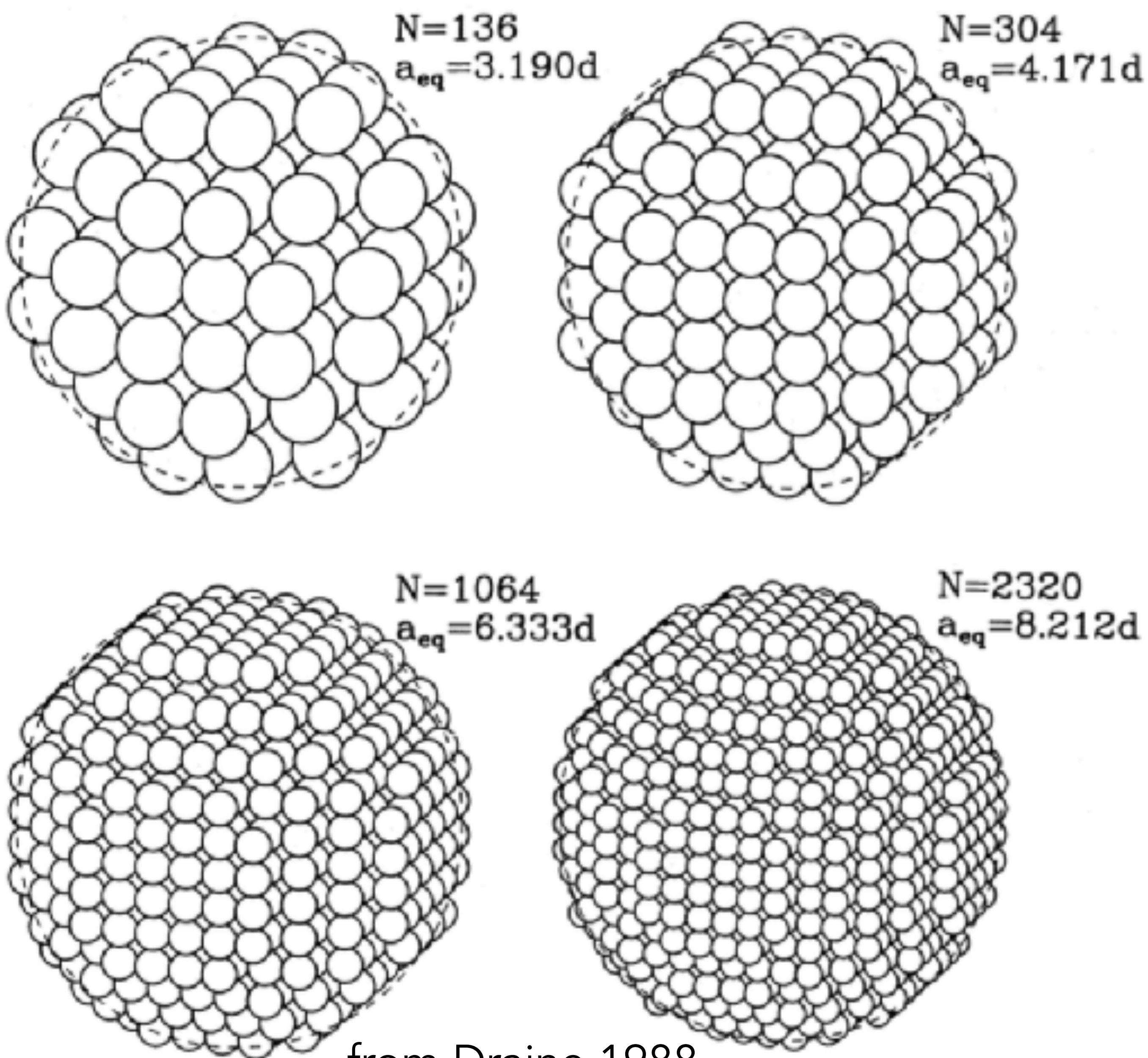
←λ



At long wavelengths $Q_{abs}/a \propto \lambda^{-2}$

Non-Spherical Grains?

Discrete Dipole approximation



from Draine 1988

replace solid particle
by N individual dipoles

dipoles oscillate in response
to incident wave and the
all E fields from the other
dipoles, solve set of
coupled linear equations

Purcell & Pennypacker 1975
Draine 1988

Dust Thermal Balance

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \; \frac{u_\nu}{h\nu} \; c \; Q_{\rm abs}(\nu) \pi a^2 d\nu$$

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \, \boxed{\frac{u_\nu}{h\nu} \, c \, Q_{\rm abs}(\nu)\pi a^2} d\nu$$

n_{ph} v σ

Dust Thermal Balance

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$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \, \boxed{\frac{u_\nu}{h\nu} \, c \, Q_{\rm abs}(\nu)\pi a^2} d\nu$$

energy per absorbed photon

n_{ph} v σ

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energy per absorbed photon

$n_{ph} \; v \; \sigma$

rate a dust grain of size a emits energy

$$\left(\frac{dE}{dt}\right)_{\rm emit} = \int 4\pi B_\nu(T) \; Q_{\rm em}(\nu) \; \pi a^2 d\nu$$

Dust Thermal Balance

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energy per absorbed photon

n_{ph} v σ

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$$\left(\frac{dE}{dt}\right)_{\rm emit} = \int 4\pi B_\nu(T)\ Q_{\rm em}(\nu)\ \pi a^2 d\nu$$

blackbody emitting over 4π str with efficiency Q_{em}

Dust Thermal Balance

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rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \, \frac{u_\nu}{h\nu} \, c \, Q_{\rm abs}(\nu) \pi a^2 d\nu$$

<u>in LTE</u>

$$u_\nu = \frac{4\pi}{c} B_\nu(T)$$

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \left(\frac{dE}{dt}\right)_{\rm emit}$$

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu\ \frac{u_\nu}{h\nu}\ c\ Q_{\rm abs}(\nu)\pi a^2 d\nu$$

in LTE

$$\int \frac{4\pi}{c} B_\nu(T)\ c\ Q_{\rm abs}(\nu)\pi a^2 d\nu = \int 4\pi B_\nu(T)\ Q_{\rm em}(\nu)\pi a^2 d\nu$$

Dust Thermal Balance

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<u>in LTE</u>

$$\int \frac{4\pi}{c} B_\nu(T) \, c \, Q_{\rm abs}(\nu) \pi a^2 d\nu = \int 4\pi B_\nu(T) \, Q_{\rm em}(\nu) \pi a^2 d\nu$$

Therefore: $Q_{\rm abs} = Q_{\rm em}$

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \ \frac{u_\nu}{h\nu} \ c \ Q_{\rm abs}(\nu)\pi a^2 d\nu$$

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \; \frac{u_\nu}{h\nu} \; c \; Q_{\rm abs}(\nu) \pi a^2 d\nu$$

Define “spectrum averaged absorption cross section”

$$\langle Q_{\rm abs} \rangle_* \equiv \frac{\int u_{*\nu} Q_{\rm abs}(\nu) d\nu}{\int u_{*\nu} d\nu}$$

Dust Thermal Balance

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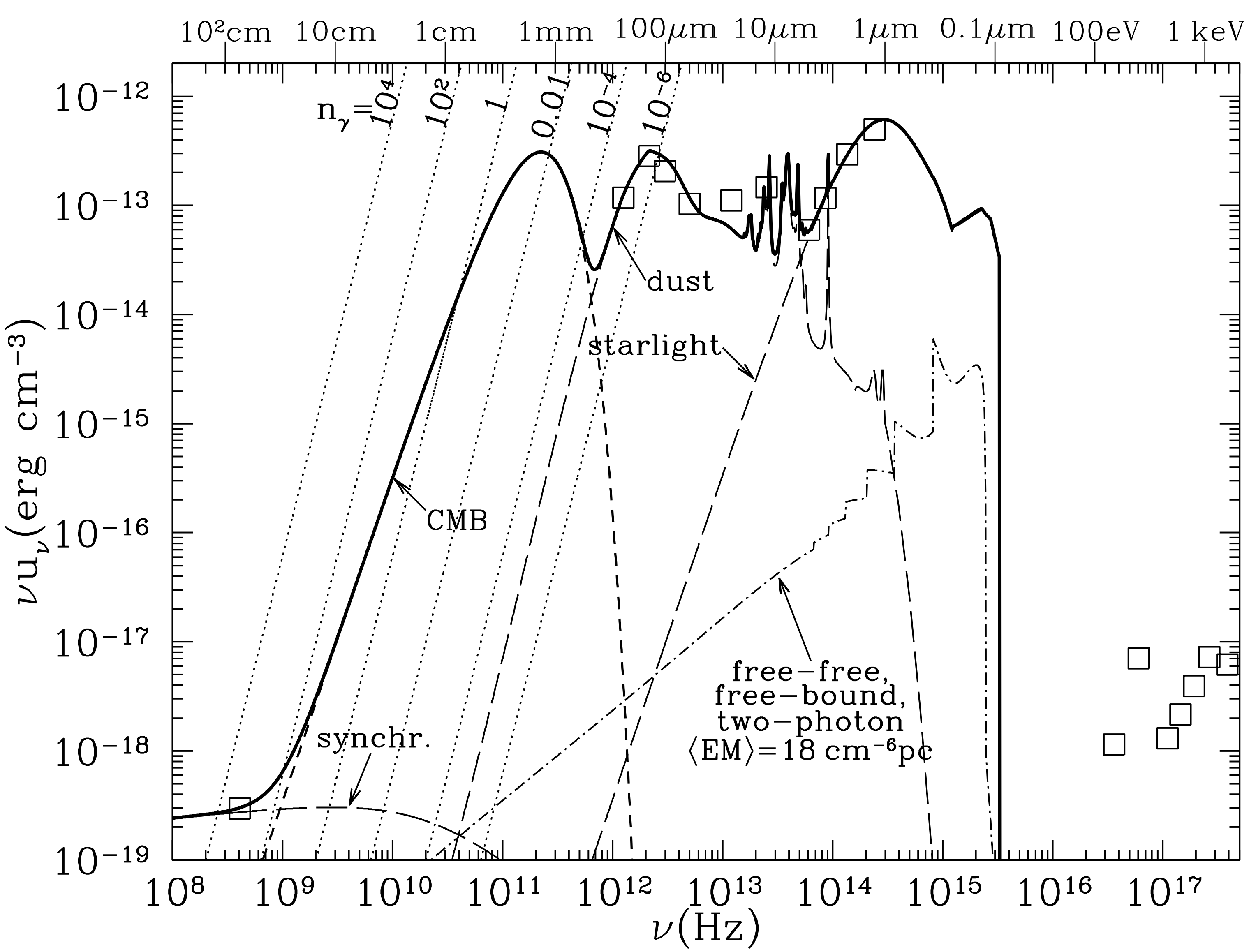
so that:

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \langle Q_{\rm abs} \rangle_* \pi a^2 \; u_* \; c$$

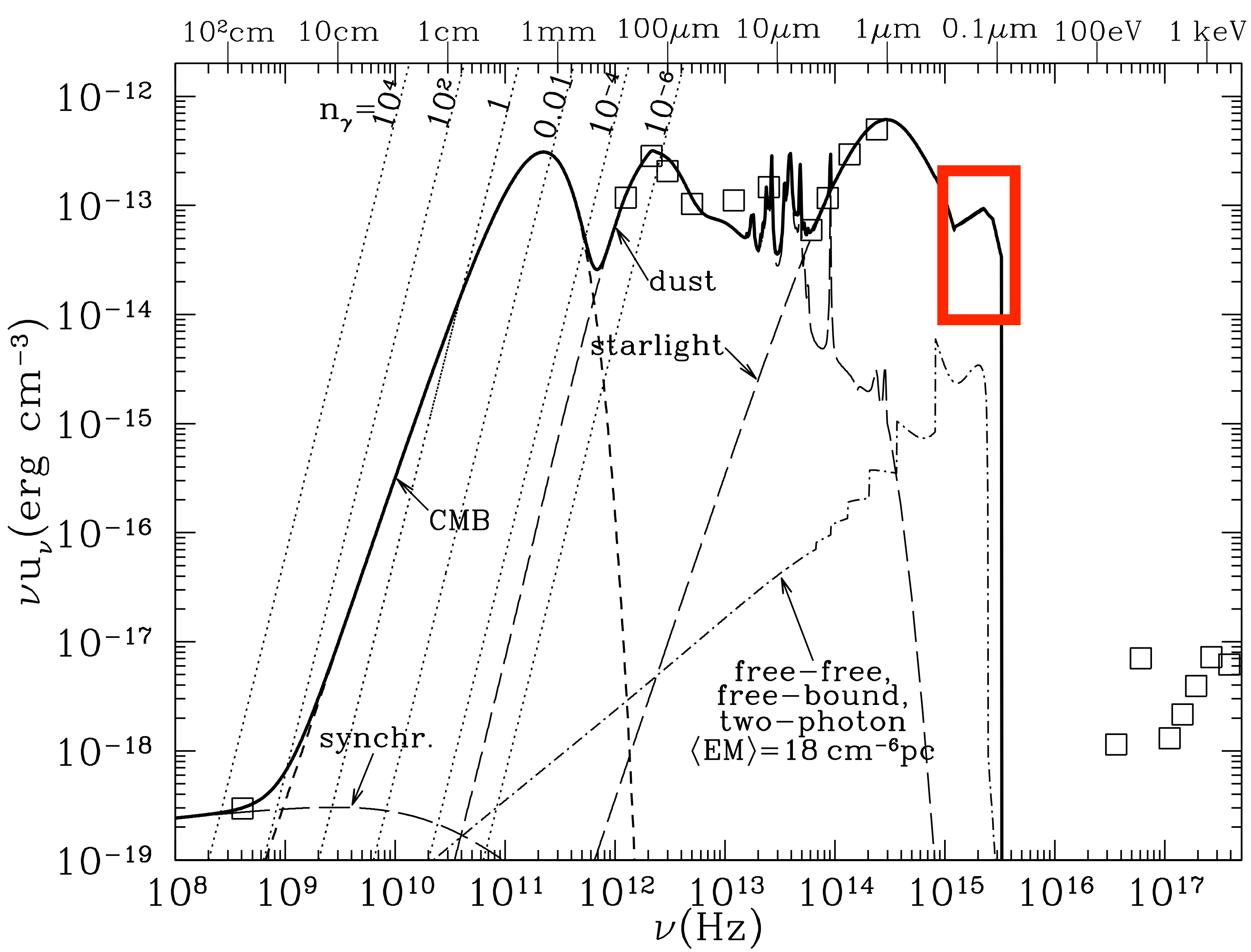
Dust Thermal Balance

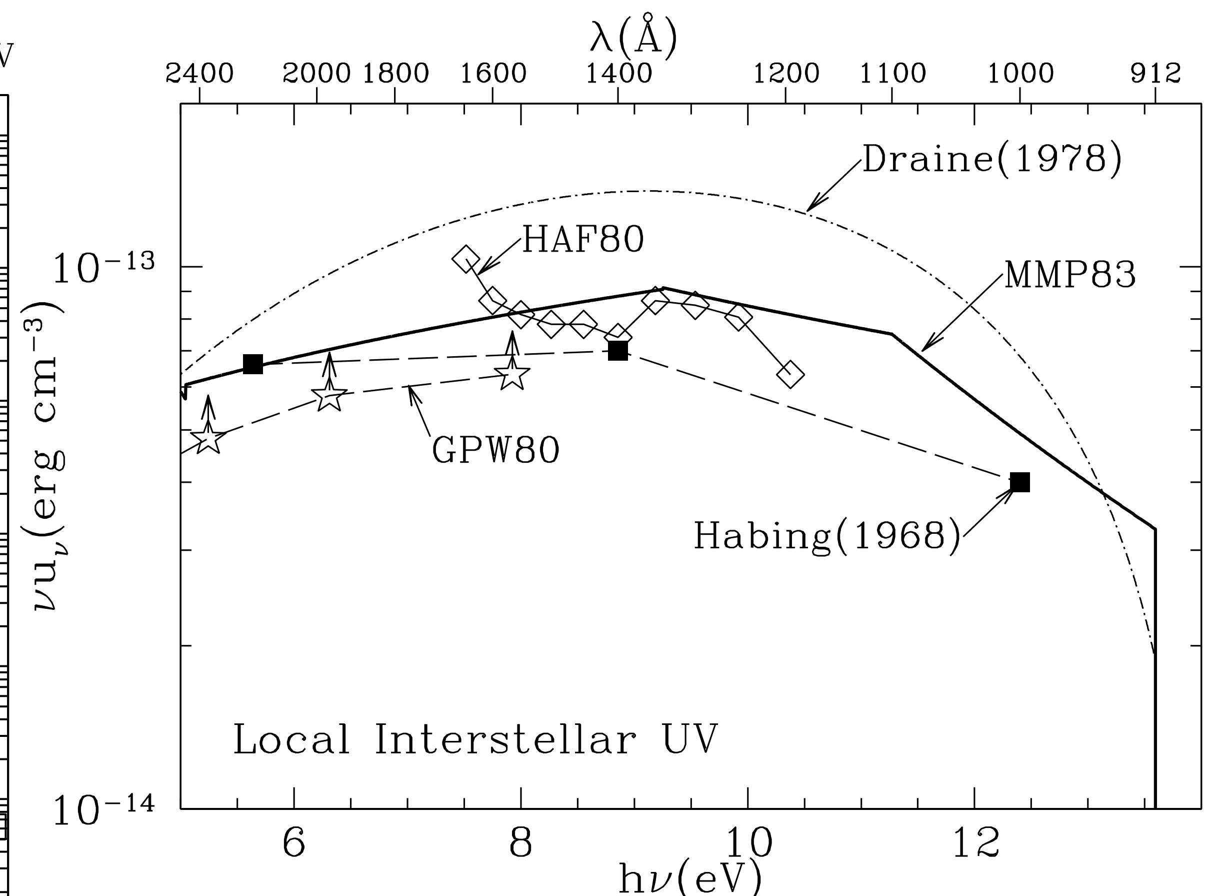
MW Average Radiation Field (u*)



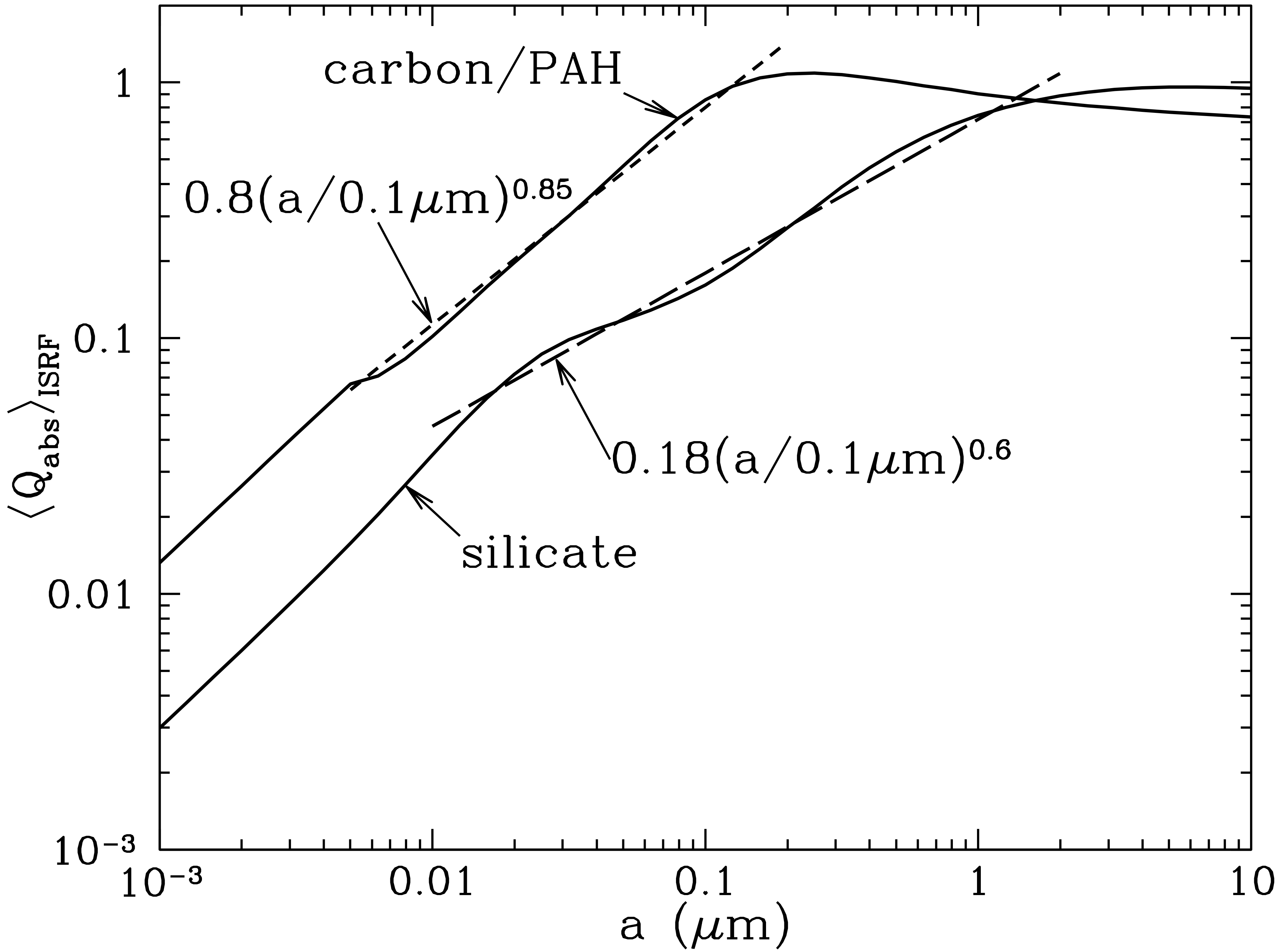
Dust Thermal Balance

MW Average Radiation Field (u*)





Dust Thermal Balance



$<Q_{abs}>_*$ for the average interstellar radiation field in the MW, and two astronomical dust analogs.

Dust Thermal Balance

rate a dust grain of size a emits energy

$$\left(\frac{dE}{dt}\right)_{\rm emit} = \int 4\pi B_\nu(T)\ Q_{\rm em}(\nu)\ \pi a^2 d\nu$$

$$Q_{\rm abs} = Q_{\rm em}$$

Dust Thermal Balance

rate a dust grain of size a emits energy

$$\left(\frac{dE}{dt}\right)_{\text{emit}} = \int 4\pi B_\nu(T)\ Q_{\text{em}}(\nu)\ \pi a^2 d\nu$$

$$Q_{\text{abs}} = Q_{\text{em}}$$

Define "Planck averaged emission efficiency"

$$\langle Q_{\text{abs}} \rangle_{\text{T}} \equiv \frac{\int B_\nu(T) Q_{\text{abs}} d\nu}{\int B_\nu(T) d\nu}$$

Dust Thermal Balance

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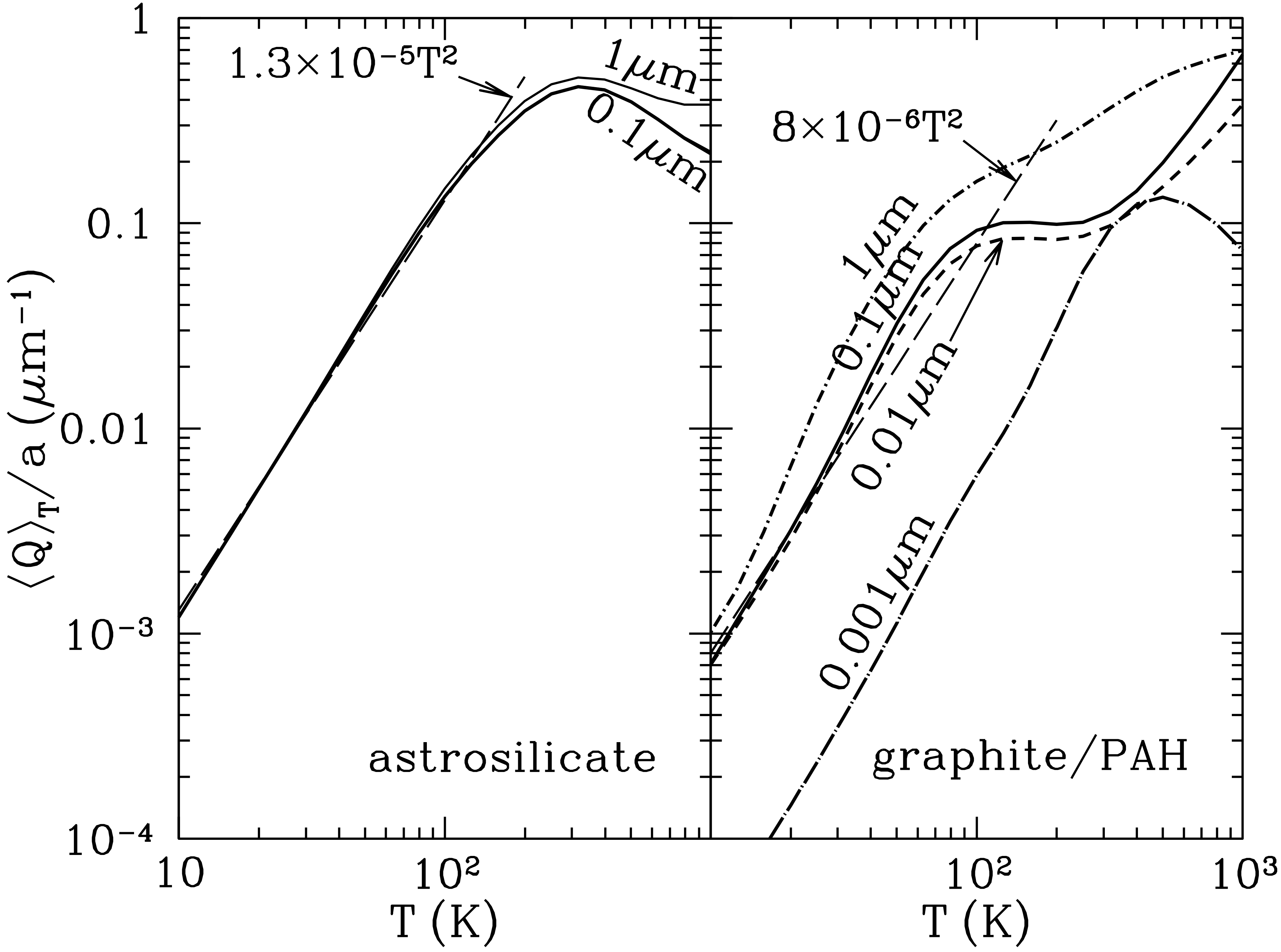
so that: $$\left(\frac{dE}{dt}\right)_{\rm em} = 4\pi a^2 \langle Q_{\rm abs}\rangle_{\rm T}\, \sigma T^4$$

Dust Thermal Balance



$<Q_{abs}>_T/a$ for the a range of blackbody temperatures

Below ~100 K $<Q_{abs}>_T \sim T^2$

Dust Thermal Balance

In steady state, emission = absorption.

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \langle Q_{\rm abs} \rangle_* \, \pi a^2 \; u_* \; c$$

$$\left(\frac{dE}{dt}\right)_{\rm em} = 4\pi a^2 \, \langle Q_{\rm abs} \rangle_{\rm T} \, \sigma T^4$$

Dust Thermal Balance

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$$\left(\frac{dE}{dt}\right)_{\rm em} = 4\pi a^2 \, \langle Q_{\rm abs}\rangle_{\rm T} \, \sigma T^4$$

for MW interstellar radiation field and dust properties we found:

$\langle Q_{\rm abs}\rangle_* \sim 0.8(a/0.1\mu m)^{0.85}$ carbon

$\langle Q_{\rm abs}\rangle_* \sim 0.18(a/0.1\mu m)^{0.6}$ silicate

Dust Thermal Balance

In steady state, emission = absorption.

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \langle Q_{\rm abs}\rangle_* \, \pi a^2 \; u_* \; c$$

$$\left(\frac{dE}{dt}\right)_{\rm em} = 4\pi a^2 \, \langle Q_{\rm abs}\rangle_{\rm T} \, \sigma T^4$$

for MW interstellar radiation field and dust properties we found:

$$\langle Q_{\rm abs}\rangle_T = 8 \times 10^{-7} (a/0.1\mu{\rm m})(T/{\rm K})^2$$ carbon

$$\langle Q_{\rm abs}\rangle_T = 1.3 \times 10^{-6} (a/0.1\mu{\rm m})(T/{\rm K})^2$$ silicate

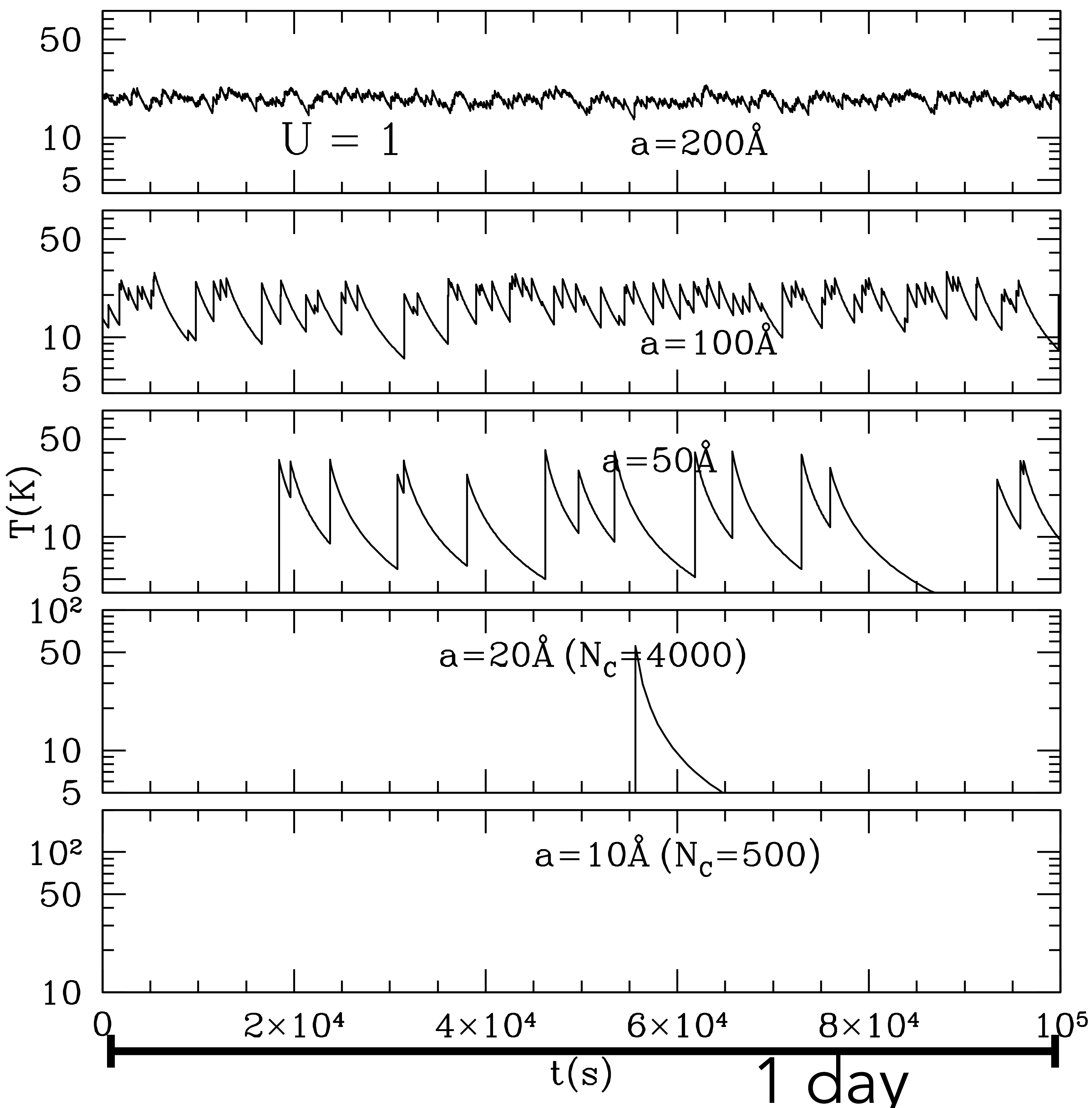
Dust Thermal Balance

Very weak dependence of equilibrium temperature on grain size.

$$T \approx 22.3(a/0.1\mu m)^{-1/40} U^{1/6} K$$ carbon

$$T \approx 16.4(a/0.1\mu m)^{1/15} U^{1/6} K$$ silicate

Dust Thermal Balance



Not all grains are in steady state…

When:

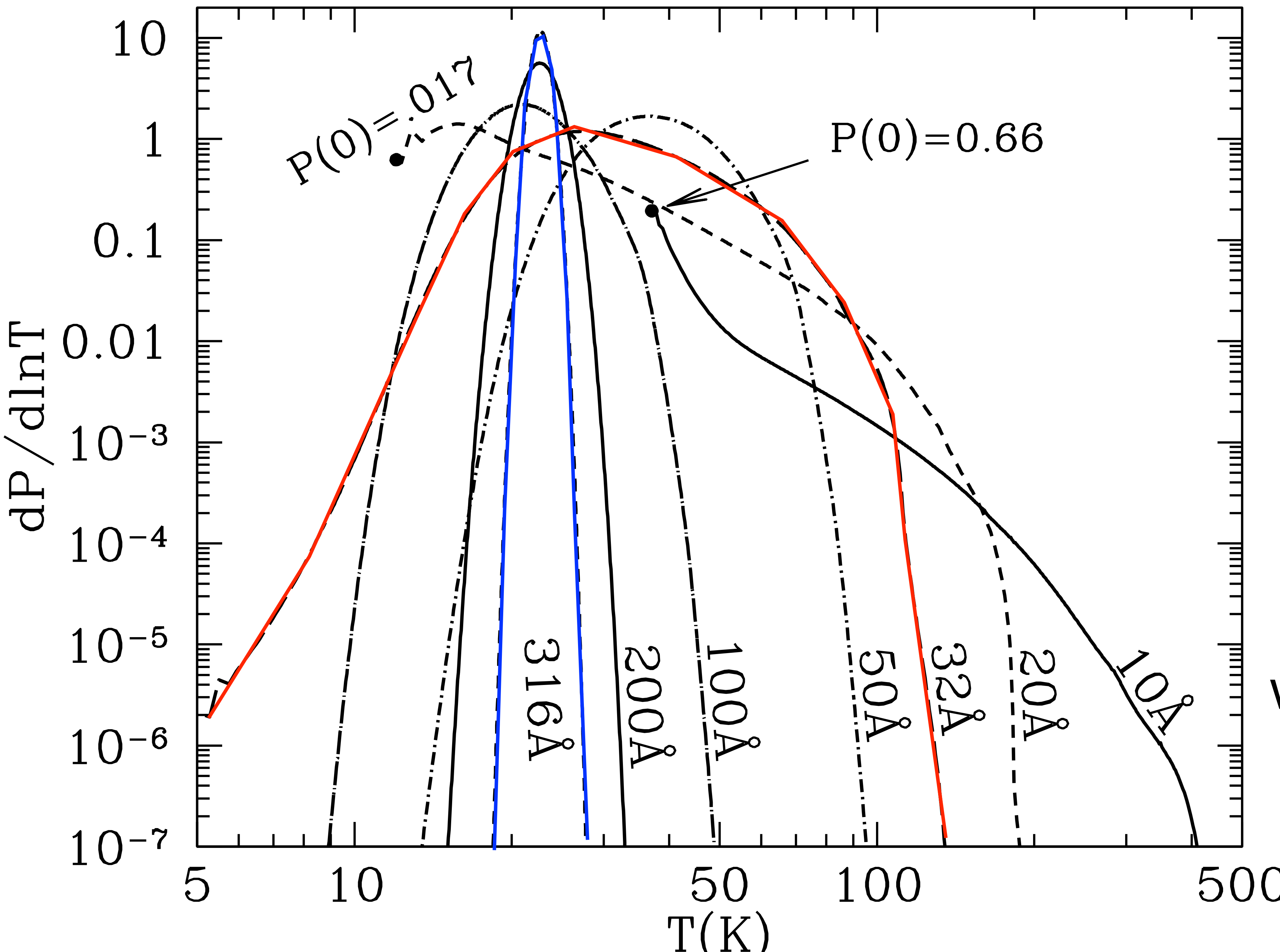
$(dE/dt)_{cool}$ << photon absorption rate

and/or

$h\nu >> E_{ss}$

Need to consider non-steady state

Dust Thermal Balance



Probability of finding grain with temp T in average MW ISRF.

PDF narrows with increasing size.

Dust Thermal Balance

While it is unlikely to find a small grain at
very high temperatures, most energy is emitted there!

$$\left(\frac{dE}{dt}\right)_{\rm em} = 4\pi a^2 \left\langle Q_{\rm abs}\right\rangle_{\rm T} \sigma T^4$$

$$\langle Q_{\rm abs}\rangle_T \sim 1.3 \times 10^{-5} T^2$$ silicate

dE/dt ~ T^6

Dust Thermal Balance

Is collisional heating important?

absorption $$\left(\frac{dE}{dt}\right)_{\rm abs} = \langle Q_{\rm abs}\rangle_* \pi a^2 \; u_* \; c$$

collisions $$\left(\frac{dE}{dt}\right)_0 = \quad n_{\rm H}\pi a^2 \langle v_{\rm H}\rangle 2kT \, \alpha$$

factor ~unity
for energy transfer from
collider to grain

Dust Thermal Balance

Is collisional heating important?

$$\frac{(dE/dt)_{\rm col}}{(dE/dt)_{\rm abs}} = \frac{3.8\times10^{-6}}{U}\frac{\alpha}{\langle Q_{\rm abs}\rangle_*}\left(\frac{n_H}{30cm^{-3}}\right)\left(\frac{T}{10^2K}\right)^{3/2}$$

radiation field strength normalized to MW average ISRF

collisional heating important in dense and/or hot gas

Dust Thermal Balance

Is collisional heating important?

More generally:

density of colliders

if grain and/or collider is charged, Coulomb focusing factor

$$\frac{(dE/dt)_{\rm coll}}{(dE/dt)_{\rm abs}} \approx \frac{2nkT}{u_*} \times \frac{\gamma}{\langle Q_{\rm abs} \rangle_*} \times \frac{(8kT/\pi m_e)^{1/2}}{c}$$

Dust Thermal Balance

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velocity of colliders relative to speed of light

Dust Thermal Balance

Is collisional heating important?

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thermal pressure (thermal energy density) relative to starlight energy density

velocity of colliders relative to speed of light

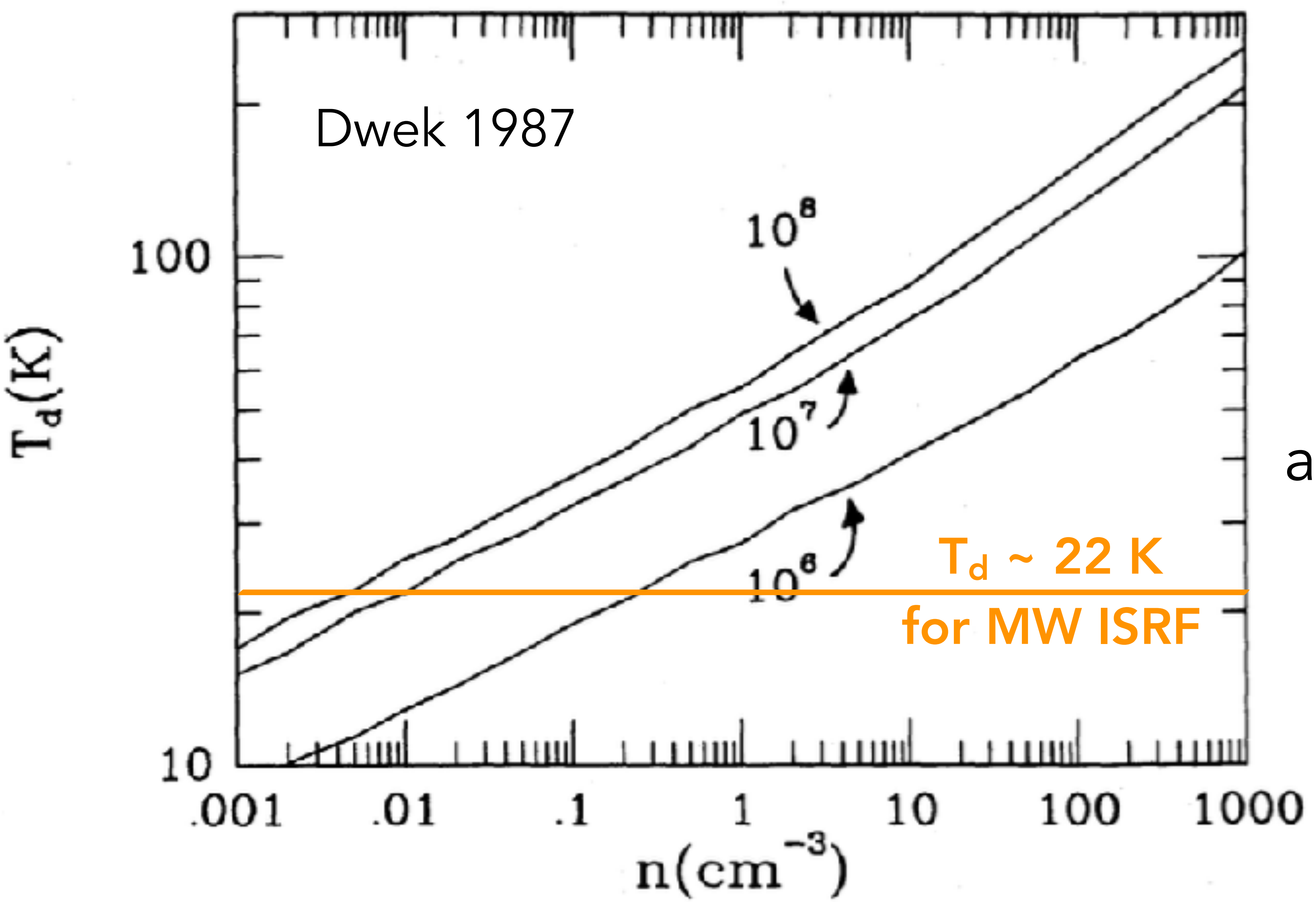
Dust Thermal Balance

Is collisional heating important?

- in places where radiation energy density is very low, (e.g. cores of molecular clouds)
- in places where thermal pressure is very high (e.g. hot plasma behind shock waves in SNe)

Dust Thermal Balance

Collisional heating in hot, dense plasmas



Temperature of an 0.1 μm graphite particle for various gas temperatures as a function of density

Dust Emission

$$j_\nu = \sum_i \int da \frac{dn_i}{da} \int dT \left(\frac{dP}{dT} \right)_{i,a} Q_{\rm abs}(\nu; i, a)\ \pi a^2 B_\nu(T)$$

Dust Emission

Emissivity
[erg/s/cm^3/Hz/sr]

$$j_\nu = \sum_i \int da \frac{dn_i}{da} \int dT \left(\frac{dP}{dT}\right)_{i,a} Q_{\rm abs}(\nu; i, a)\ \pi a^2 B_\nu(T)$$

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energy/time/
solid angle/freq
emitted by a grain
of size *a* and
composition *i*

Dust Emission

Emissivity
[erg/s/cm^3/Hz/sr]

$$j_\nu = \sum_i \int da \frac{dn_i}{da} \int dT \left(\frac{dP}{dT}\right)_{i,a} Q_{\rm abs}(\nu; i, a)\ \pi a^2 B_\nu(T)$$

integral over temperature probability distribution function for grain of size *a* and composition *i*

energy/time/ solid angle/freq emitted by a grain of size *a* and composition *i*

Dust Emission

Emissivity

[erg/s/cm^3/Hz/sr]

integral over grain size distribution

$$j_\nu = \sum_i \int da \frac{dn_i}{da} \int dT \left(\frac{dP}{dT}\right)_{i,a} Q_{\rm abs}(\nu; i, a)\ \pi a^2 B_\nu(T)$$

integral over temperature probability distribution function for grain of size *a* and composition *i*

energy/time/solid angle/freq emitted by a grain of size *a* and composition *i*

Dust Emission

Emissivity
[erg/s/cm^3/Hz/sr]

integral over
grain size distribution

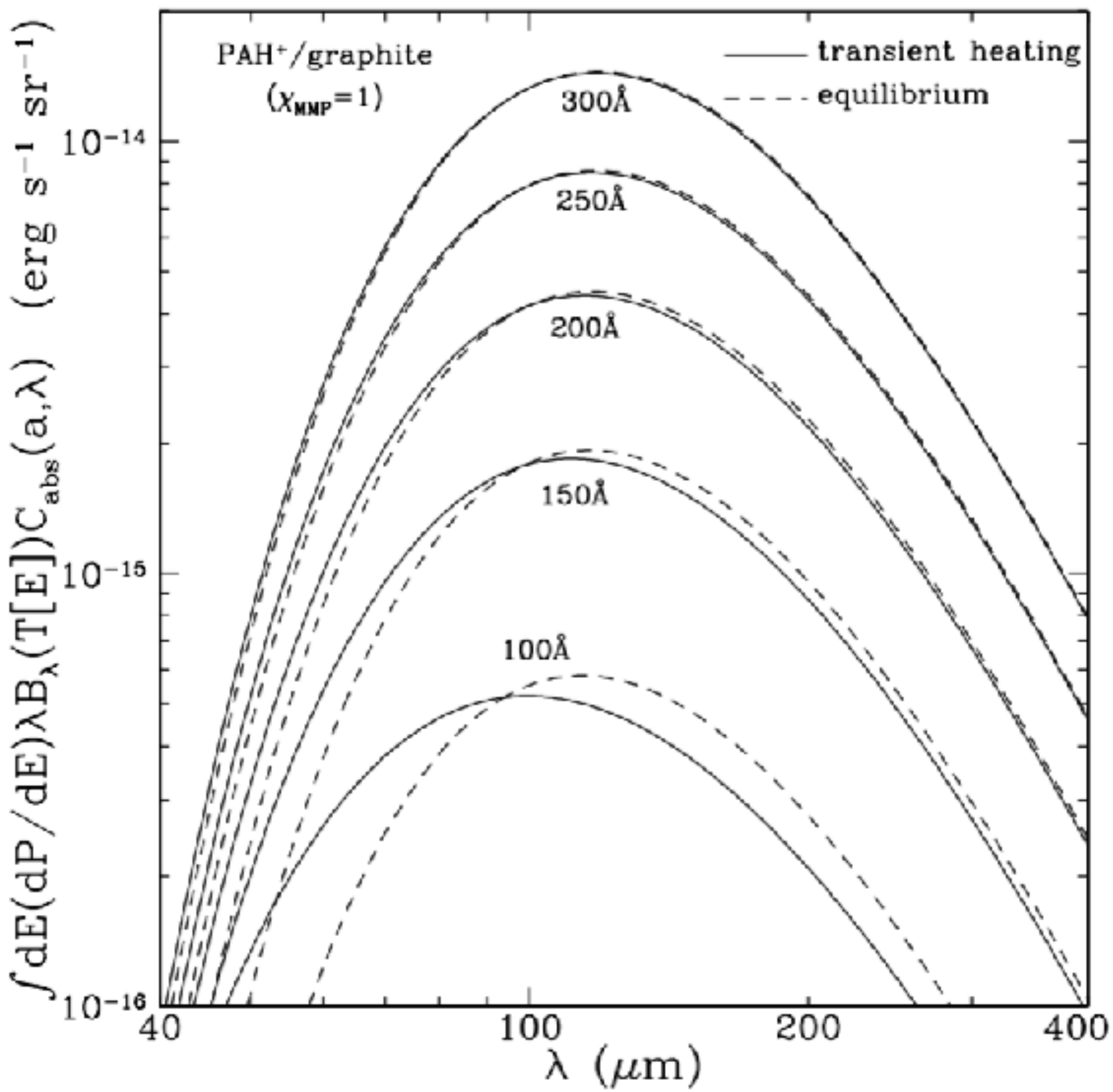
$$j_\nu = \sum_i \int da \frac{dn_i}{da} \int dT \left(\frac{dP}{dT}\right)_{i,a} Q_{\rm abs}(\nu; i, a)\ \pi a^2 B_\nu(T)$$

sum over
different grain
compositions

integral over
temperature probability
distribution function
for grain of size *a*
and composition *i*

energy/time/
solid angle/freq
emitted by a grain
of size *a* and
composition *i*

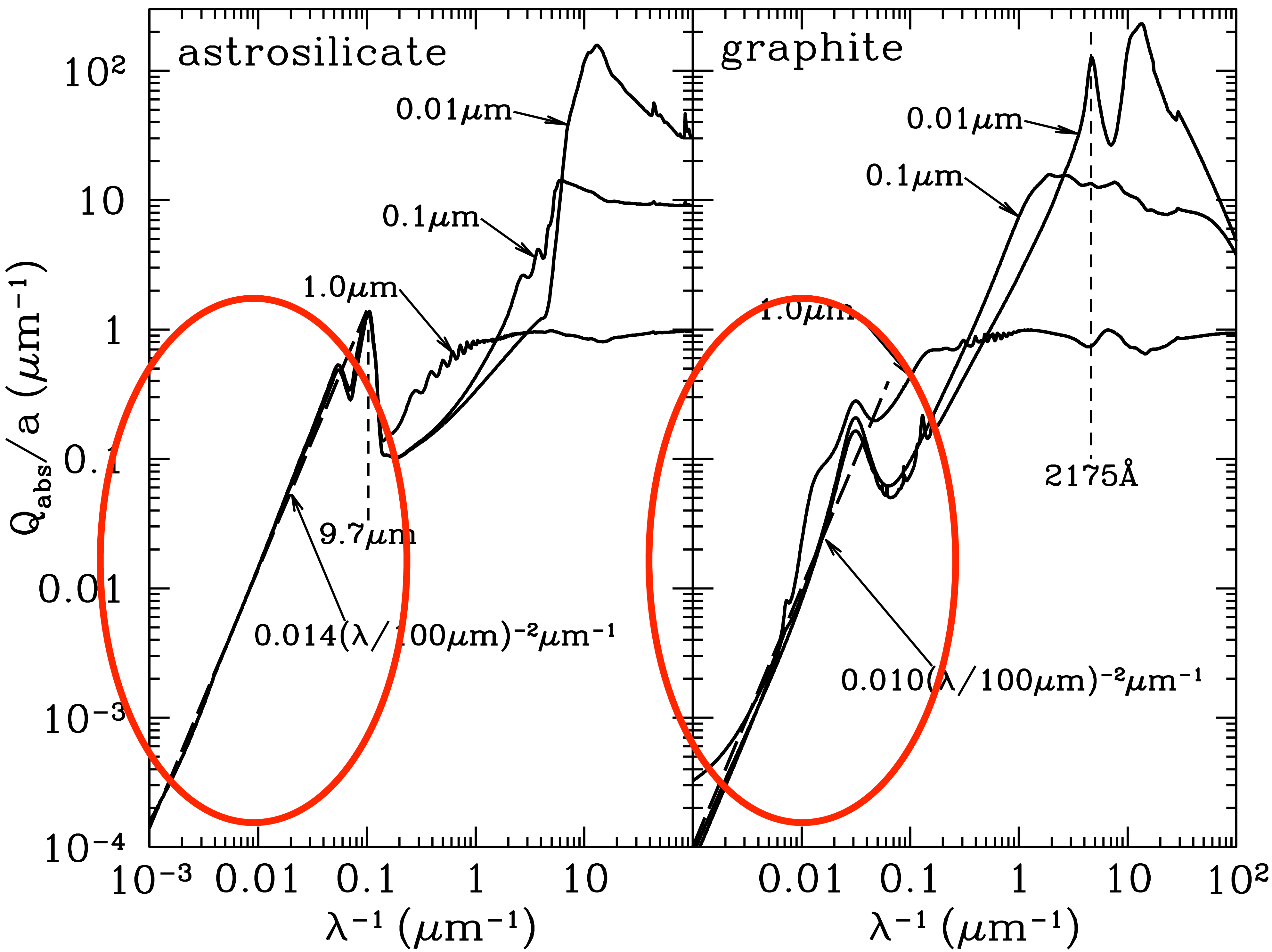
Dust Emission



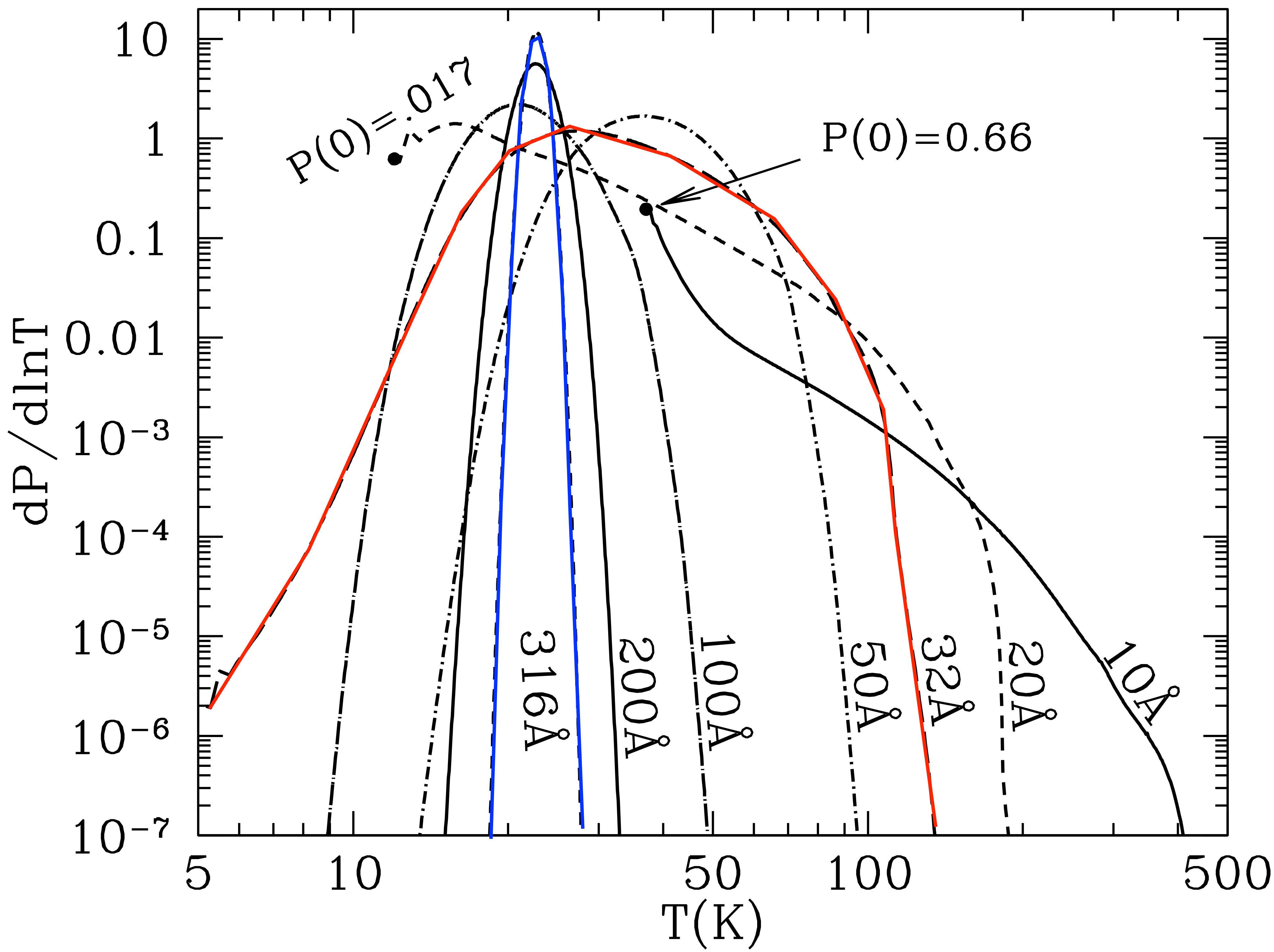
For grains that are large enough, dP/dT is ~delta function & Q_{abs} is smooth and prop to λ^{-2}.

Also T_{SS} is ~independent of grain size.

←λ



At long wavelengths $Q_{abs}/a \propto \lambda^{-2}$ i.e. $Q_{abs} \propto a\lambda^{-2}$



10
1
0.1
0.01
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
dP/dlnT
5
10
50
100
500
T(K)
P(0)=.017
P(0)=0.66
316Å
200Å
100Å
50Å
32Å
20Å
10Å

Dust Emission

For "equilibrium" grain emission

$$j_\nu = \sum_i \int da \frac{dn_i}{da} \int dT \left(\frac{dP}{dT}\right)_{i,a} Q_{\rm abs}(\nu; i, a)\ \pi a^2 B_\nu(T)$$

Dust Emission

For "equilibrium" grain emission

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delta function at T_{SS}

Dust Emission

For "equilibrium" grain emission

$$j_\nu = \sum_i \int da \frac{dn_i}{da} \int dT \left(\frac{dP}{dT}\right)_{i,a} \boxed{Q_{\rm abs}(\nu; i, a)\ \pi a^2 B_\nu(T)}$$

delta function at T_{SS}

$\pi a^3\, Q_{abs,0}\, \lambda^{-2}\, B_\nu(T_{SS})$

Dust Emission

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can go outside integral over size distribution

Dust Emission

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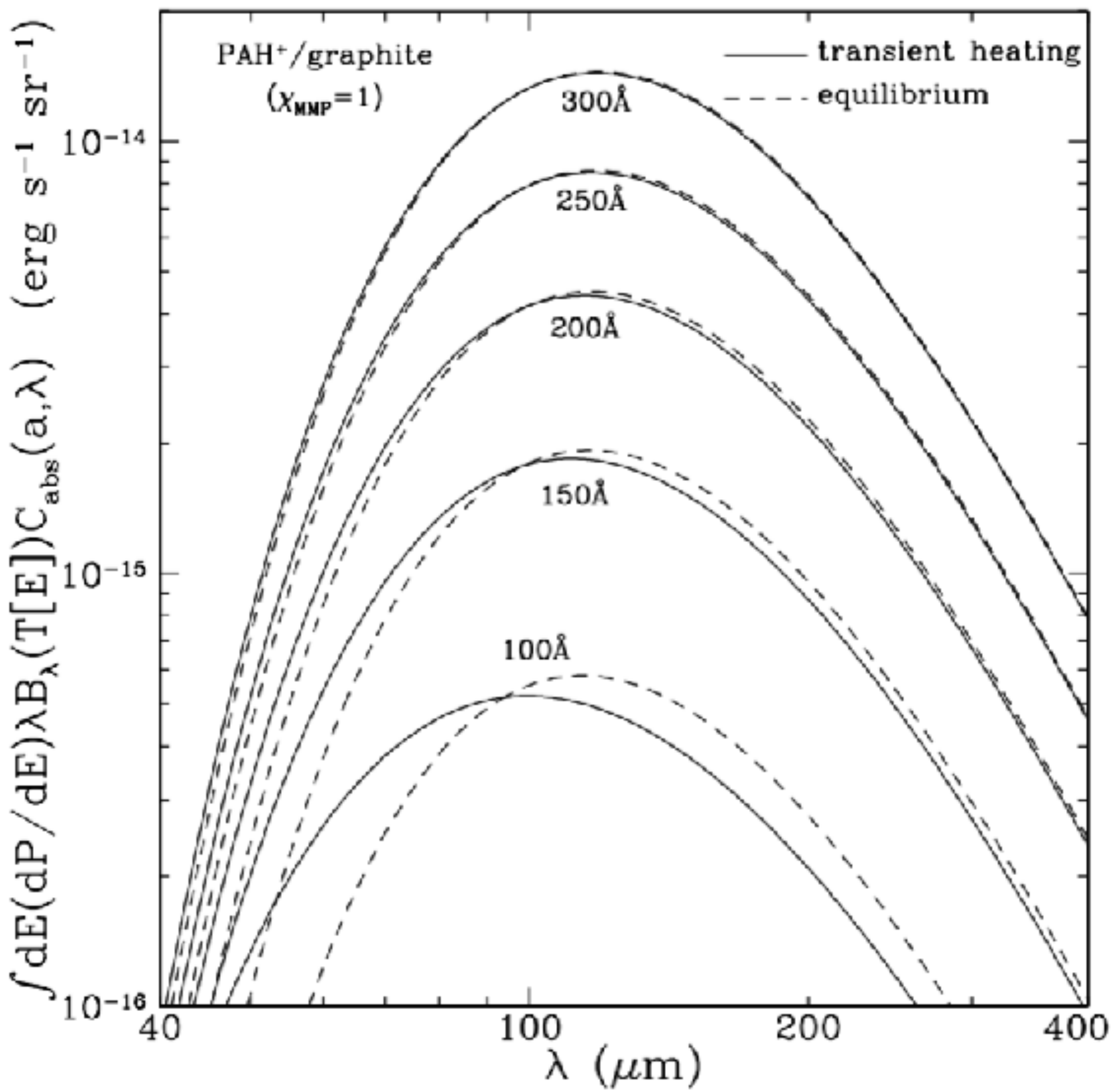
delta function at T_{SS}

$\pi a^3 Q_{abs,0} \lambda^{-2} B_\nu(T_{SS})$

can go outside integral over size distribution

End up with: $j_\nu =$ function that depends on grain pop × $B_\nu(T_{SS})$

Dust Emission



For grains that are large enough, dP/dT is ~delta function & Q_{abs} is smooth and prop to λ^{-2}.

Also T_{SS} is ~independent of grain size.

Change of units:
S_λ = surface brightness
(typical unit: MJy/sr or Jy/arsec^2)

"Modified Blackbody"

Only works for equilibrium emission!

$$\kappa_\lambda = \frac{\kappa^{S}_{\mathrm{eff},160}}{160^{-\beta_{\mathrm{eff}}}} \lambda^{-\beta_{\mathrm{eff}}}$$

from Gordon et al. 2014

In general, the surface brightness of dust with temperature, T_d, is

$$S_\lambda = \tau_\lambda B_\lambda(T_d) \quad (1)$$

$$= N_d \pi a^2 Q_\lambda B_\lambda(T_d) \quad (2)$$

$$= \frac{\Sigma_d}{m_d} \pi a^2 Q_\lambda B_\lambda(T_d) \quad (3)$$

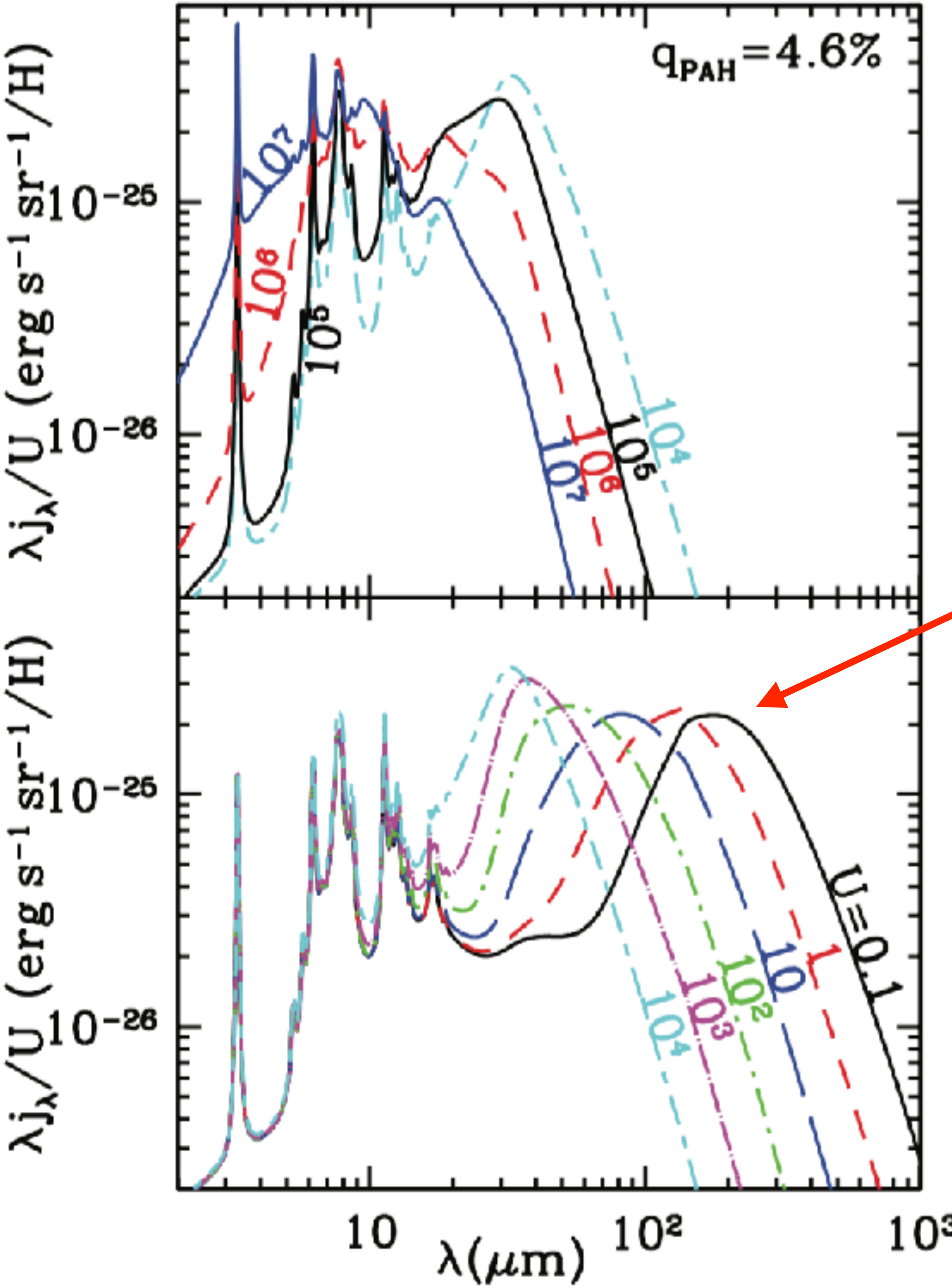
$$= \frac{\Sigma_d}{\frac{4}{3} a^3 \rho} \pi a^2 Q_\lambda B_\lambda(T_d) \quad (4)$$

$$= \frac{3}{4 a \rho} \Sigma_d Q_\lambda B_\lambda(T_d) \quad (5)$$

$$= \kappa_\lambda \Sigma_d B_\lambda, \quad (6)$$

where τ_λ is the dust optical depth, N_d is the dust column density, a is the grain radius, Q_λ is the dust emissivity, B_λ is the Planck function, Σ_d is the dust surface mass density, m_d is the mass of a single dust grain, ρ is the grain density, κ_λ is the grain absorption cross section per unit mass. These equations can be evaluated in standards units (e.g., cgs or MKS). We found it convenient to express Σ_d in $M_\odot$ pc^{-2}, κ_λ in cm^2 g^{-1}, and B_λ and S_λ in MJy sr^{-1} and then Equation (6) is

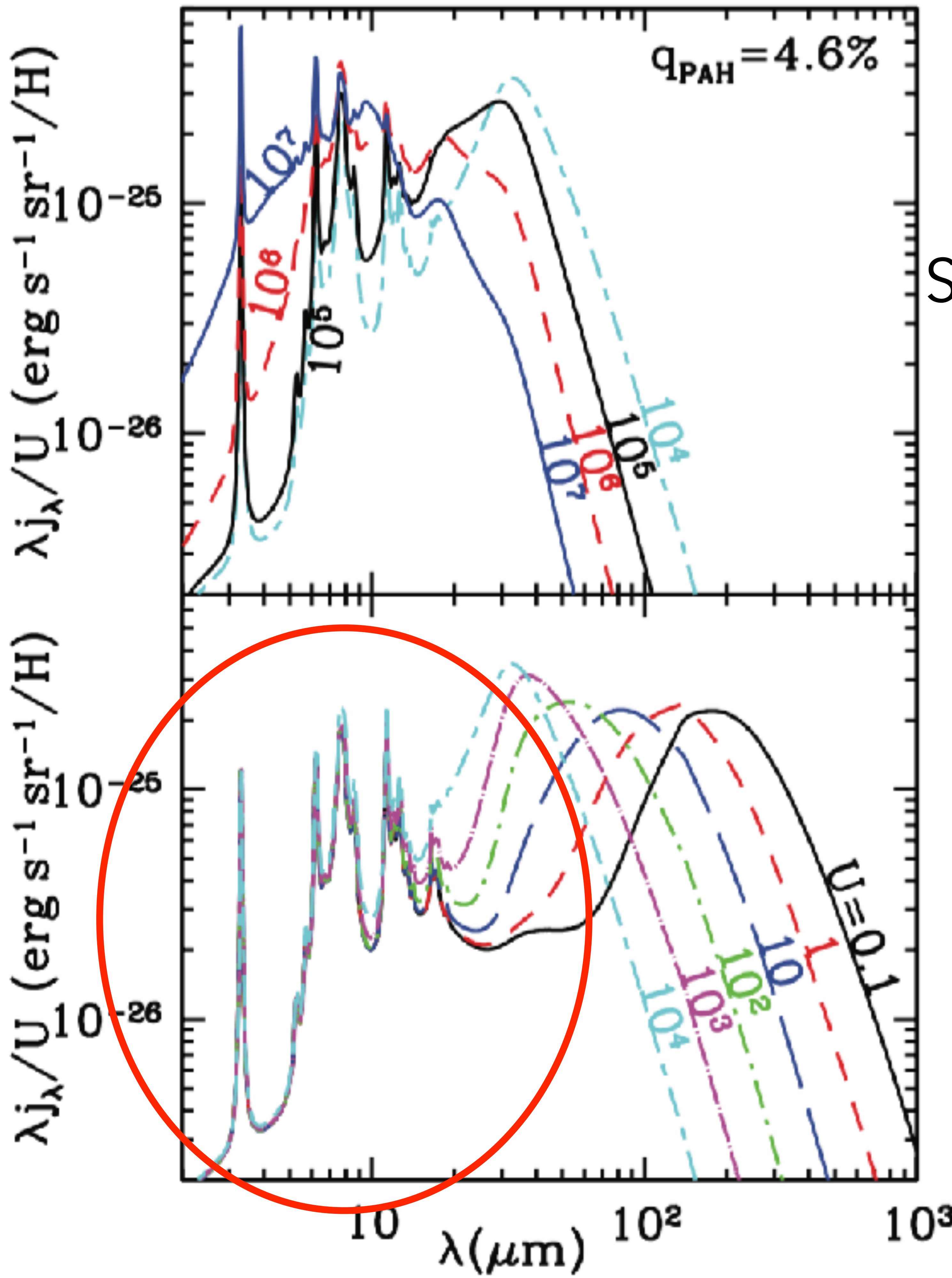
$$S_\lambda = (2.0891 \times 10^{-4}) \kappa_\lambda \Sigma_d B_\lambda. \quad (7)$$



Draine & Li 2007 dust model

As strength of radiation field increases, $T_{d,ss}$ goes up like $U^{1/6}$.

This part of the spectrum is well-described by "modified blackbody"



Draine & Li 2007 dust model

Stochastically Heated Dust: Intensity of radiation field doesn't change shape of spectrum and $j_\nu \propto U$

why:

- temp of small grains depends on average photon energy which isn't changing here (i.e. dP/dT doesn't depend on U)
- grains cool completely between photon absorptions