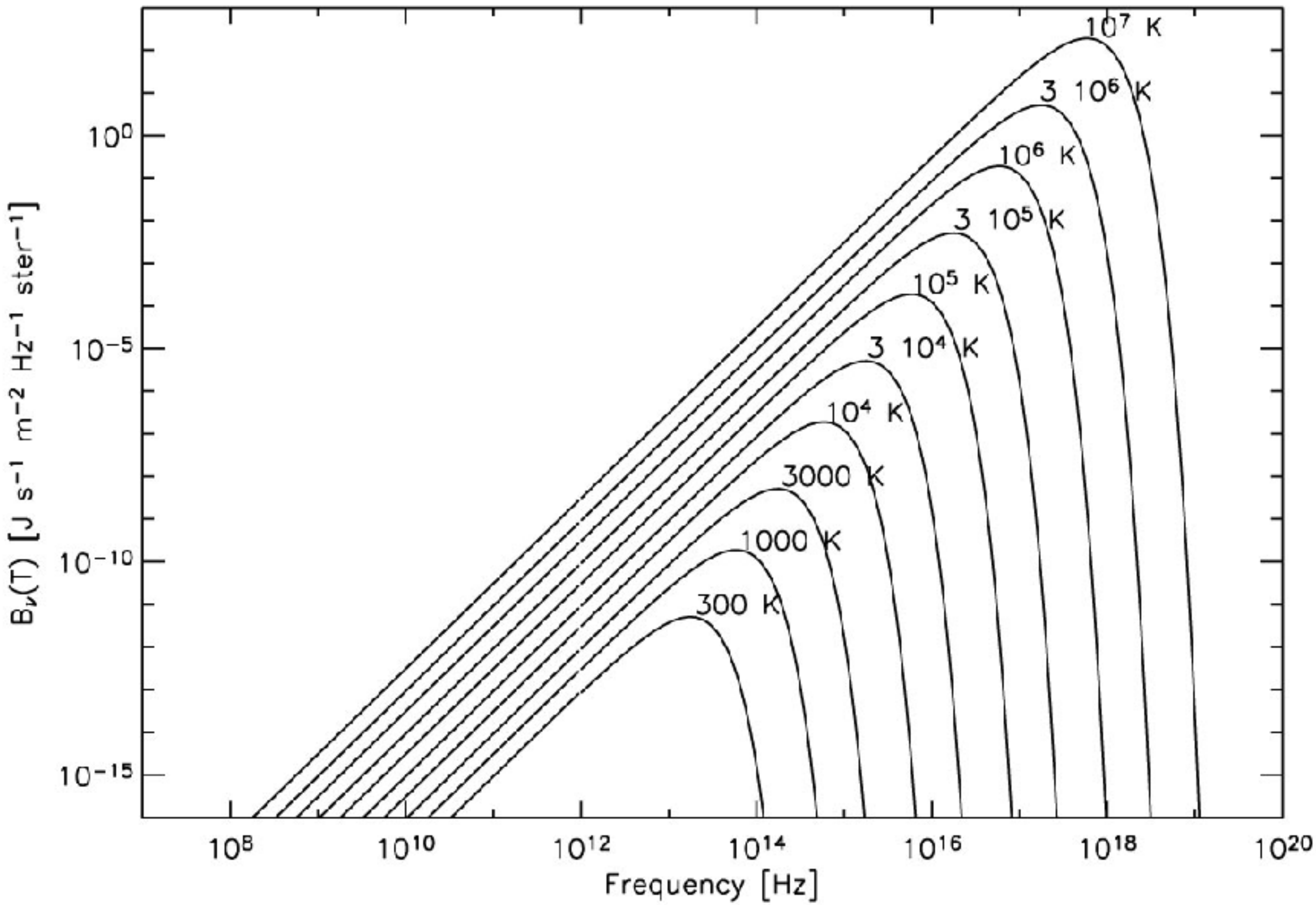
Physics 224
The Interstellar Medium

Lecture #7

- Part I: A note on random walks
- Part II: HI radiative transfer
- Part III: Absorption Lines, more generally
- Part IV (if time): Ionization Processes

Brightness temperature

$$T_b = \frac{h\nu}{k} \ln^{-1}\left(1 + \frac{2h\nu^3}{I_\nu c^2}\right)$$



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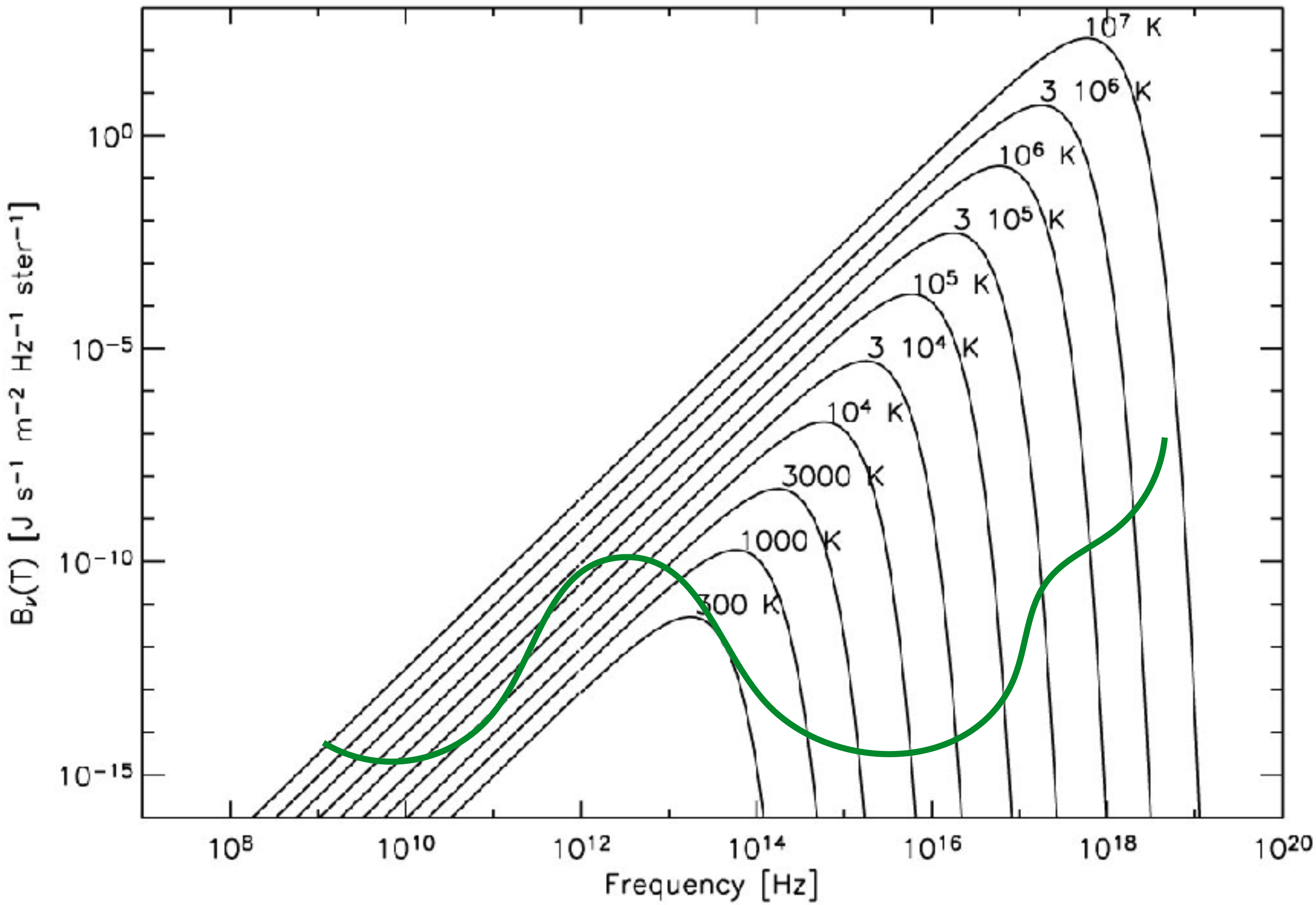

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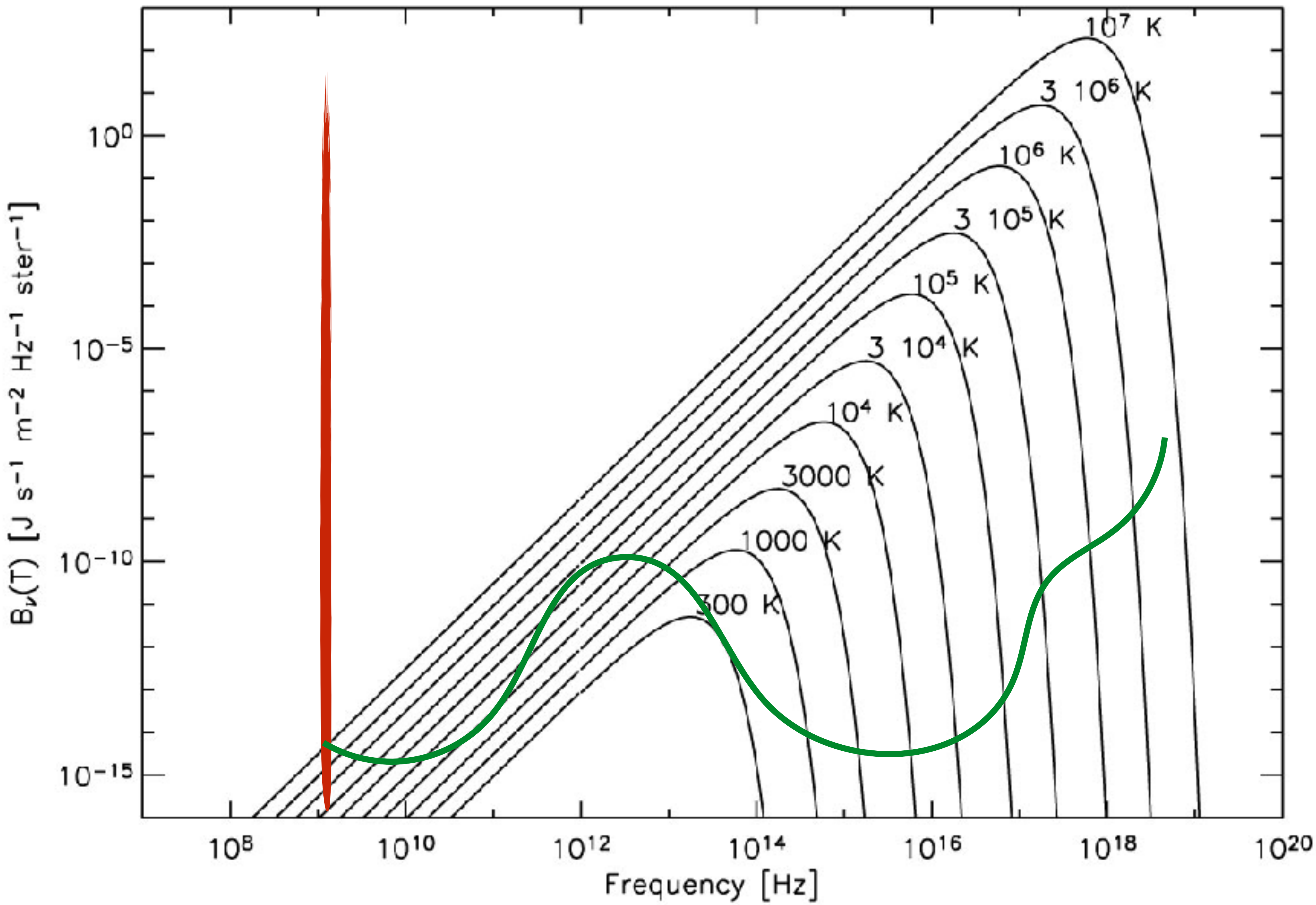

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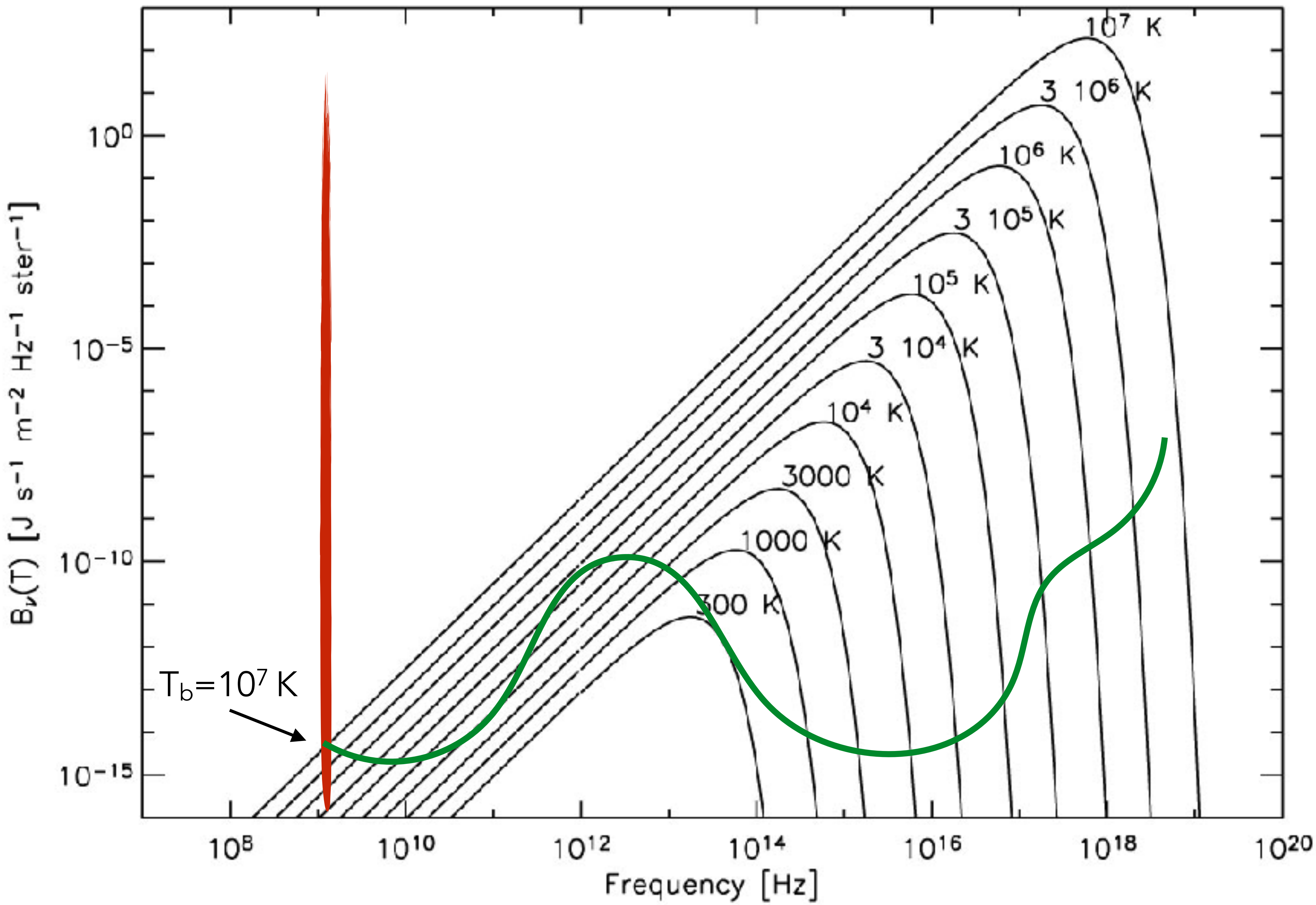

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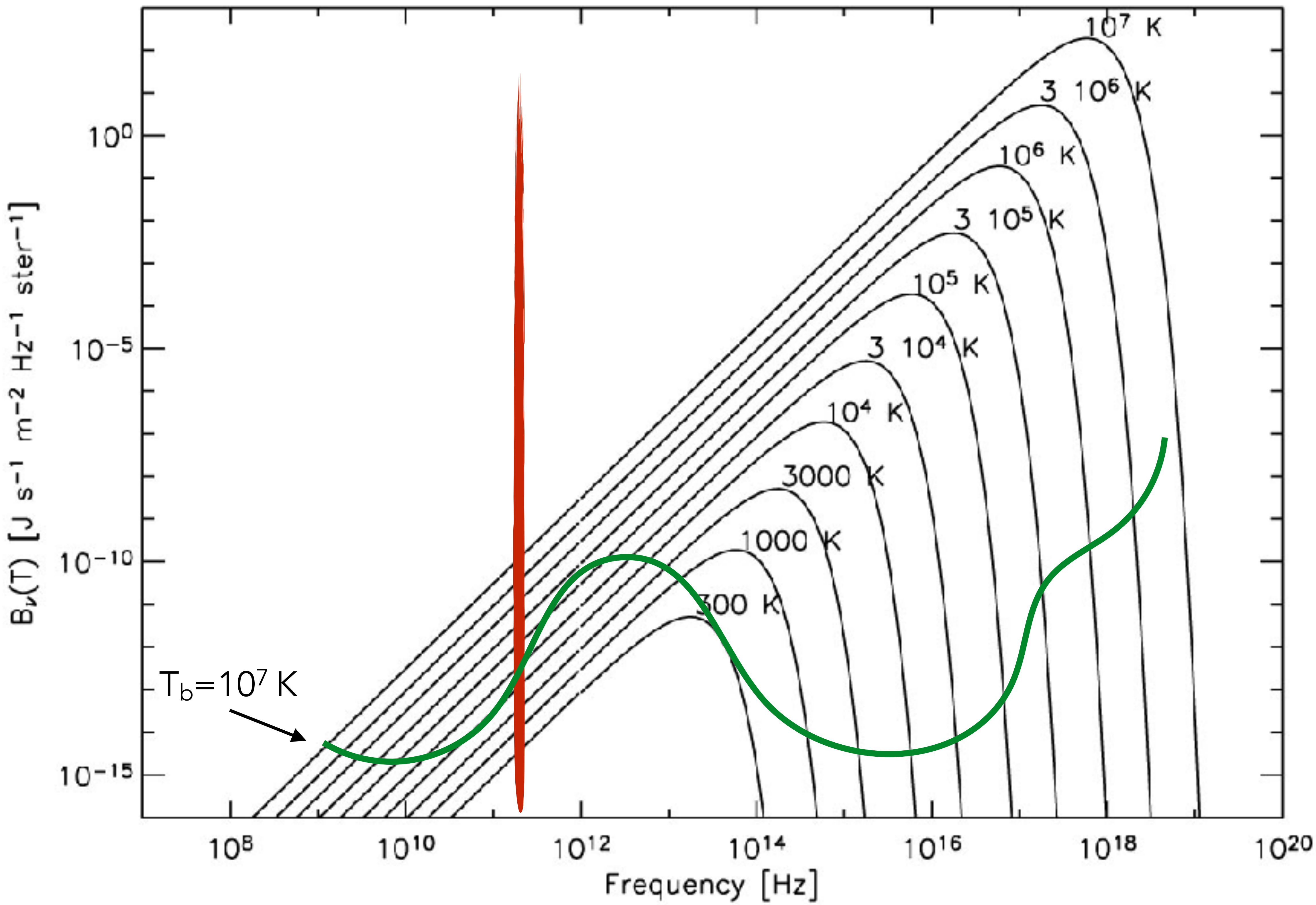

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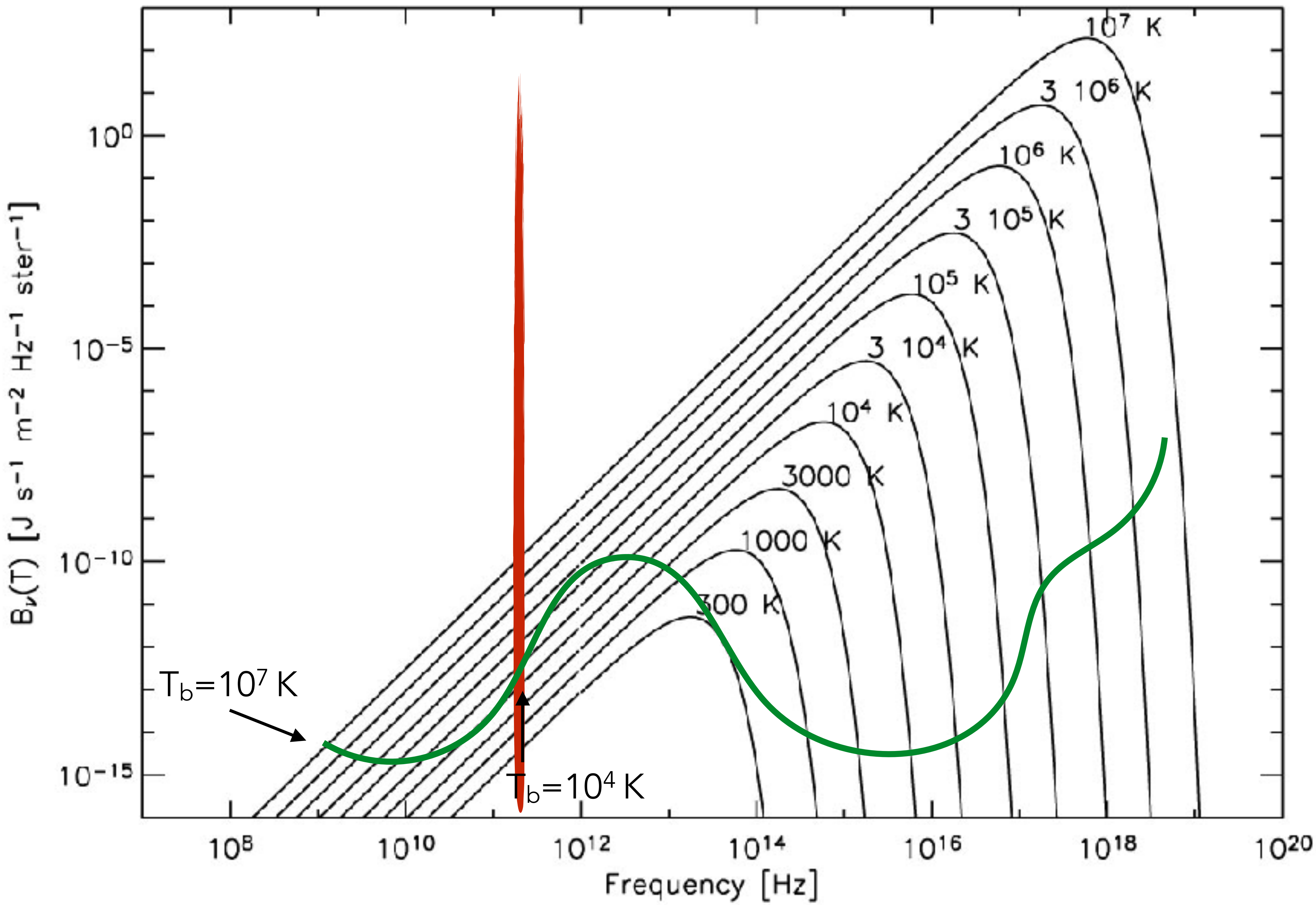

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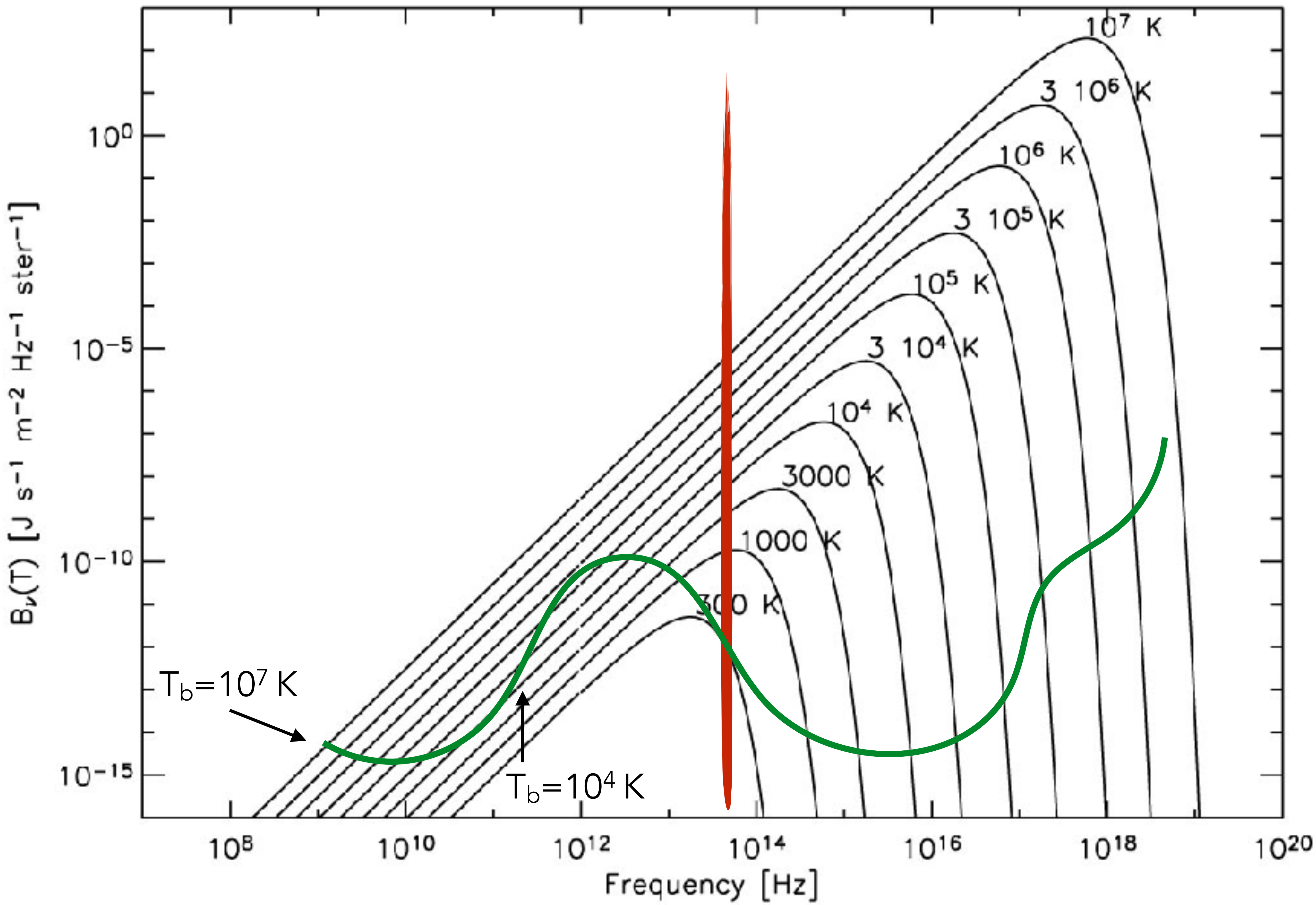

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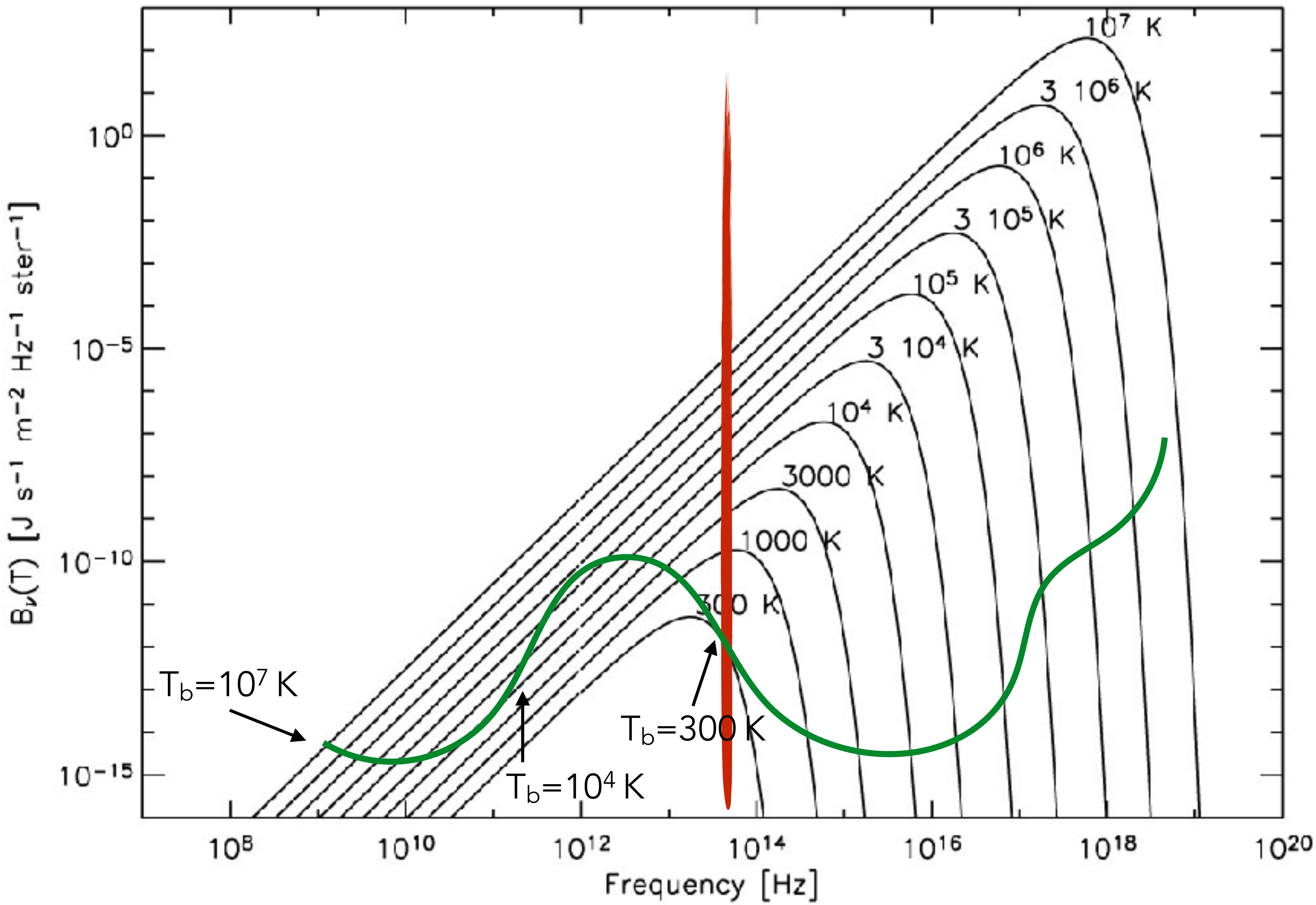

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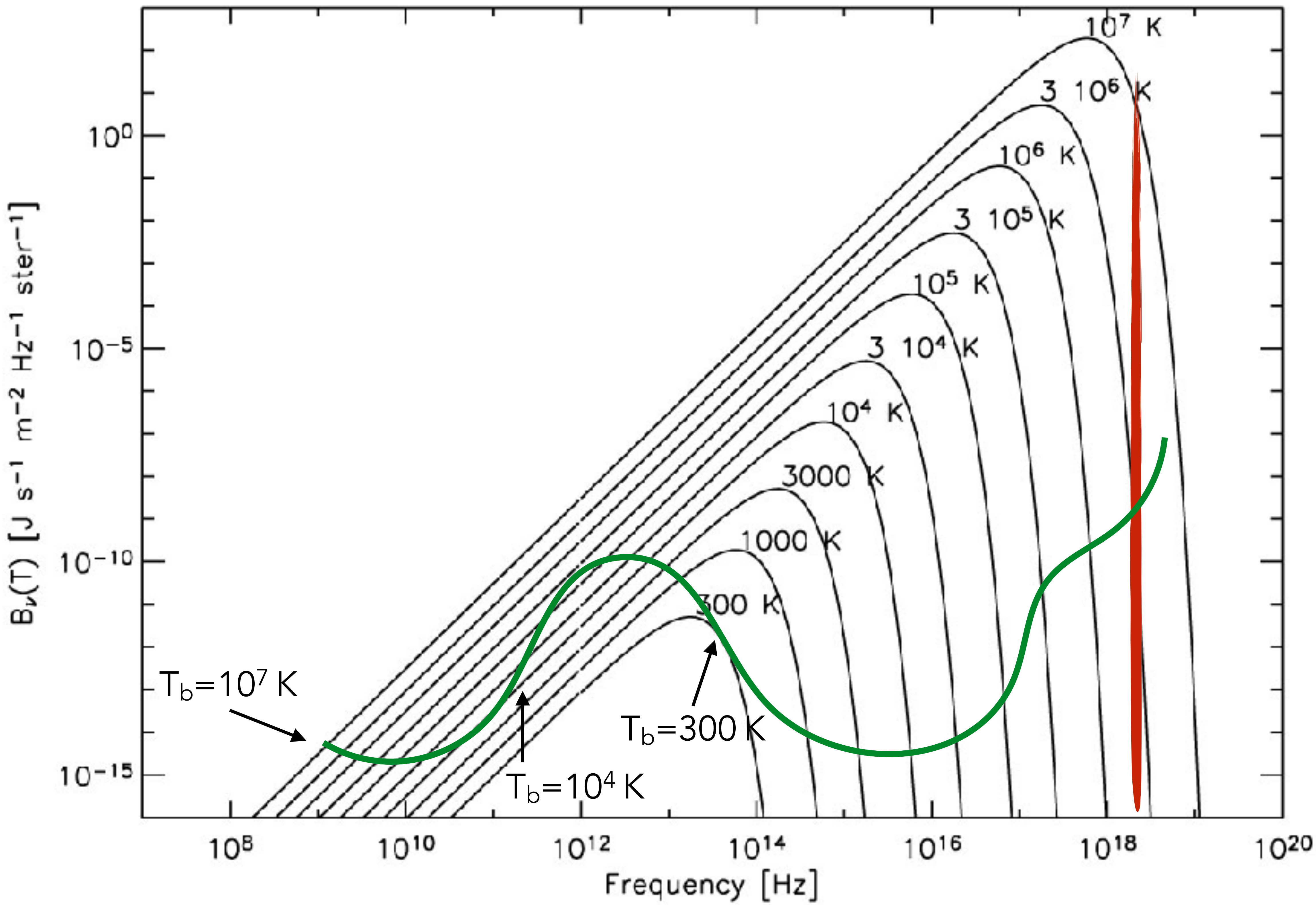

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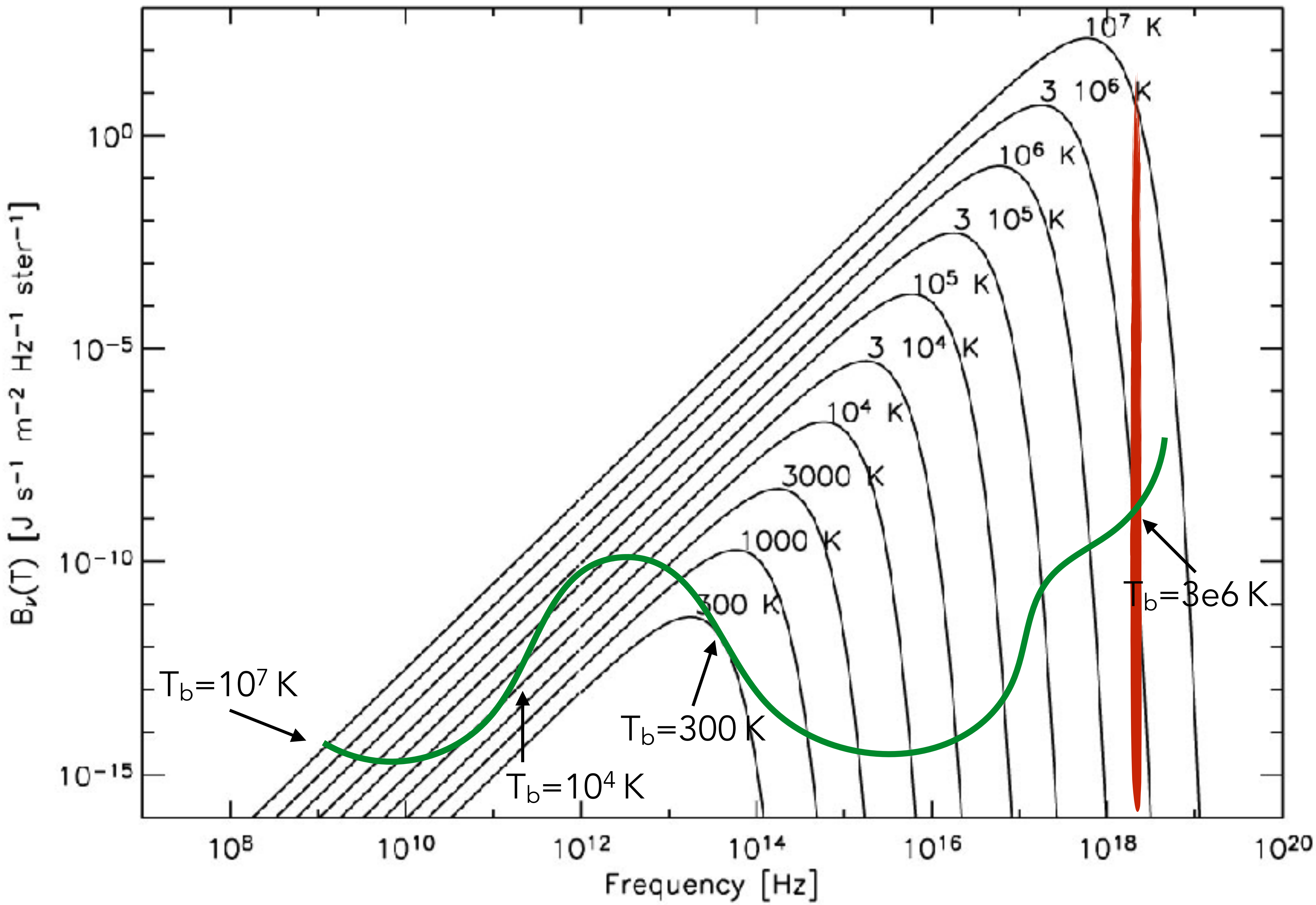

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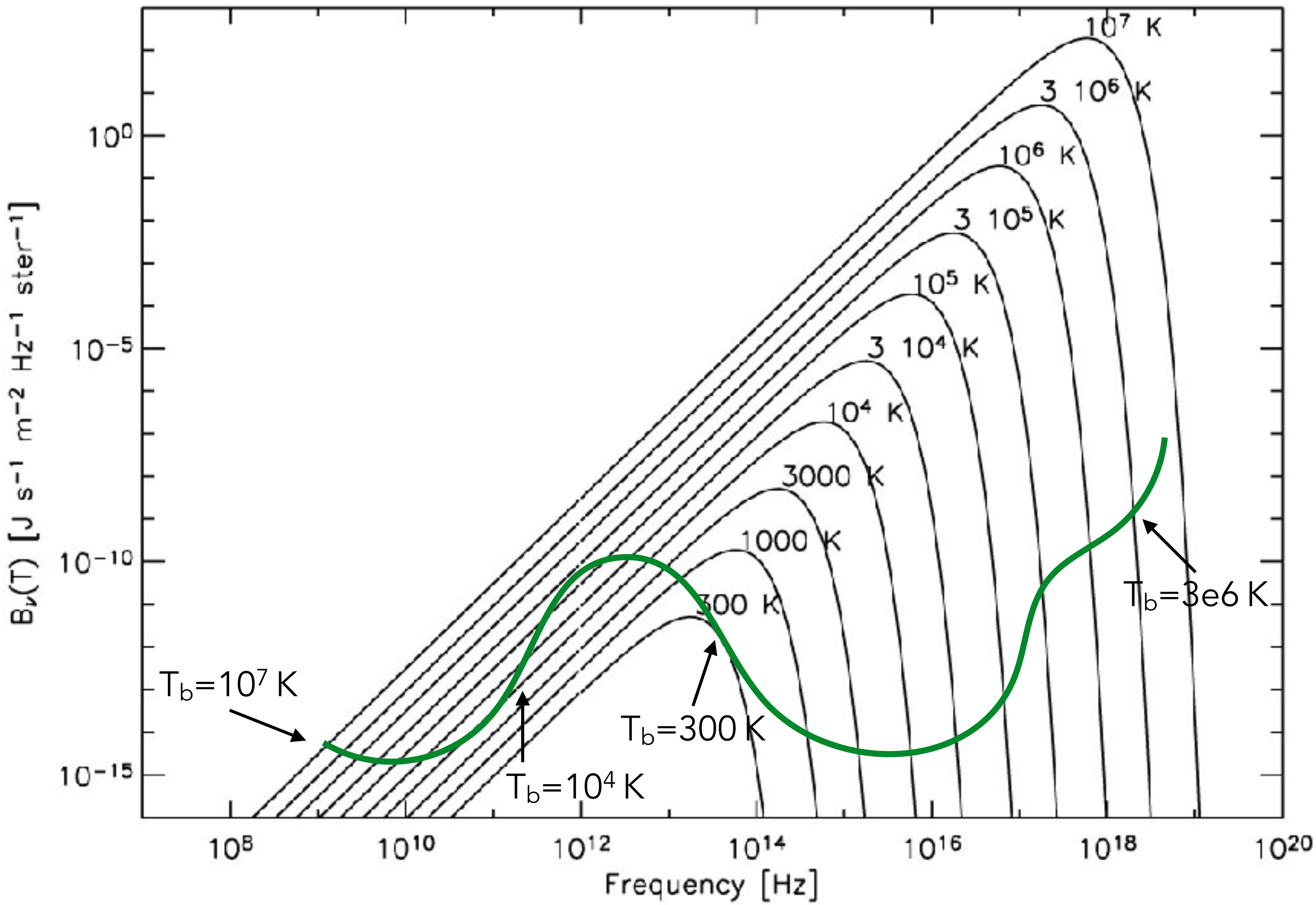
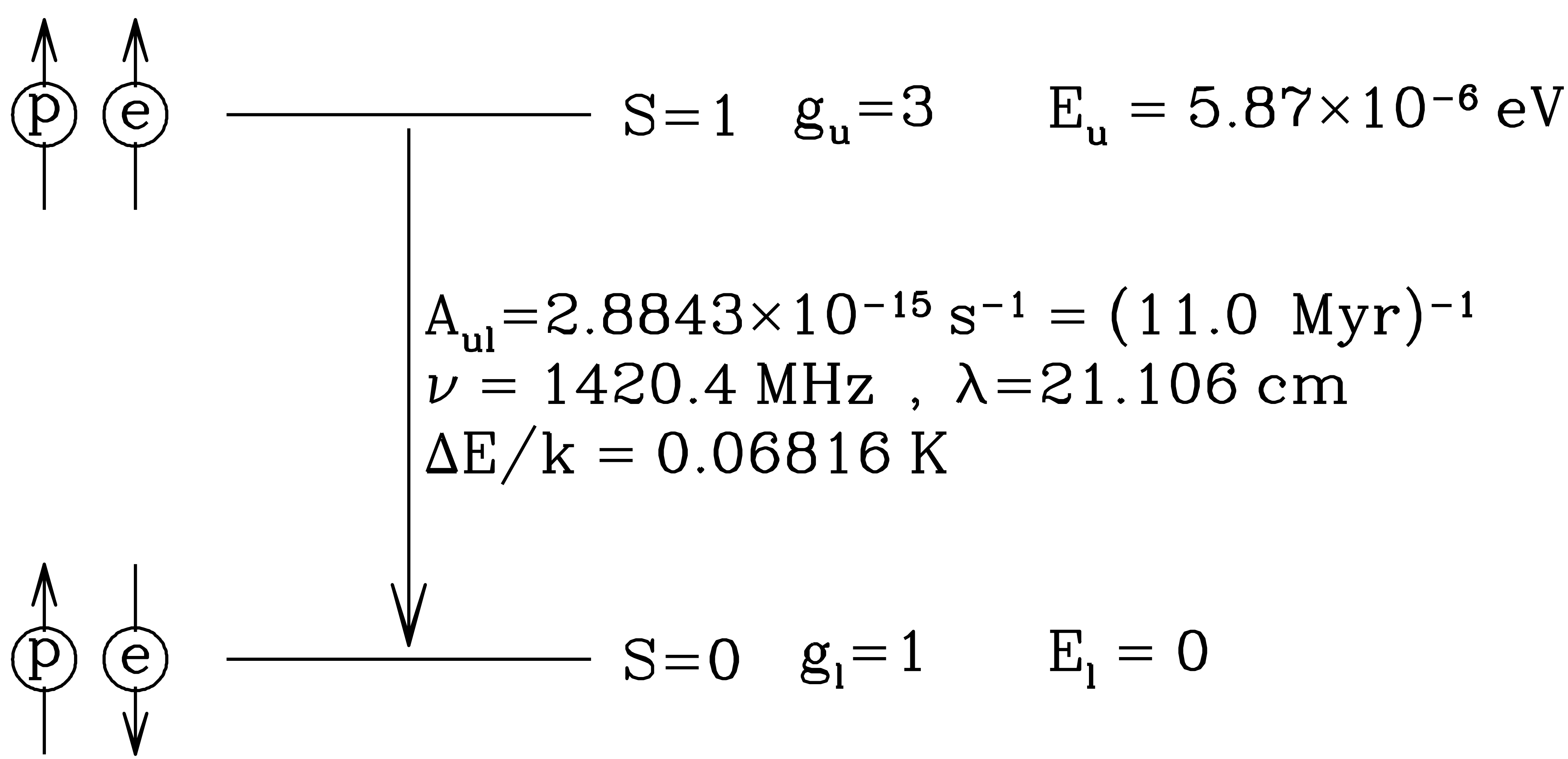


HI 21-cm Radiative Transfer

Hyperfine splitting due to interaction of electron spin and nuclear spin.



*remember: degeneracy = 2(S+1) = 3 for upper

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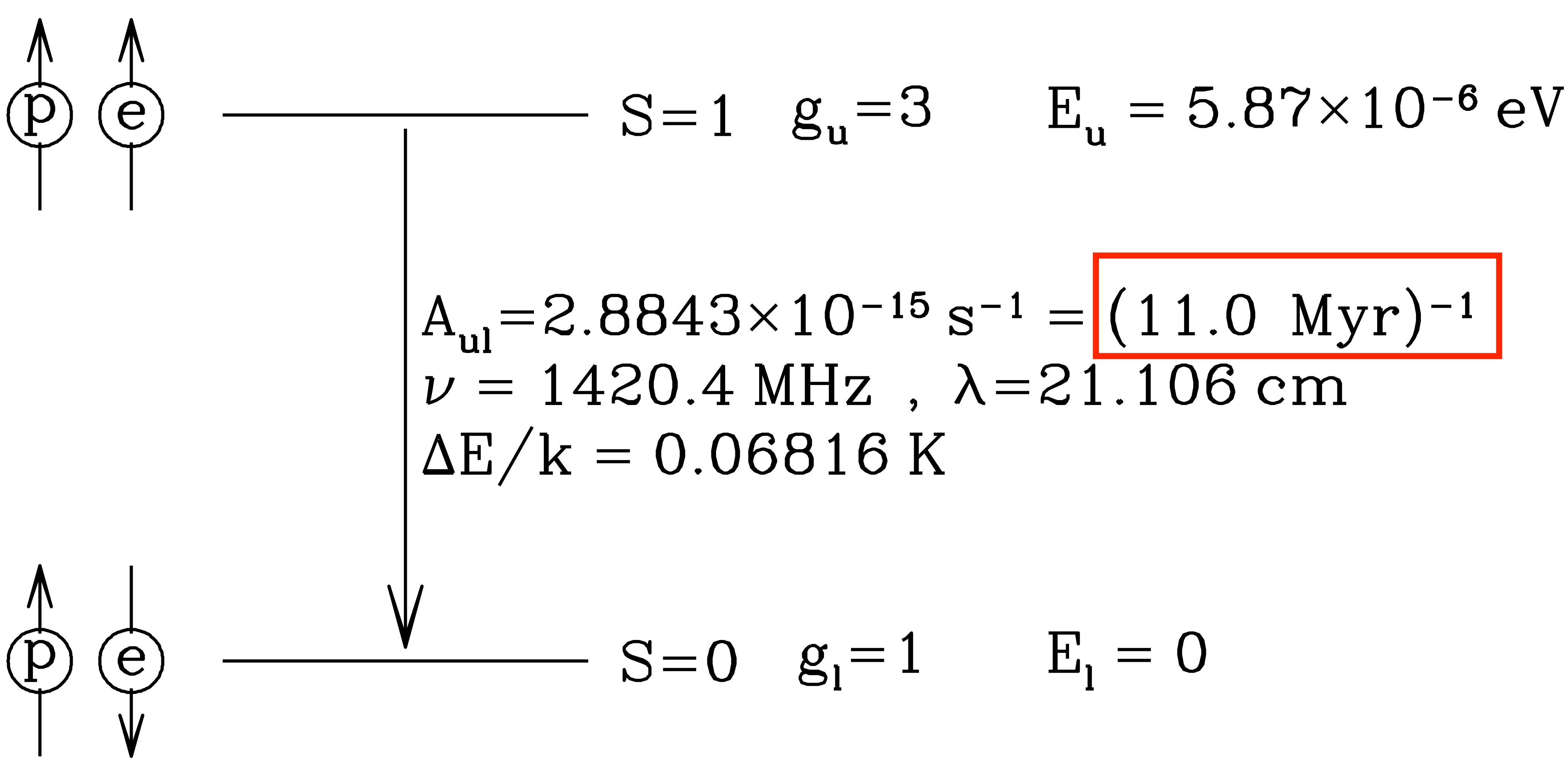

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define $T_{exc} \equiv T_{spin}$

$$\frac{n_u}{n_l} \equiv \frac{g_u}{g_l} e^{-h\nu_{ul}/kT_{exc}}$$

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define $T_{exc} \equiv T_{spin} \gg 0.0682$ K

because cosmic microwave background can populate levels

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Under all ISM conditions, 75% of HI is in upper level.

$$j_\nu = n_u \frac{A_{ul}}{4\pi} h\nu_{ul}\phi_\nu = \frac{3}{16\pi} A_{ul}\ h\nu_{ul}\ n(\mathrm{H\ I})\ \phi_\nu$$

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Emissivity is independent of T_{spin}!!

HI 21-cm Radiative Transfer

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from last week:

$$\frac{g_l B_{lu}}{g_u B_{ul}} = 1$$

$$B_{ul} = \frac{c^3}{8\pi h \nu^3} A_{ul}$$

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to get:

$$e^{-E_{ul}/kT_{spin}} = 1 - h\nu_{ul}/kT_{spin}$$

HI 21-cm Radiative Transfer

put all this together and we find:

$$\kappa_\nu \approx \frac{3}{32\pi} A_{ul} \frac{hc\lambda_{ul}}{kT_{spin}} n(\mathrm{H\ I})\phi_\nu$$

absorption coefficient depends inversely on T_{spin}
this is a consequence of stimulated emission not being negligible!

Line Profile ϕ_ν determined by two processes:

1) Natural Broadening
2) Doppler Broadening

Natural Broadening: from Uncertainty principle $\Delta E \Delta t \geq \hbar$

Δt = lifetime of state

Doppler Broadening: from spread in velocity of particles in the gas

Natural Broadening results in a Lorentz profile (approximately)

$$\phi_\nu \approx \frac{4\gamma_{u\ell}}{16\pi^2(\nu - \nu_{u\ell})^2 + \gamma_{u\ell}^2},$$

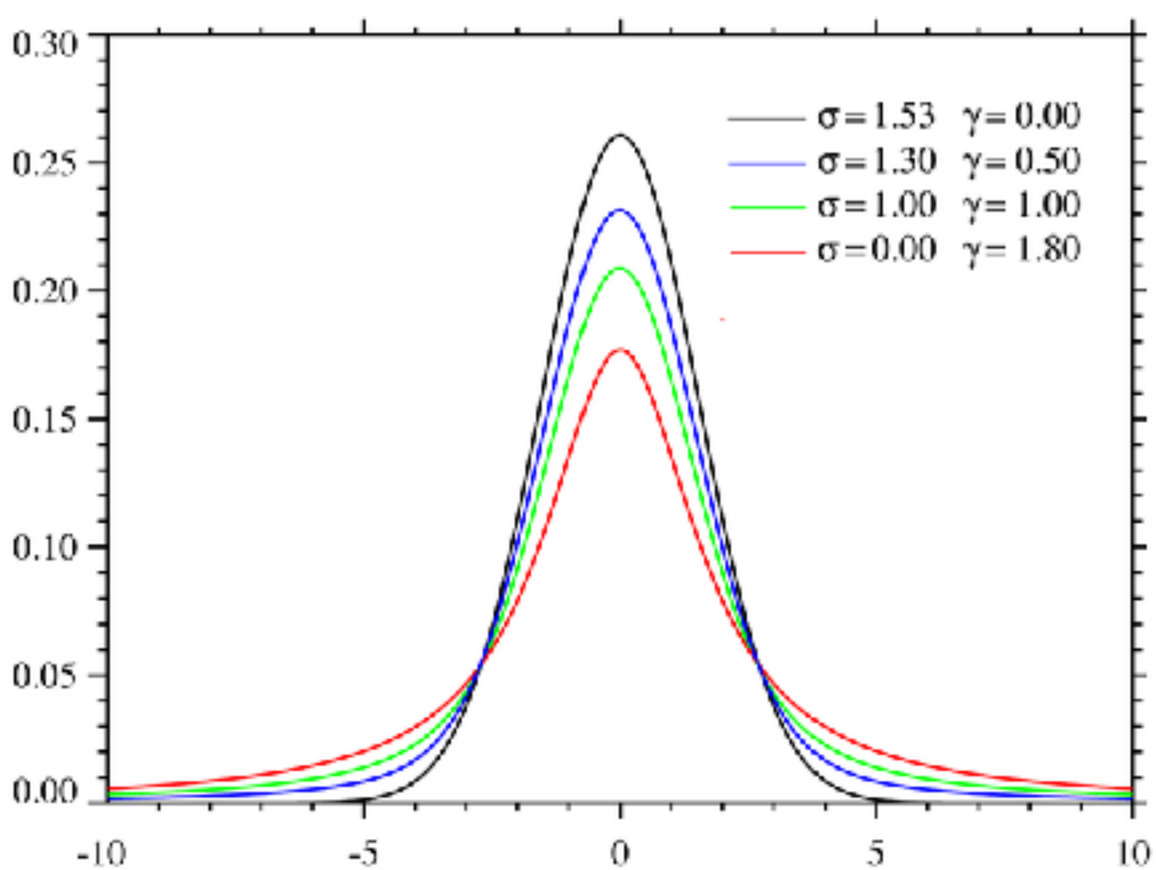
where:

$$\gamma_{u\ell} = \sum_{j<u} A_{uj} + \sum_{j<\ell} A_{\ell j}.$$

Is a sum of all of the relevant lifetimes (~1/A) for the energy levels you are transitioning between.

Doppler Broadening means that Lorentz profile is convolved with the velocity dispersion of the gas.

$$\phi_\nu = \frac{1}{\sqrt{2\pi\sigma_v^2}} \int_{-\infty}^{\infty} e^{-v^2/2\sigma_v^2} \frac{4\gamma_{u\ell}}{16\pi^2(\nu - (1 - v/c)\nu_{u\ell})^2 + \gamma_{u\ell}^2} dv$$



This gives you a "Voigt" profile.

HI 21-cm Radiative Transfer

two parts of line profile ϕ_ν:

- natural broadening ← Depends on lifetime (i.e. Einstein A value) of transition

$$(\Delta v)^{\text{intr.}}_{\text{FWHM}} = \frac{\gamma_{ul}}{2\pi} \sim \frac{A_{ul}}{2\pi}$$

- Doppler broadening

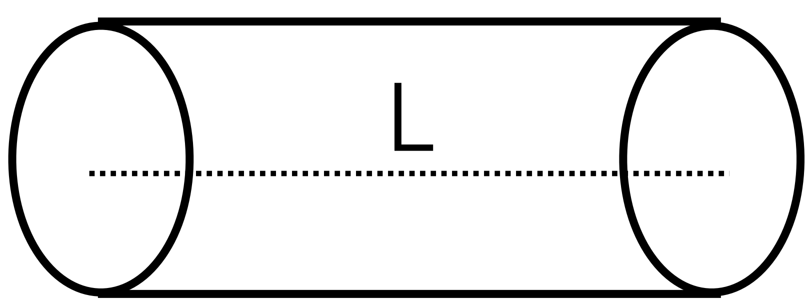
Because lifetime of 21-cm transition is SO LONG natural broadening is negligible.

Line profile depends on velocity dispersion.

Column Density

$$N(\mathrm{H\ I}) \equiv \int ds\ n(\mathrm{H\ I})$$

for uniform volume density N = nL



n = # particles cm^{-3}

column density is
number of particles
per unit area
along a path length L

Column Density - Optical Depth

$$N(\mathrm{H\ I}) \equiv \int ds\ n(\mathrm{H\ I})$$

$$\tau_\nu = \int \kappa_\nu ds$$

$$\kappa_\nu \approx \frac{3}{32\pi} A_{ul} \frac{hc\lambda_{ul}}{kT_{spin}} n(\mathrm{H\ I})\phi_\nu$$

assume T_{spin}, σ

are constant along path length L

$$\tau_\nu \sim \mathrm{constant}\ \frac{1}{T_{spin}} N(\mathrm{HI})\phi_\nu$$

HI 21-cm Radiative Transfer

$$\tau_\nu = 2.19 \left(\frac{N(\mathrm{H\ I})}{10^{21}\mathrm{cm}^{-2}}\right)\left(\frac{T_{spin}}{100K}\right)^{-1}\left(\frac{\sigma_v}{\mathrm{kms}^{-1}}\right)^{-1} e^{-u^2/2\sigma_v}$$

u = velocity difference from line center

Lots of regions where optical depth in HI can be significant.

HI 21-cm Radiative Transfer

In the optically thin case $\tau_\nu \ll 1$

$$I_\nu = I_\nu(0) + \int j_\nu ds = I_\nu(0) + \frac{3}{16\pi} A_{ul} \; h\nu_{ul} \; \phi_\nu \; N(\mathrm{H\ I})$$

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Say $I_\nu(0) = 0$

$$\boxed{\int I_\nu\ d\nu = \frac{3}{16\pi} A_{ul}\ h\nu_{ul}\ N(\mathrm{H\ I})}$$

can get N(HI) from integral of line!

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Note: radio astronomers often convert I_ν to brightness temperature

UV/Opt/NIR Transitions

For most such lines $E_{ul} \gg kT_{exc}$

$$\kappa_\nu = \frac{h\nu}{4\pi} n_l B_{lu} \phi_\nu \left(1 - e^{-E_{ul}/kT_{exc}}\right)$$

This means upper level is generally not populated, so

- stimulated emission is negligible!
- $n \sim n_l$ (i.e. few atoms in upper level)
- not seen in emission

Look in <u>absorption</u> against background light sources.

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Absorption Lines

In that case, we can integrate κ_ν over the path length (s) to get optical depth and show:

$$\tau_\nu = \frac{\pi e^2}{m_e c} f_{lu} \; N_l \; \phi_\nu$$

"oscillator strength" related to Einstein coeff

column density of absorbers

$$A_{ul} = \frac{8\pi^2 e^2 \nu_{lu}^2}{m_e c^3} \frac{g_l}{g_u} f_{lu}$$

Absorption Lines

General line w/o stimulated emission:

$$\tau_\nu = \text{const.}\ N_l\ \phi_\nu$$

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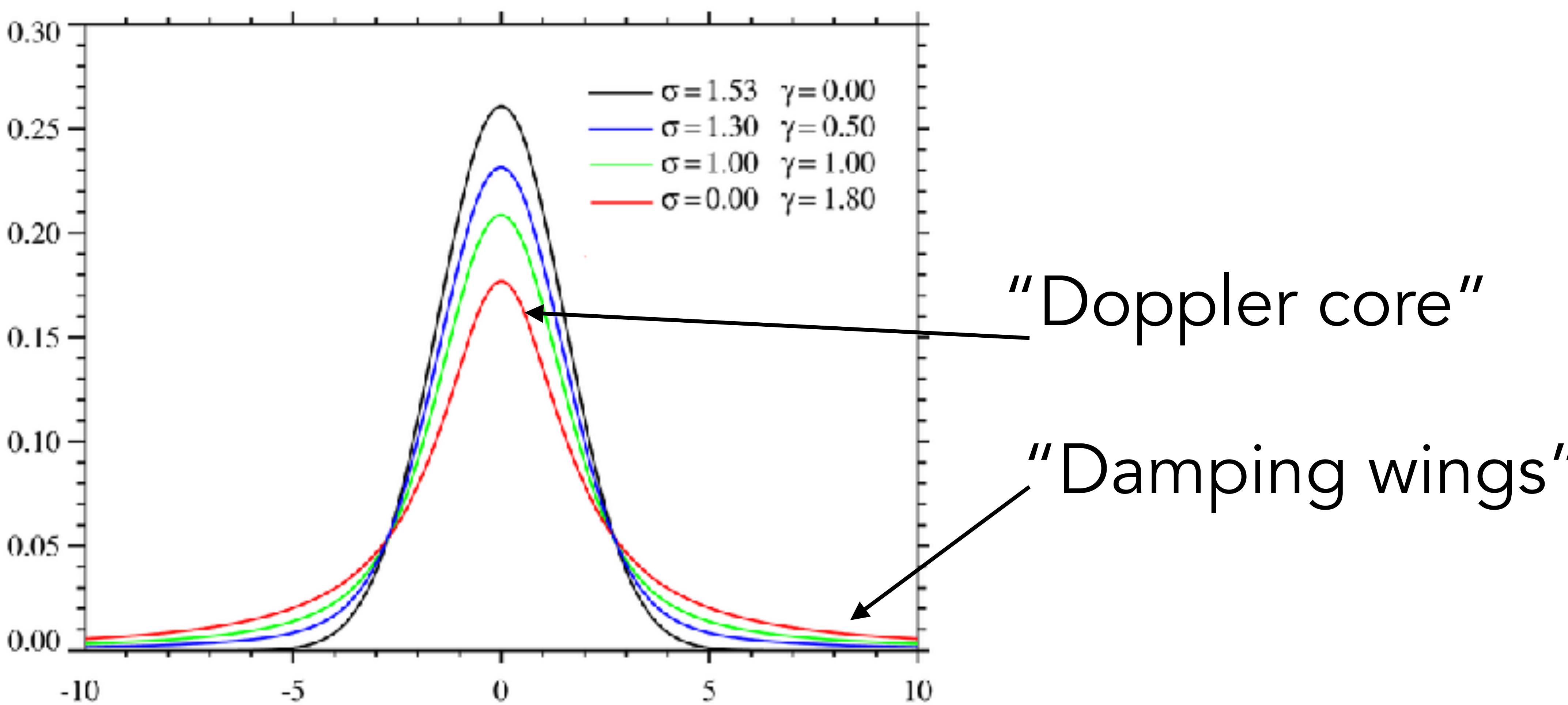
line profile

Absorption Lines

Voigt Profile: convolution of Lorentz & Gaussian

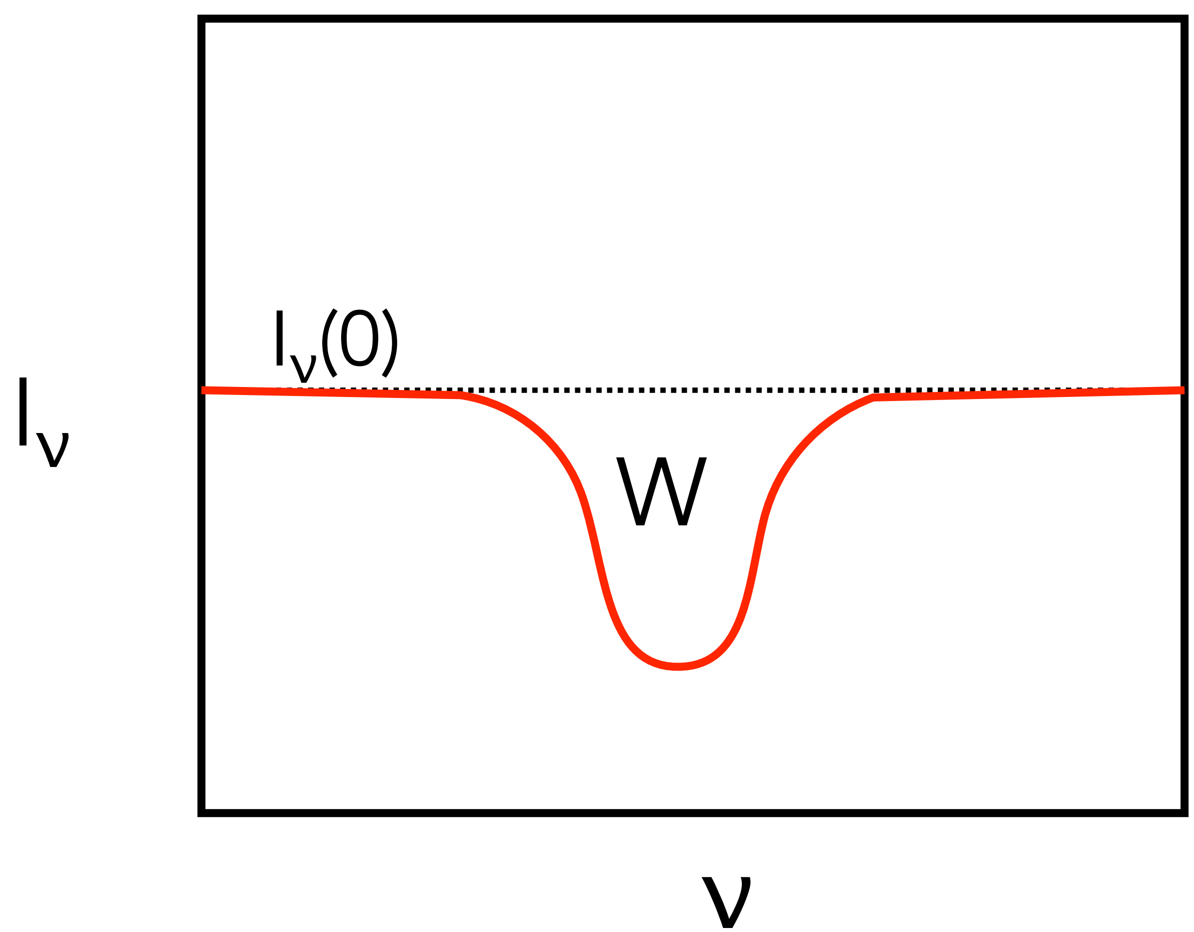
$$\phi_\nu = \frac{1}{\sqrt{2\pi\sigma_v^2}} \int_{-\infty}^{\infty} e^{-v^2/2\sigma_v^2} \frac{4\gamma_{u\ell}}{16\pi^2(\nu - (1 - v/c)\nu_{u\ell})^2 + \gamma_{u\ell}^2} \, dv$$



Absorption Lines

Define "equivalent width" of a line:

$$W_\nu \equiv \int \frac{I_\nu(0) - I_\nu}{I_\nu(0)} d\nu = \int (1 - e^{-\tau_\nu}) d\nu$$



Why is this useful?
if we know what $I_\nu(0)$ is and absorption is happening in a narrow freq range,
we can relate EW to τ_ν

Absorption Lines

When $\tau_\nu \ll 1$, Taylor expansion of $1 - e^{-\tau_\nu}$

$$W_\nu \approx \int 1 - (1 - \tau_\nu) d\nu \approx \int \tau_\nu d\nu$$

often use wavelength units in optical astronomy:

$$W_\lambda = W_\nu \frac{\lambda}{\nu} \qquad \frac{W_\lambda}{\lambda_0} = \int \tau_\nu \frac{d\nu}{\nu_0}$$

line center

Absorption Lines

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Absorption Lines

When $\tau_\nu \ll 1$, Taylor expansion of $1 - e^{-\tau_\nu}$

For Gaussian: $$\tau_\nu = \tau_0 e^{-(u/b)^2}$$ with: $$u = c(\nu_0 - \nu)/\nu_0$$ $$b = \sqrt{2}\sigma_V$$

where: $$\tau_0 = \sqrt{\pi}\frac{e^2}{m_e c}\frac{N_l f_{lu} \lambda_{lu}}{b}$$ is optical depth at line center

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For H Lyman α:

$$\tau_0 = 0.7580 \left(\frac{N_l}{10^{13}\mathrm{cm}^{-2}}\right)\left(\frac{f_{lu}}{0.4164}\right)\left(\frac{\lambda_{lu}}{1215.7\mathring{A}}\right)\left(\frac{10\ \mathrm{km\ s}^{-1}}{b}\right)$$

Absorption Lines

Voigt Profile: convolution of Lorentz & Gaussian

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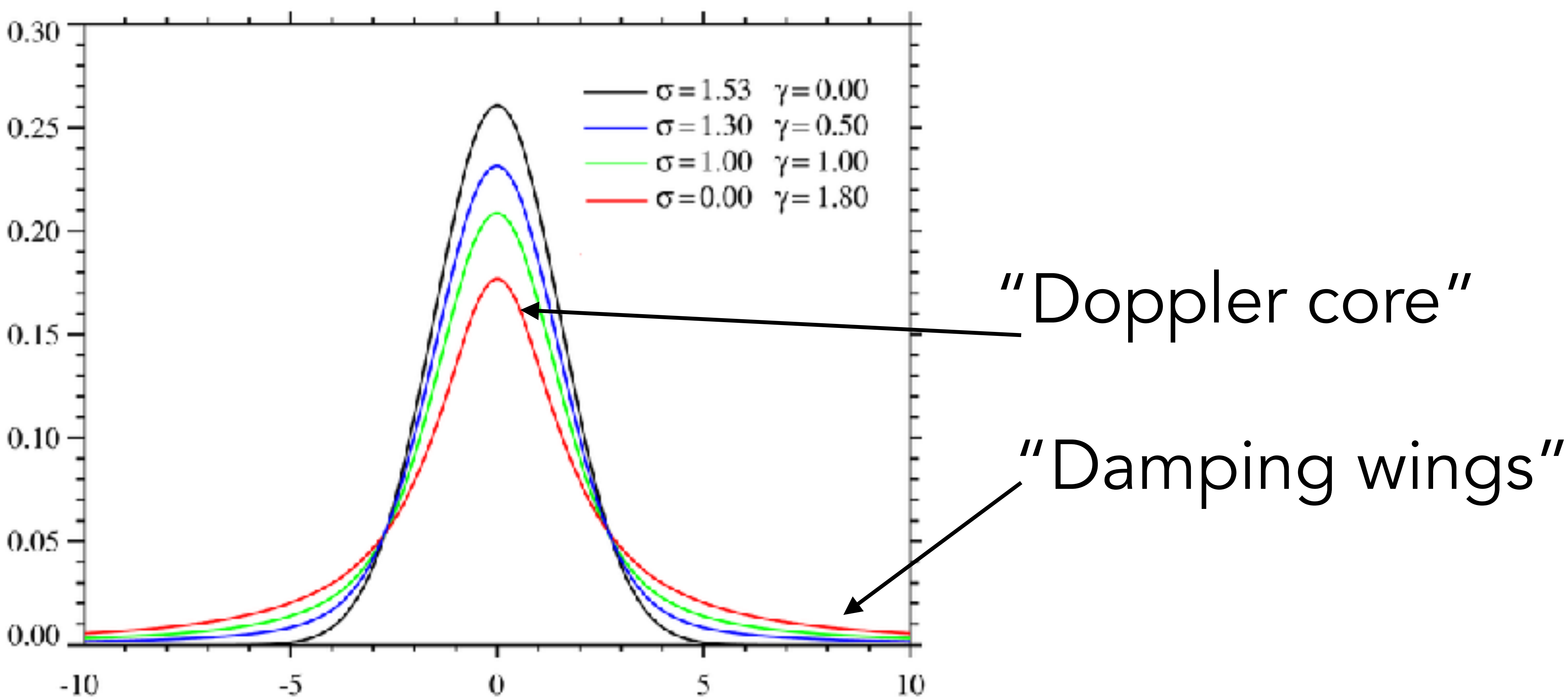
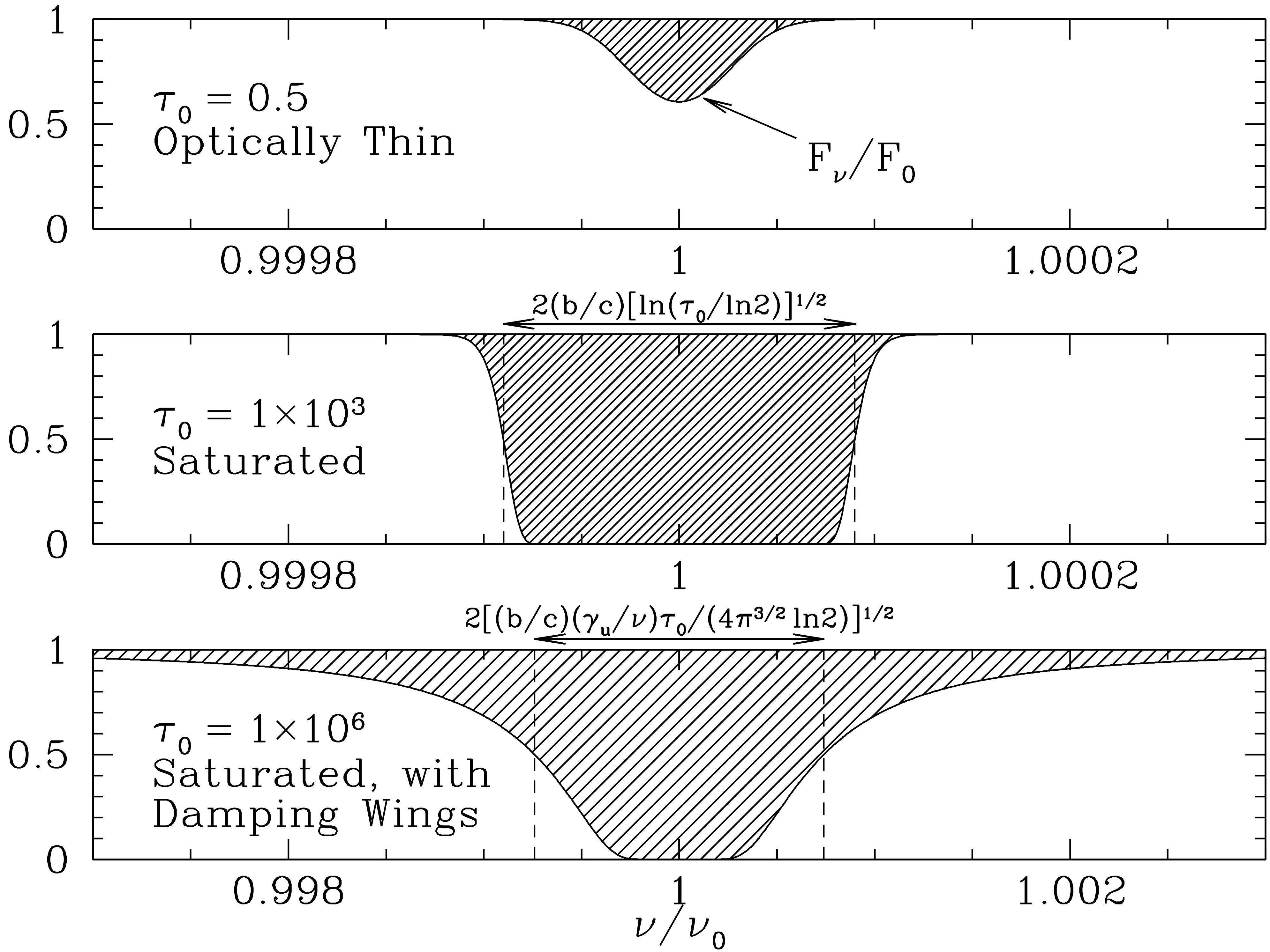


Absorption Lines



Absorption Lines

Online demo…

https://www.sns.ias.edu/~ting/lyman_alpha.html