Physics 224
The Interstellar Medium

Lecture #9

- Part I: HII Regions

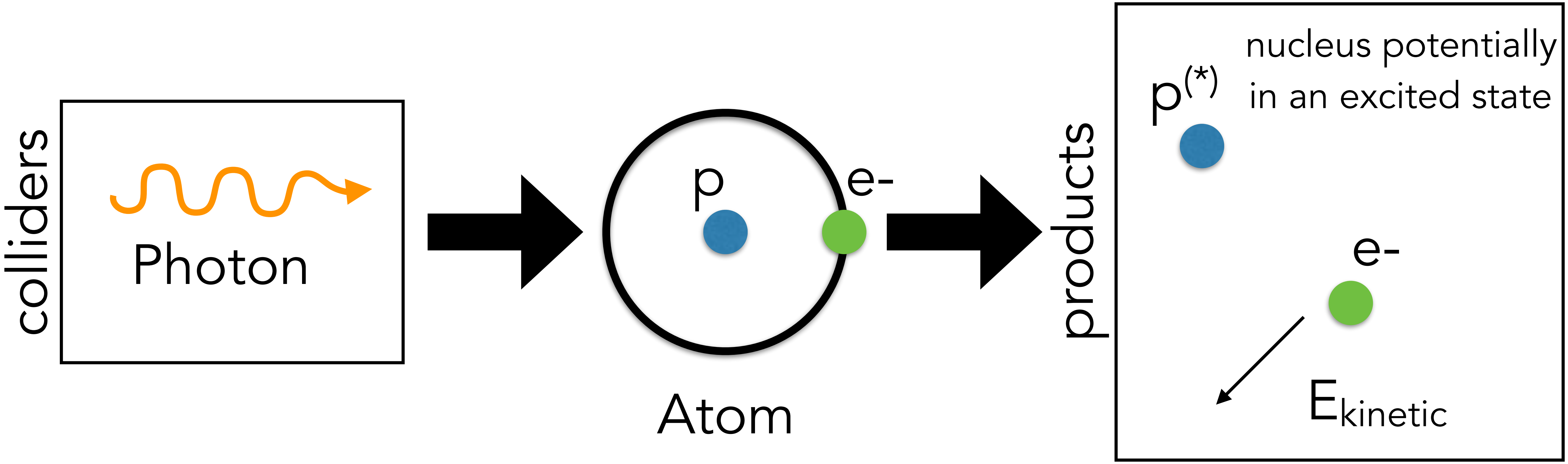
- Part II: Collisional Excitation

- Part III: Nebular Diagnostics

HII Region: Photoionized gas surrounding young, hot stars

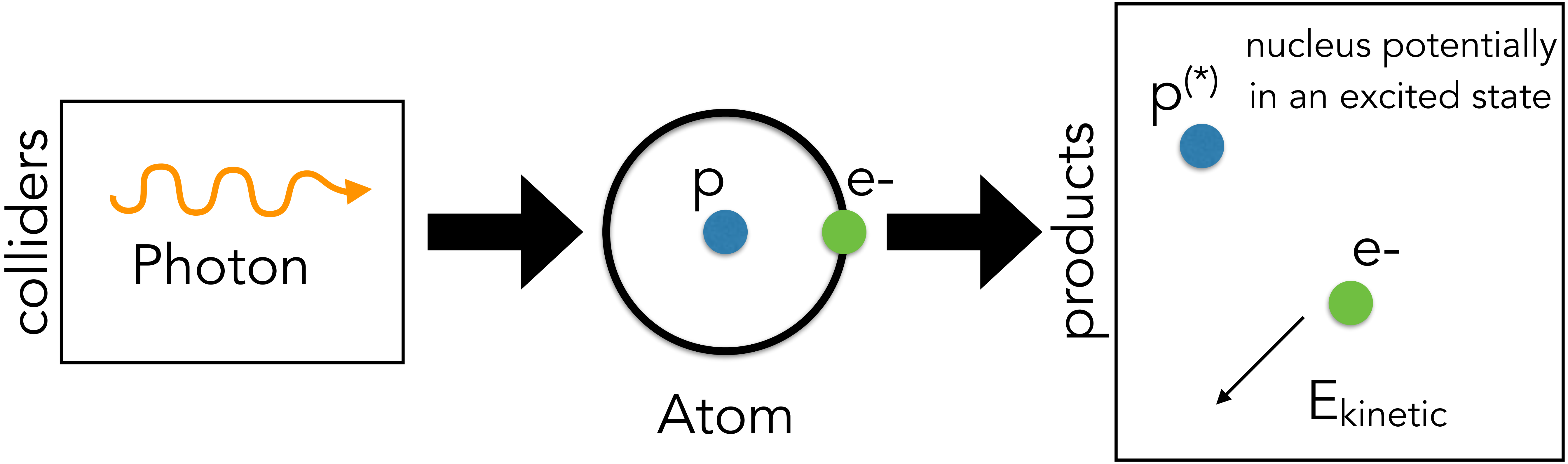
Credit: NASA,ESA, M. Robberto (Space Telescope Science Institute/ESA) and the Hubble Space Telescope Orion Treasury Project Team

Photoionization of H



rate per volume ~ $n_{atom}\, n_{collider}\, \sigma\, c$

Photoionization of H



colliders

Photon

p

e-

Atom

products

$p^{(*)}$

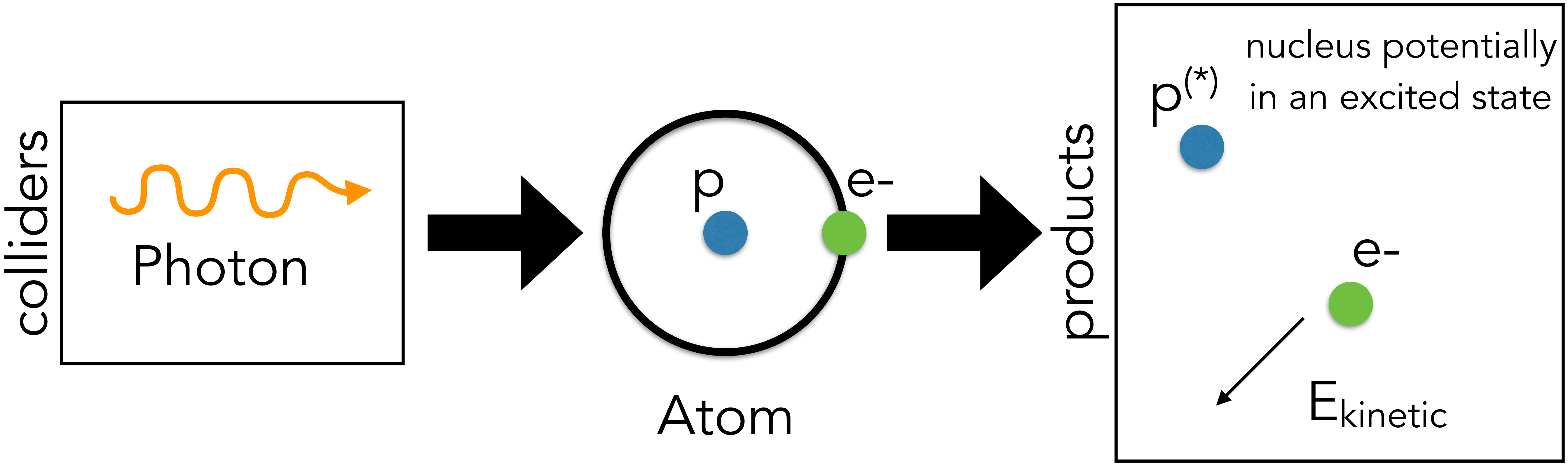
nucleus potentially in an excited state

e-

$E_{kinetic}$

$$\text{rate per volume} \sim n_{atom} \boxed{n_{collider}\ \sigma\ c}$$

Photoionization of H



$$\text{rate per volume} \sim n_{\text{atom}} \boxed{n_{\text{collider}}\, \sigma\, c}$$

$$\zeta_{\text{pi}} = \int_{\nu_1}^{\infty} \sigma_{\text{pi}}(\nu) c \frac{u_\nu}{h\nu} d\nu$$

Cross section can be determined analytically for Hydrogen (and “hydrogenic” ions - those with 1 e- remaining)

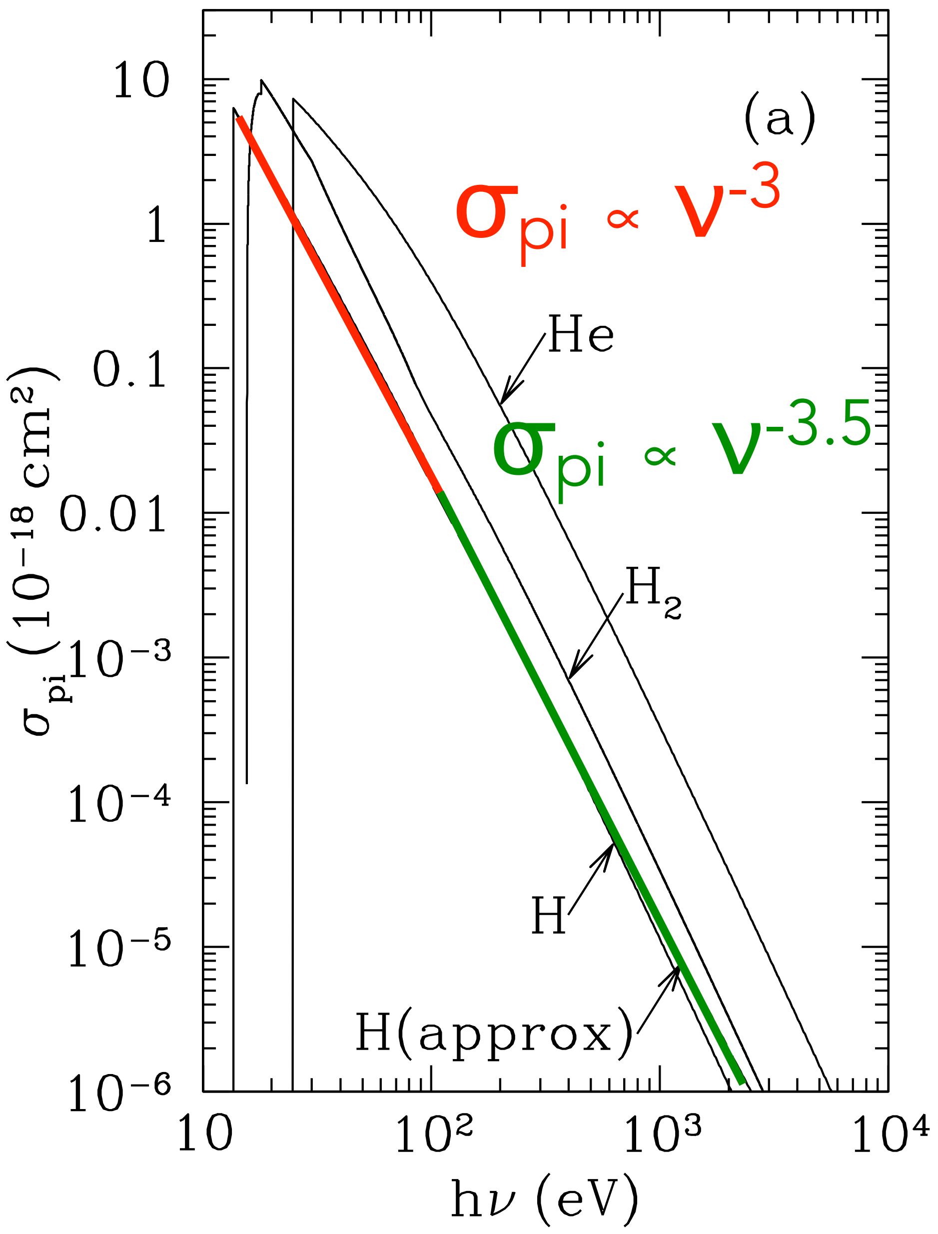
when $h\nu > 13.6\ Z^2$ eV

$$\sigma_{\rm pi}(\nu) = \sigma_0 \left(\frac{Z^2 I_{\rm H}}{h\nu}\right)^4 \frac{e^{4-(4\tan^{-1}x)/x}}{1-e^{-2\pi/x}}$$

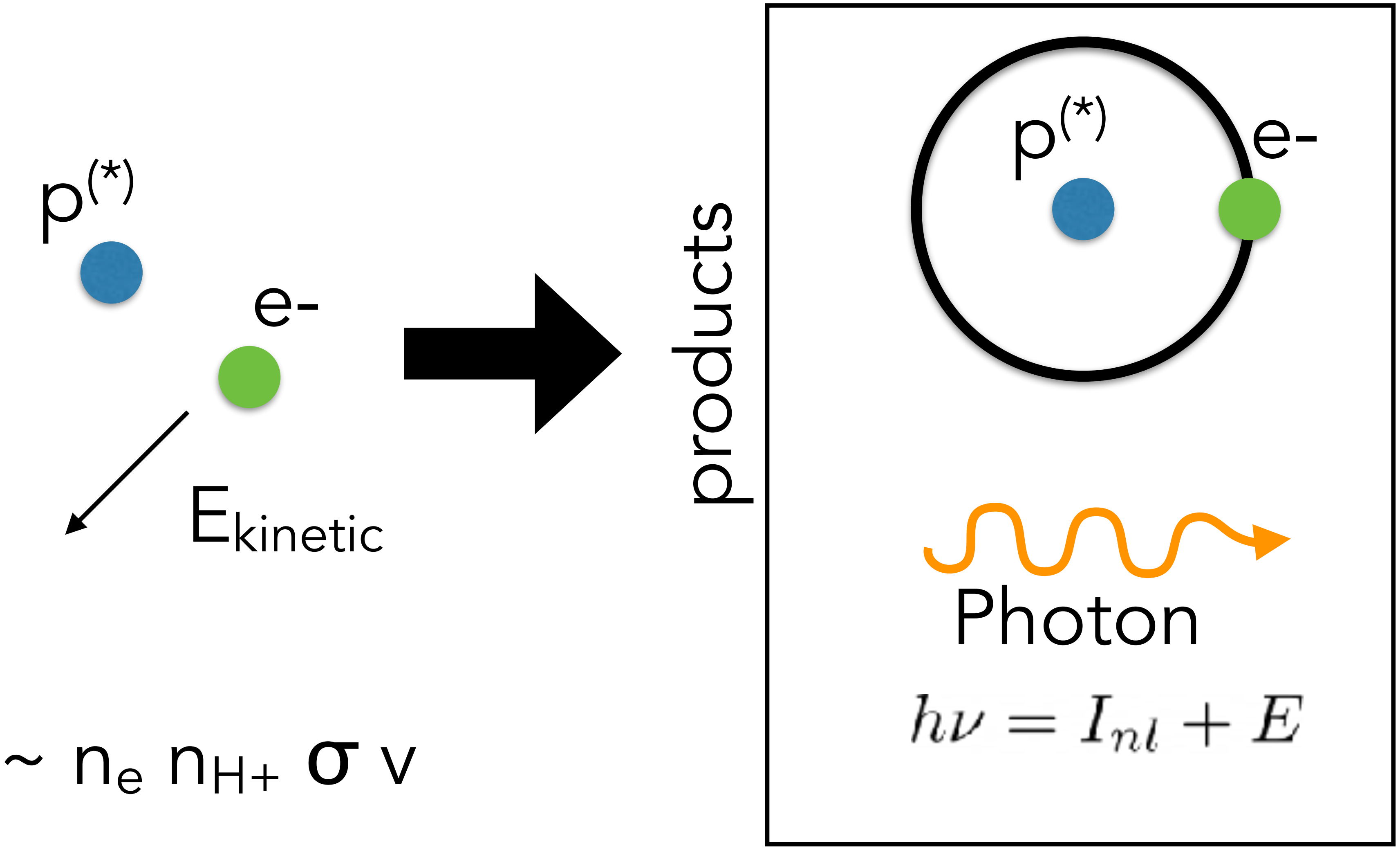
where: $x = \sqrt{\frac{h\nu}{Z^2 I_{\rm H}} - 1}$

and “cross section at threshold” is

$$\sigma_0 = \frac{2^9\pi}{3e^4} Z^{-2} \alpha \pi a_0^2 = 6.304 \times 10^{-18} Z^{-2}\ \mathrm{cm}^{-2}$$

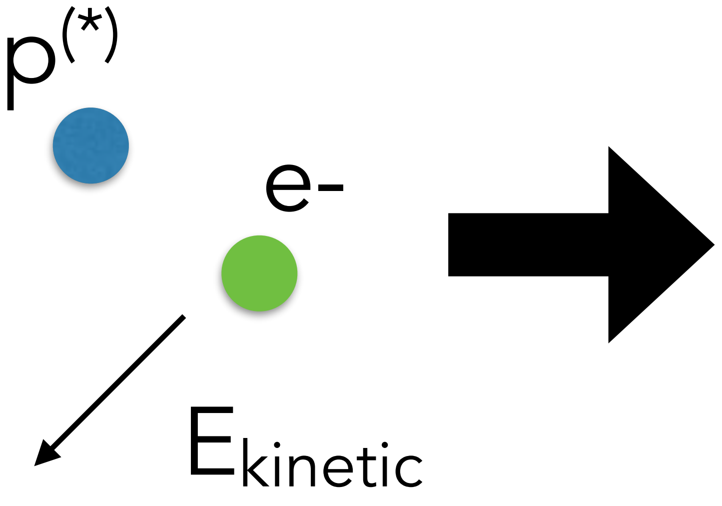


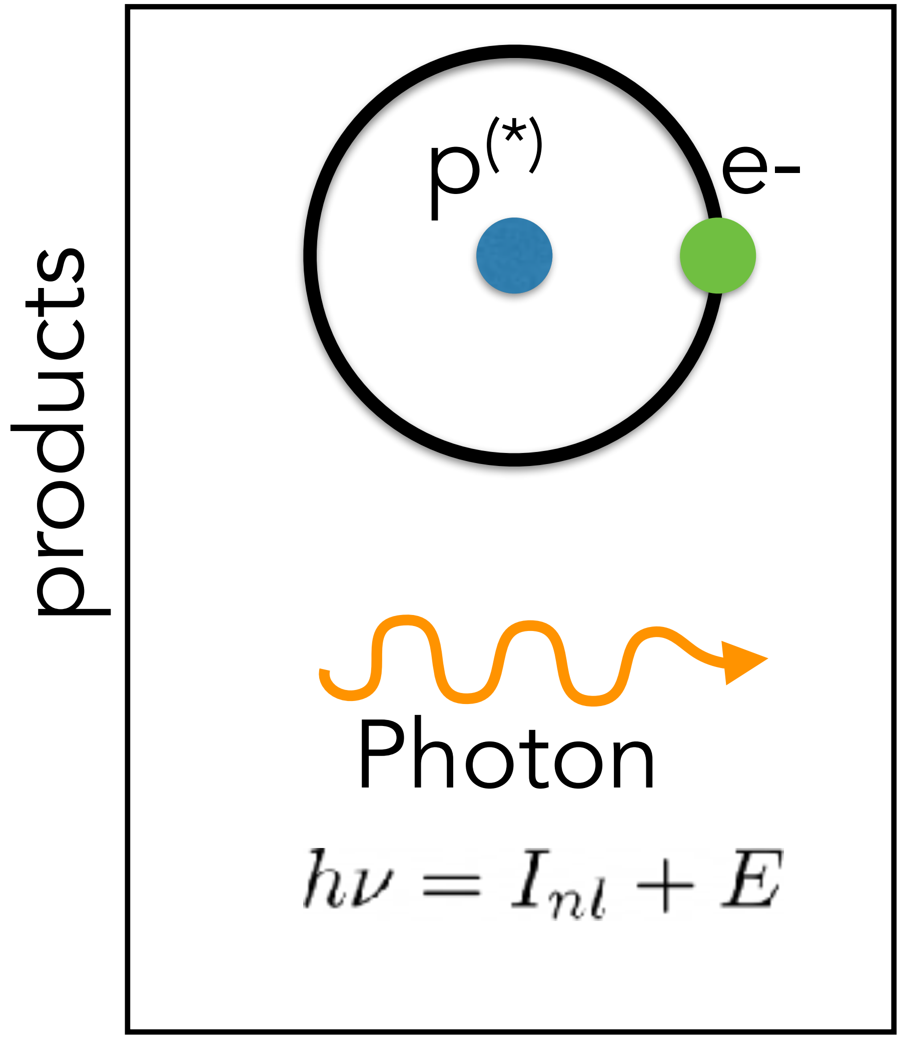
Radiative Recombination of H



rate per volume ~ $n_e \, n_{H+} \, \sigma \, v$

Radiative Recombination of H

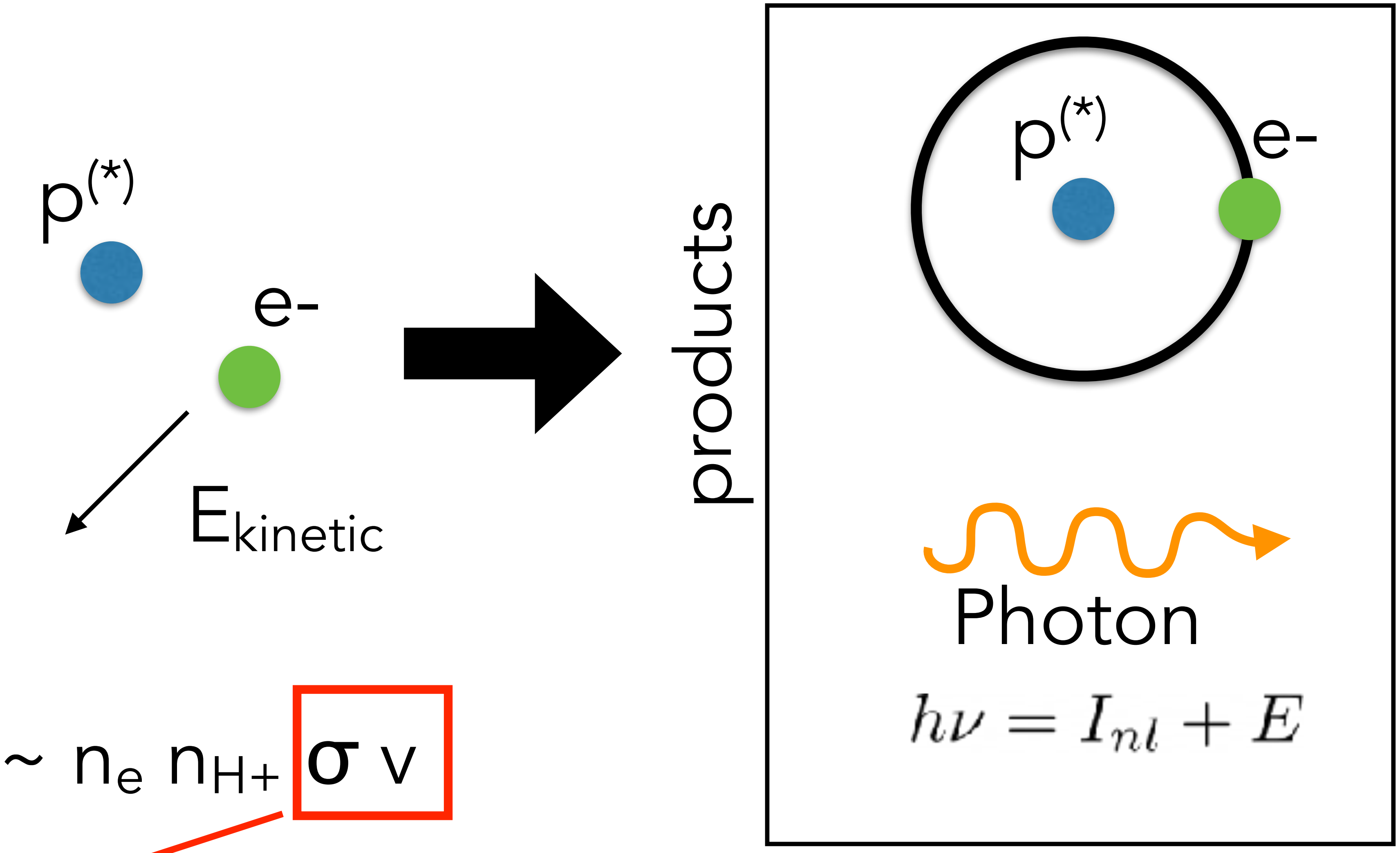




$$h\nu = I_{nl} + E$$

rate per volume ~ $n_e\, n_{H+}\, \boxed{\sigma\, v}$

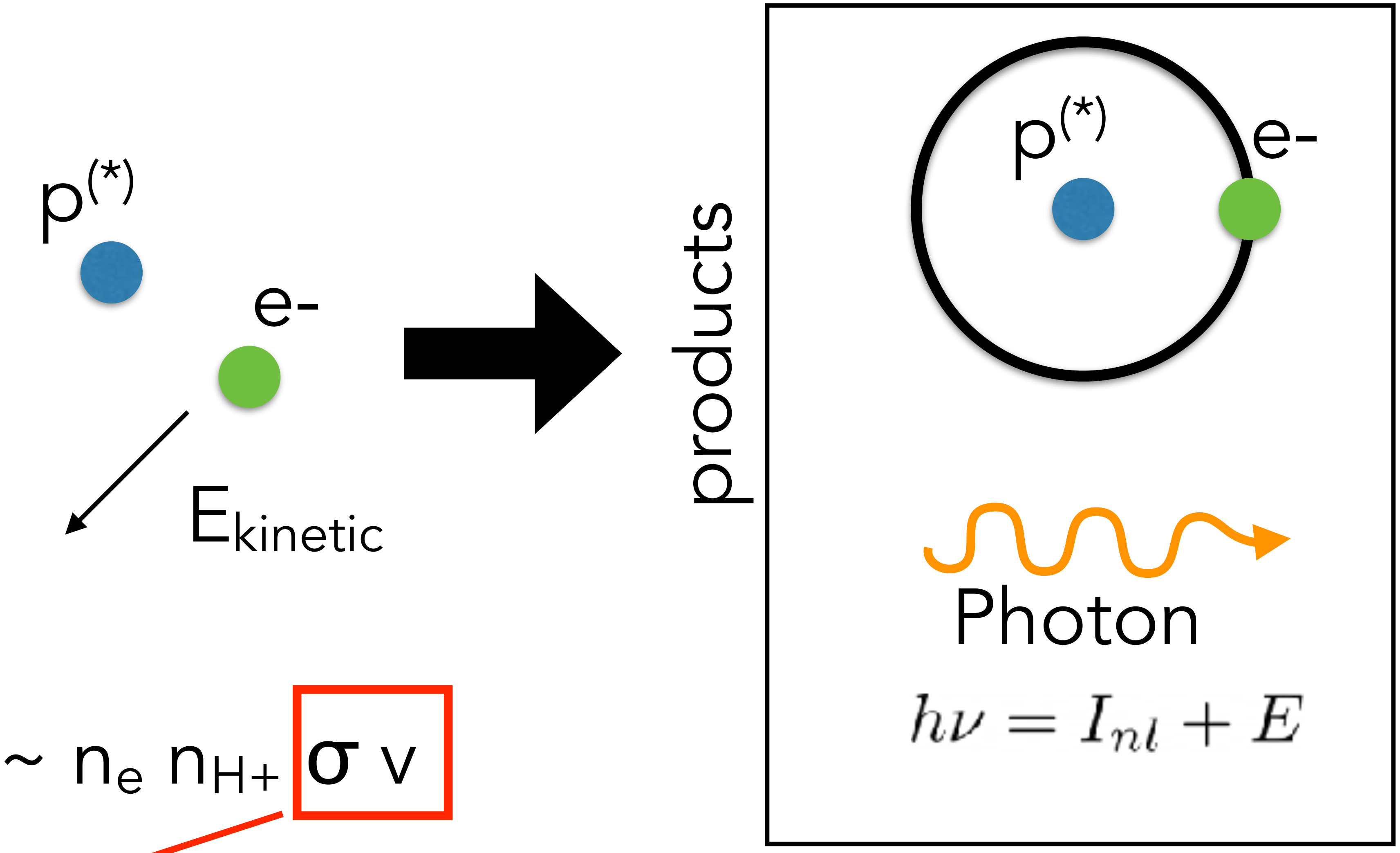
Radiative Recombination of H



rate per volume ~ $n_e \, n_{H+} \, \boxed{\sigma \, v}$

$$\alpha_{nl}(T) = \left(\frac{8kT}{\pi m_e}\right)^{1/2} \int_0^\infty \sigma_{\mathrm{rr},nl}(E) \frac{E}{kT} e^{-E/kT} \frac{dE}{kT}$$

Radiative Recombination of H



$$h\nu = I_{nl} + E$$

rate per volume ~ $n_e \, n_{H+} \, \boxed{\sigma \, v}$

$$\alpha_{nl}(T) = \left(\frac{8kT}{\pi m_e}\right)^{1/2} \int_0^{\infty} \sigma_{\mathrm{rr},nl}(E) \frac{E}{kT} e^{-E/kT} \frac{dE}{kT}$$

"Case B" $\alpha_B(T) = \sum_{n=2}^{\infty} \sum_{\ell=0}^{n-1} \alpha_{n\ell}(T) = \alpha_A(T) - \alpha_{1s}(T)$

Given photoionization cross section from before, we can use detailed balance to work out radiative recombination cross section.

Milne Relation:

$$\sigma_{\rm rr}(E) = \frac{g_\ell}{g_u}\frac{(I_{X,u\ell}+E)^2}{Em_ec^2}\sigma_{\rm pi}(h\nu = I_{X,u\ell}+E)$$

$$X_u^+ + e^- \rightarrow X_\ell + h\nu$$

HII Regions

Stromgren’s insight:

HII regions surrounding massive, young stars are regions where H is **~fully ionized** with a sharp boundary.

more to discuss Friday!

We can use this to estimate HII region properties using a steady-state approximation with constant n, where total ionizations balance recombinations.

HII Regions

Stromgren's insight:

HII regions surrounding massive, young stars are regions where H is ~fully ionized with a sharp boundary.

ionizing photon production rate ≈ recombination rate

$$Q_0 = \frac{4\pi}{3} R_{SO}^3 \; \alpha_B \; n(H^+) \; n_e$$

HII Regions

Stromgren's insight:

HII regions surrounding massive, young stars are regions where H is ~fully ionized with a sharp boundary.

ionizing photon production rate ≈ recombination rate

$$\boxed{Q_0} = \frac{4\pi}{3} R_{SO}^3 \, \alpha_B \, n(H^+) \, n_e$$

ionizing photons per sec

HII Regions

Stromgren's insight:

HII regions surrounding massive, young stars are regions where H is ~fully ionized with a sharp boundary.

ionizing photon production rate ≈ recombination rate

$$\boxed{Q_0} = \boxed{\frac{4\pi}{3} R_{SO}^3}\, \alpha_B \; n(H^+) \; n_e$$

ionizing photons per sec

volume of Stromgren sphere

HII Regions

Stromgren’s insight:

HII regions surrounding massive, young stars are regions where H is ~fully ionized with a sharp boundary.

ionizing photon production rate ≈ recombination rate

$$\boxed{Q_0} = \boxed{\frac{4\pi}{3} R_{SO}^3} \boxed{\alpha_B \; n(H^+) \; n_e}$$

ionizing photons per sec

volume of Stromgren sphere

recomb rate per volume Case B!

HII Regions

$$R_{S0} = \left(\frac{3Q_0}{4\pi n_H^2 \alpha_B} \right)^{1/3} = 9.77 \times 10^{18} Q_{0,49}^{1/3} n_2^{-2/3} T_4^{0.28} \mathrm{cm}$$

where:

$$Q_{0,49} = Q_0 / 10^{49} \mathrm{s}^{-1}$$

$$n_2 = n_H / 10^2 \mathrm{cm}^{-3}$$

$$T_4 = T / 10^4 \mathrm{K}$$

At n_2, T_4,and $Q_{0,49}$
R_{S0} ~ 3pc

Decreases in size when n increases.
Increases when Q_0 increases.

HII Regions

Stromgren’s insight:

HII regions surrounding massive, young stars are regions where H is ~fully ionized with a sharp boundary.

Transition from ionized to neutral will be approximately the mean free path of ionizing photons in HI.

$$l_{\rm mfp} = \frac{1}{n(H^0)\ \sigma_{pi}} = 3.39 \times 10^{17} \left(\frac{n(H^0)}{1\ {\rm cm}^{-3}} \right)^{-1} {\rm cm}$$

here: mfp for 18 eV photon

HII Regions

Calculation from Stromgren (& ch 15.3 in Draine) of ionization fraction as a function of radius (shows ~fully ionized is a good approximation)

$$\frac{x^2}{1-x} \approx \frac{1-y^3}{3y^2}\tau_S$$

$\tau_S = n_H \sigma_{pi} R_{S0}$

$x = n_e/n_H$

neutral frac: (1-x)

$y = r/R_{S0}$

Can calculate "typical" value from radius where 1/2 of mass is enclosed

$$(1 - x_m) = 1.1 \times 10^{-3} Q_{0,49}^{-1/3} n_2^{-1/3}$$

HII Regions

Timescale for ionization is short:

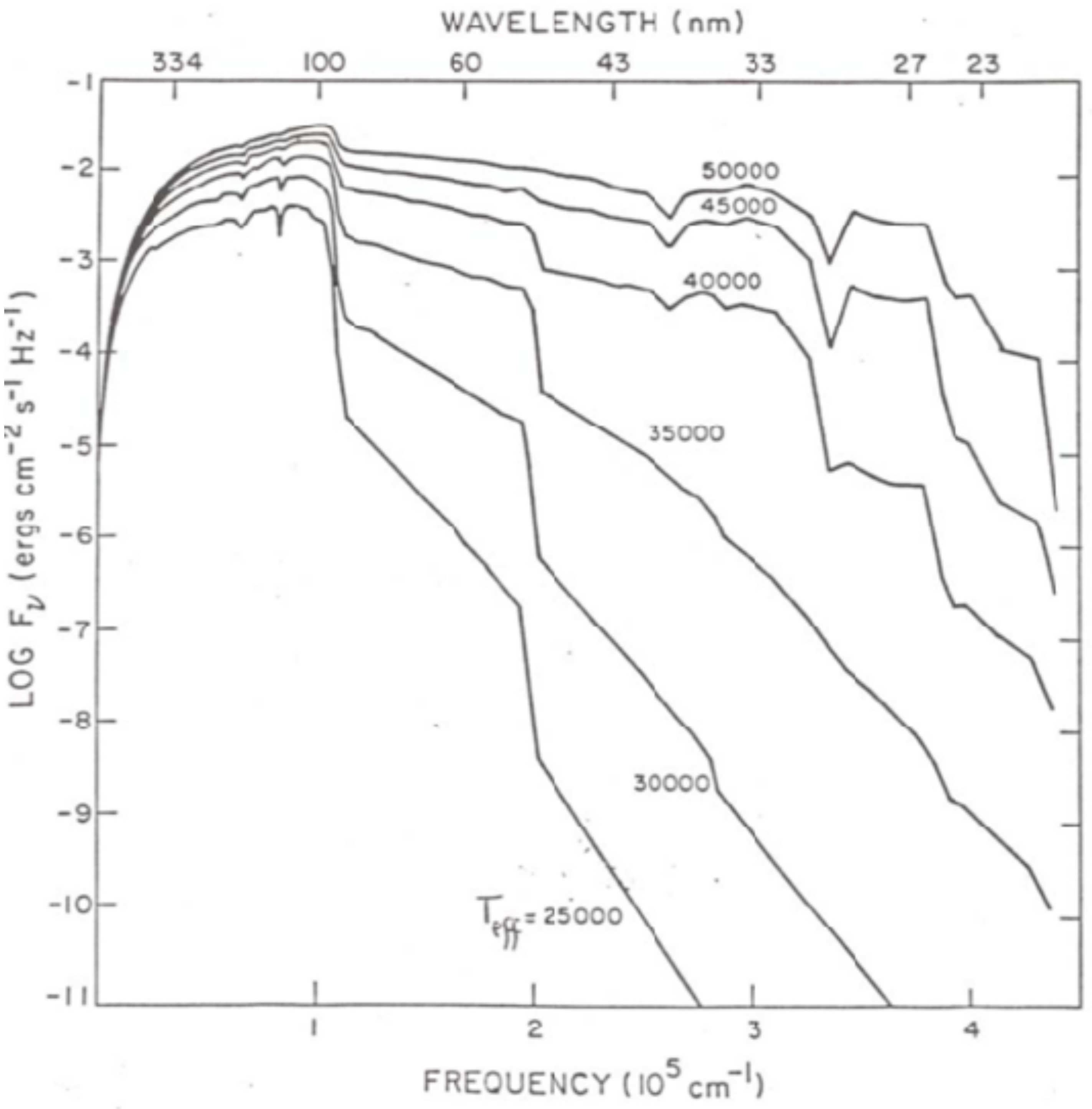
H to ionize

$$\tau_{\rm ioniz} \equiv \frac{(4/3)\pi R_{S0}^3 n_H}{Q_0} = \frac{1}{\alpha_B n_H} = \frac{1.22\times 10^3 {\rm yr}}{n_2}$$

ionizing photons per sec

Ionization equilibrium happens quickly after star turns on.

HII Regions

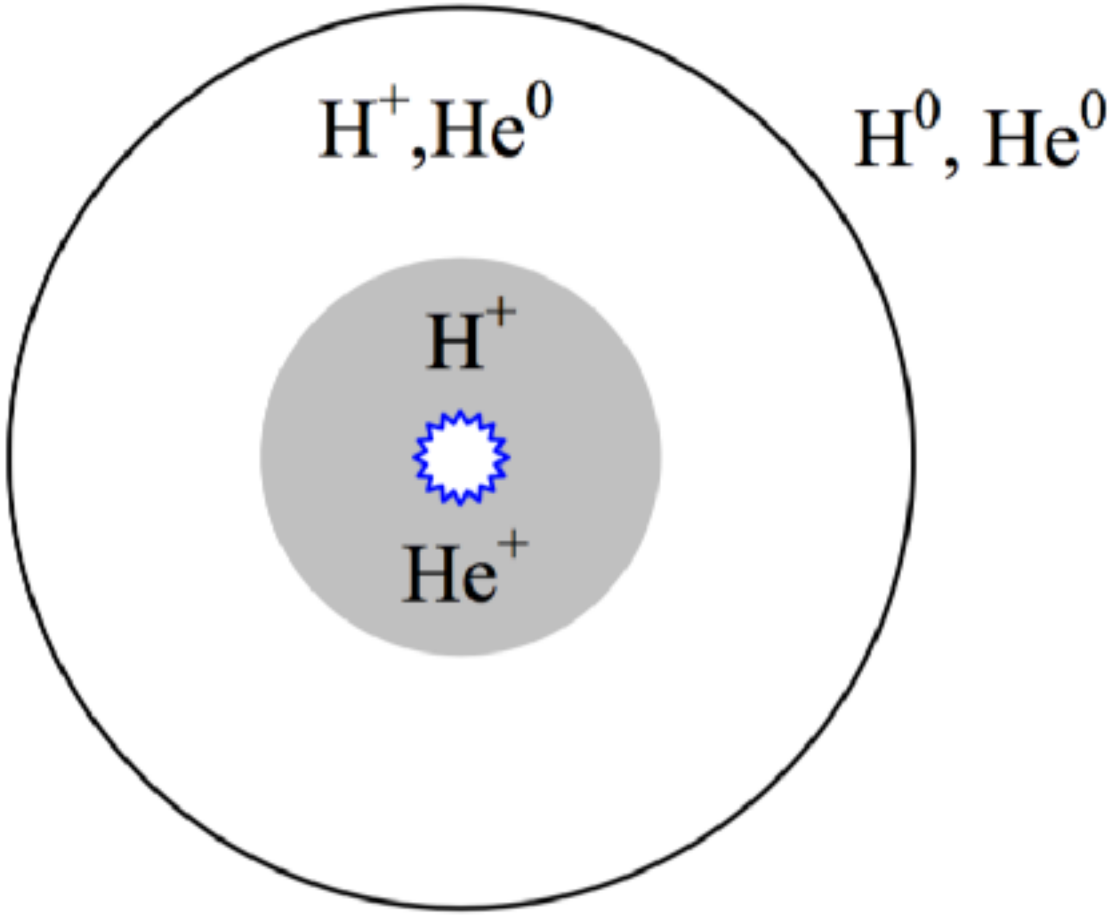


HII regions have more than just H in them, ionization structure in other elements depends on stellar spectrum and density.

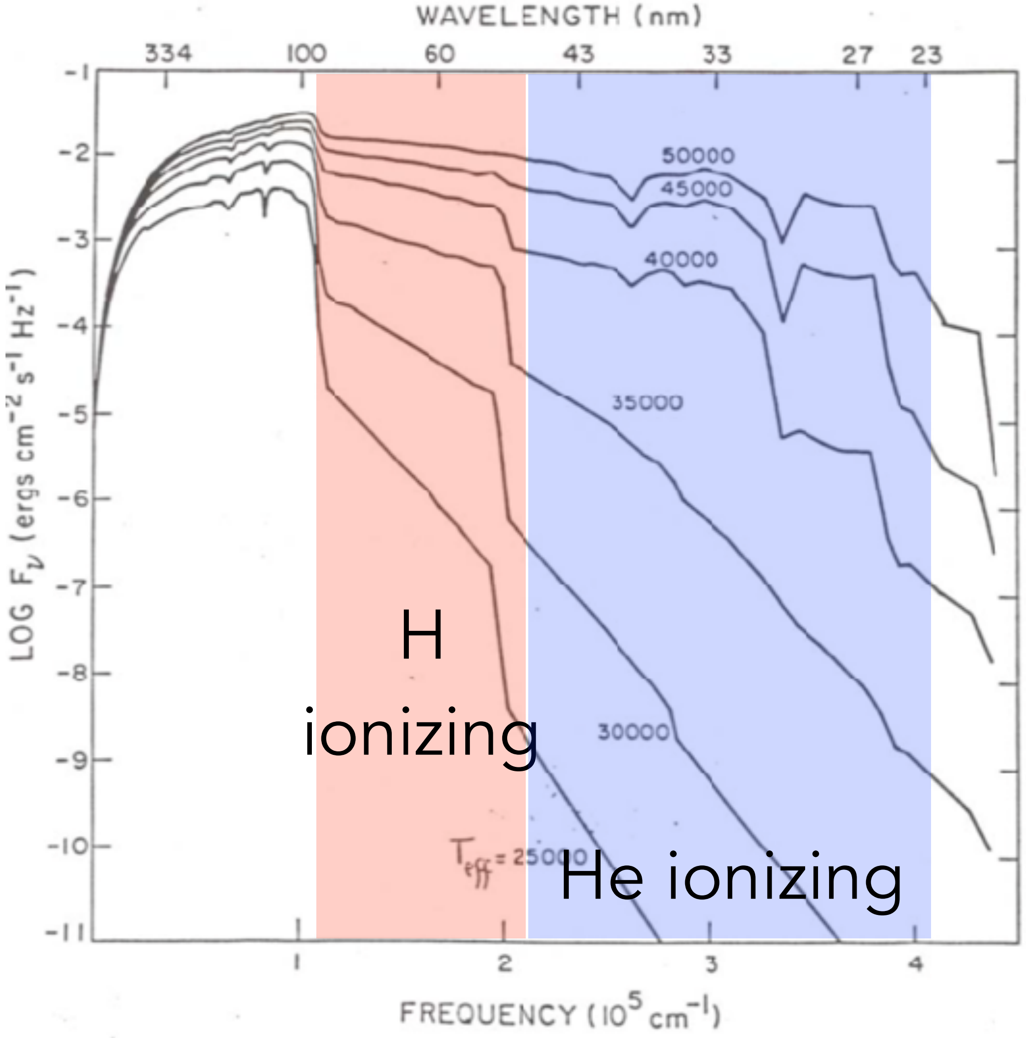
HII Regions

Next abundant element: He

Ionization potential 24.59 eV



"Cool" < 40,000 K star

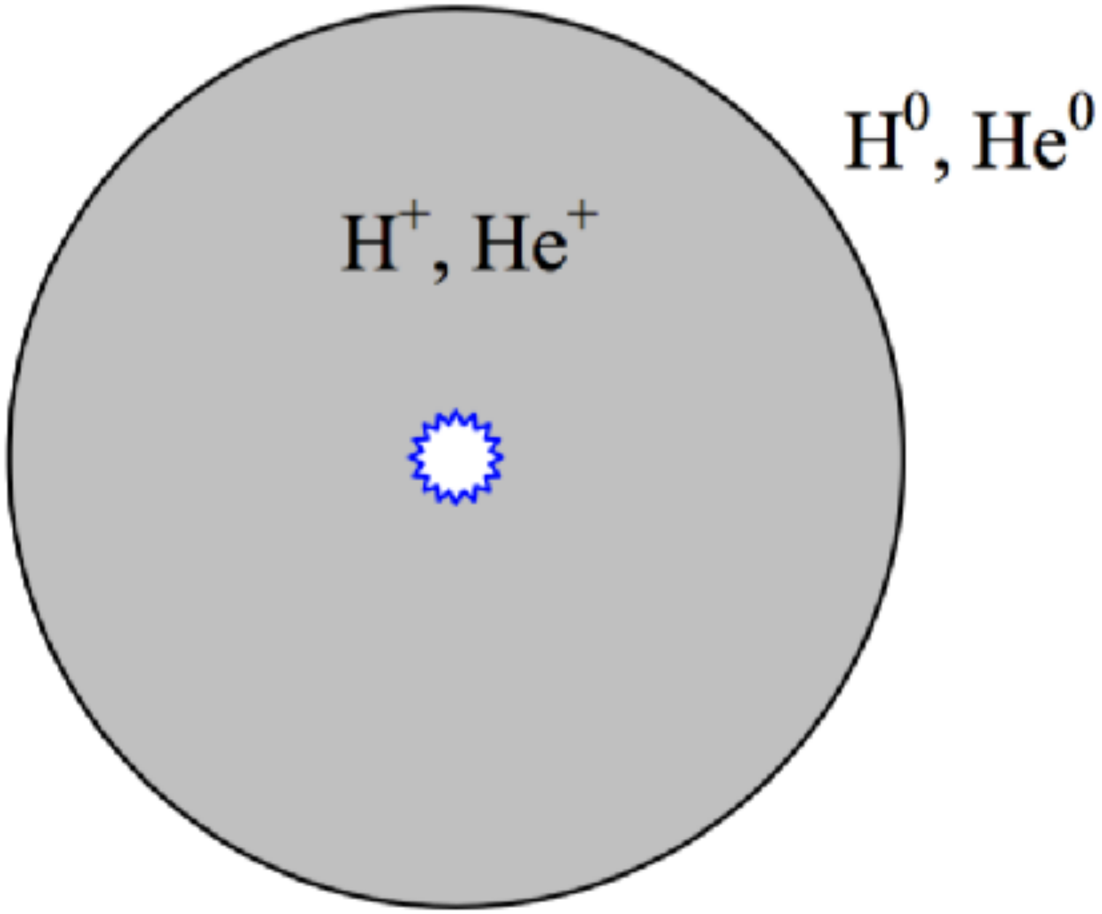


He in stellar atmospheres means not much past here

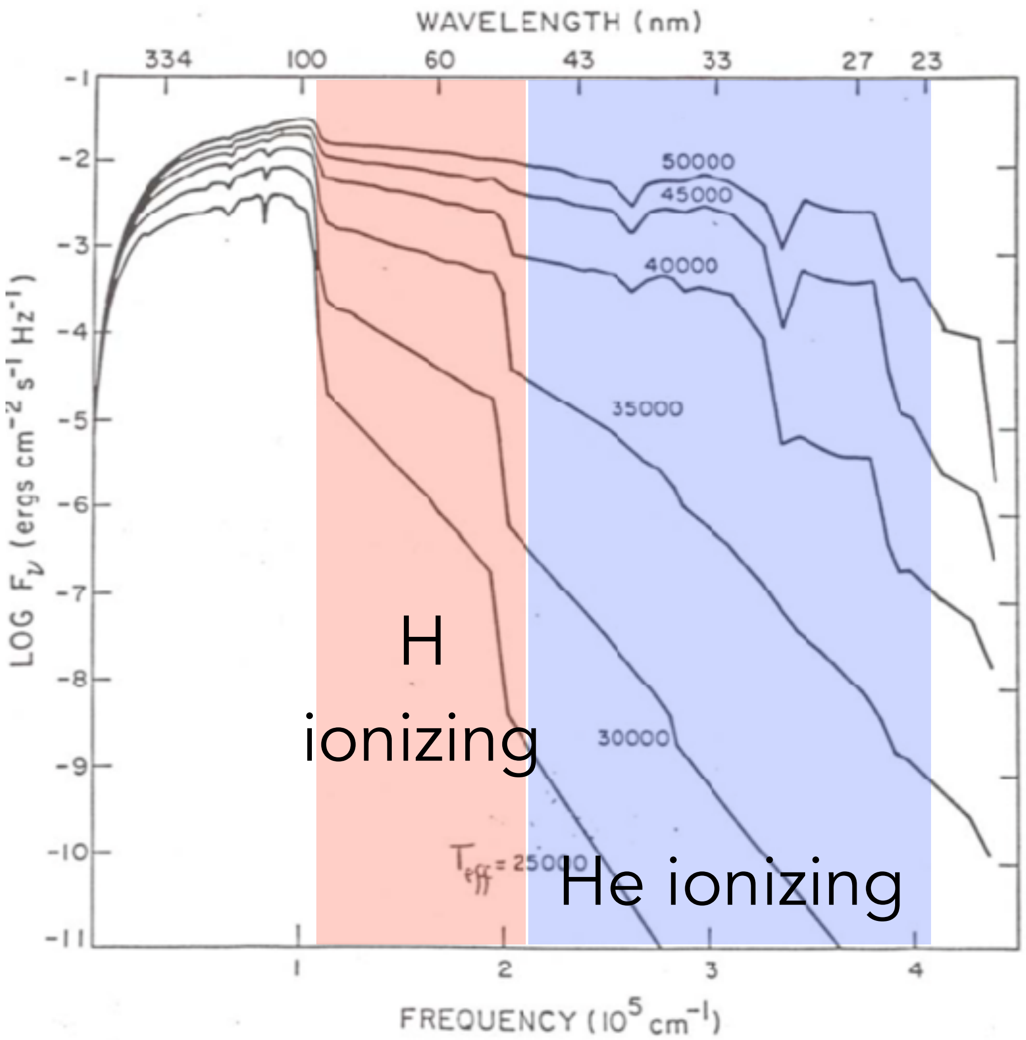
HII Regions

Next abundant element: He

Ionization potential 24.59 eV



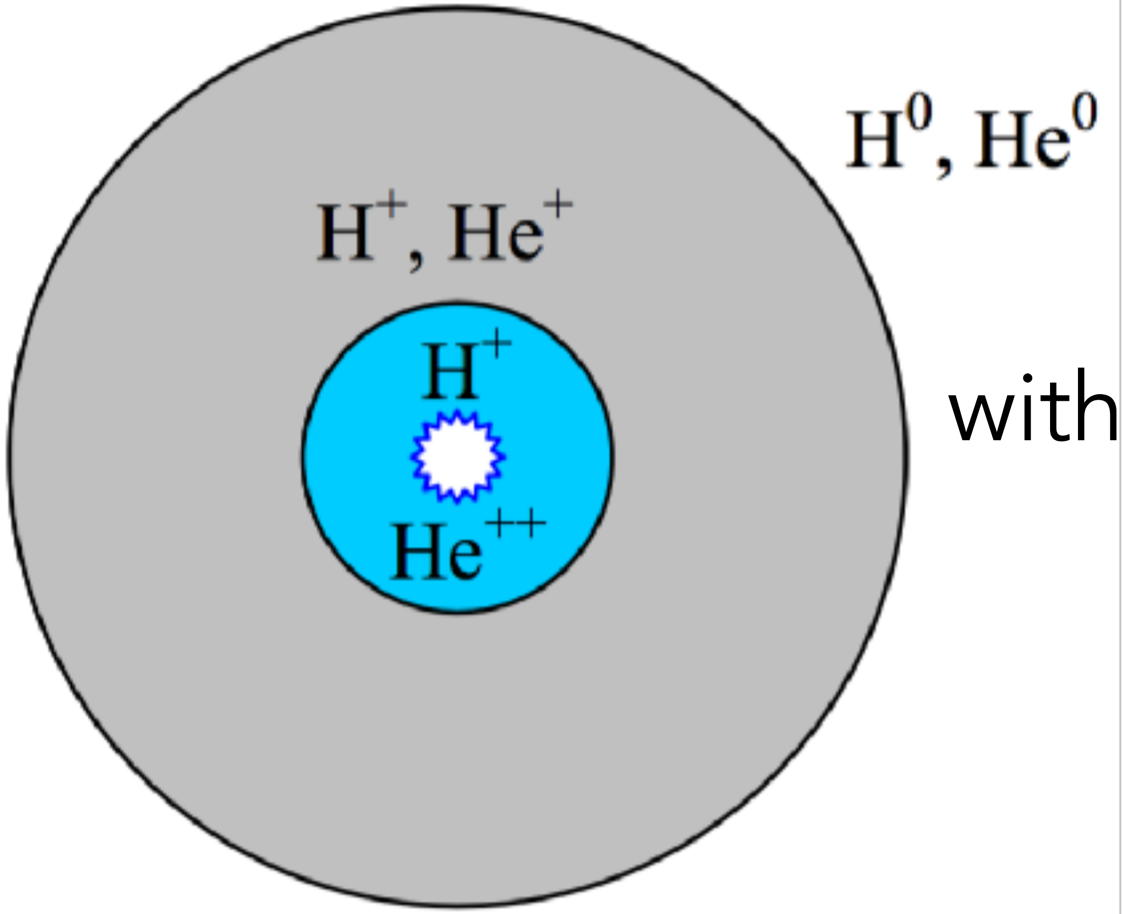
"Hot" 40-100,000 K star



He in stellar atmospheres means not much past here

HII Regions

Next abundant element: He
Ionization potential 24.59 eV



For stars or ionizing sources
with enough photons at E > 54.4 eV
get He^{++} zone

HII Regions

Photoionization Modeling:
coupling of ionization state, stellar spectrum, density, temperature, etc for multiple species

Cloudy & Associates

Photoionization Simulations for the Discriminating Astrophysicist Since 1978

Part II: Collisional Excitation

Collisional Excitation

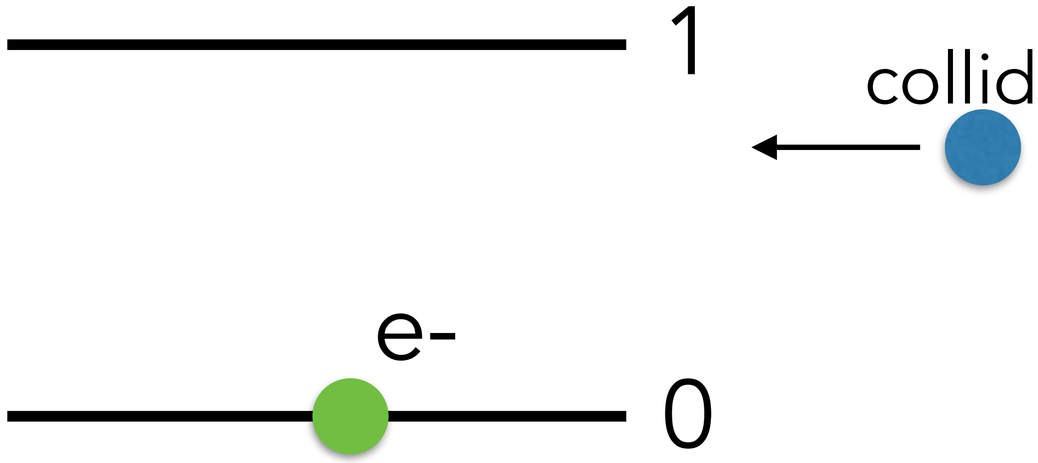
is important because:

1) it can put electrons in excited states that radiatively decay and remove energy from the gas

2) radiative transitions fed by collisional excitation give us very useful diagnostics of gas conditions

Collisional Excitation

Two Level Atom



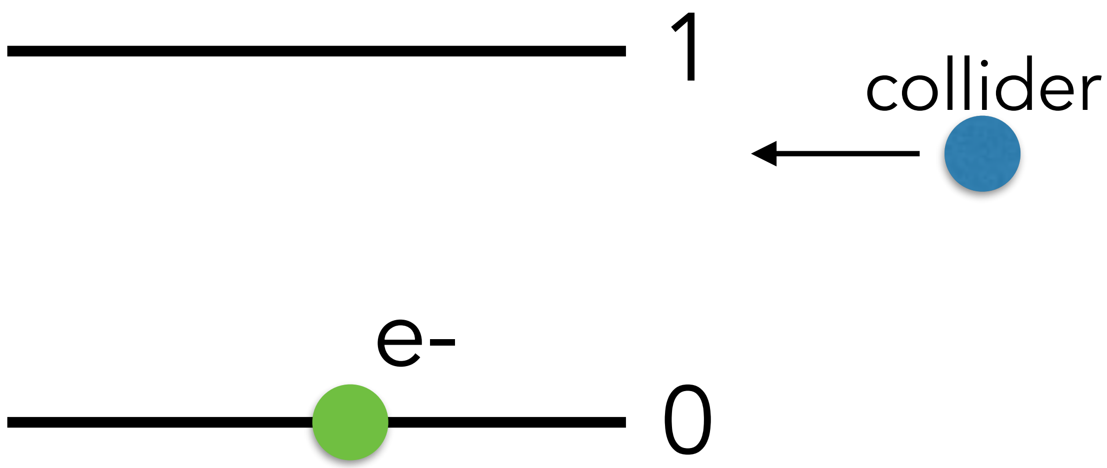
Assume no background radiation field
(i.e. ignore stimulated emission)

dn_1/dt = (rate of collisions from 0 to 1) -
(rate of collisions from 1 to 0) -
(spontaneous emission from 1 to 0)

*per volume

Collisional Excitation

Two Level Atom



Assume no background radiation field
(i.e. ignore stimulated emission)

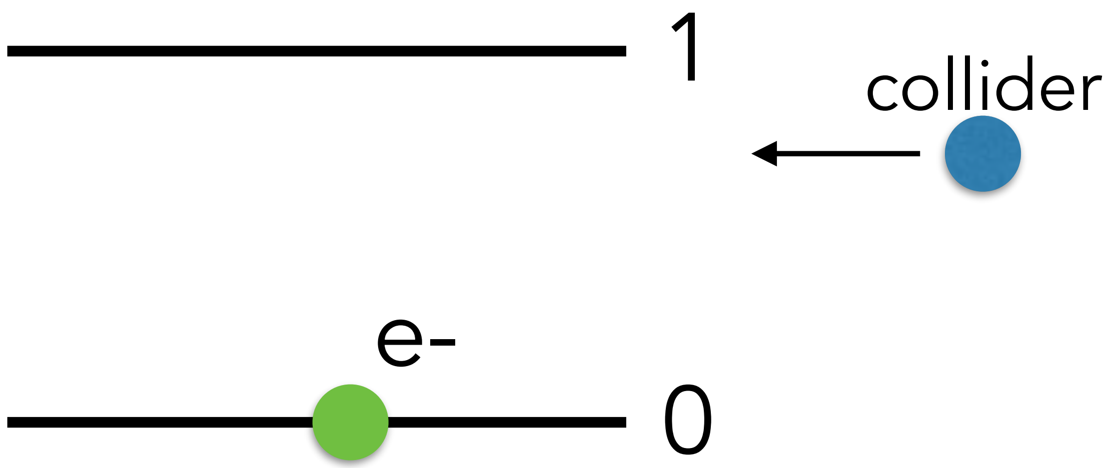
dn_1/dt = (rate of collisions from 0 to 1) -
(rate of collisions from 1 to 0) -
(spontaneous emission from 1 to 0)

$dn_1/dt = n_c\, n_0\, k_{01}$ -
$n_c\, n_1\, k_{10}$ -
$n_1\, A_{10}$

*per volume

Collisional Excitation

Two Level Atom



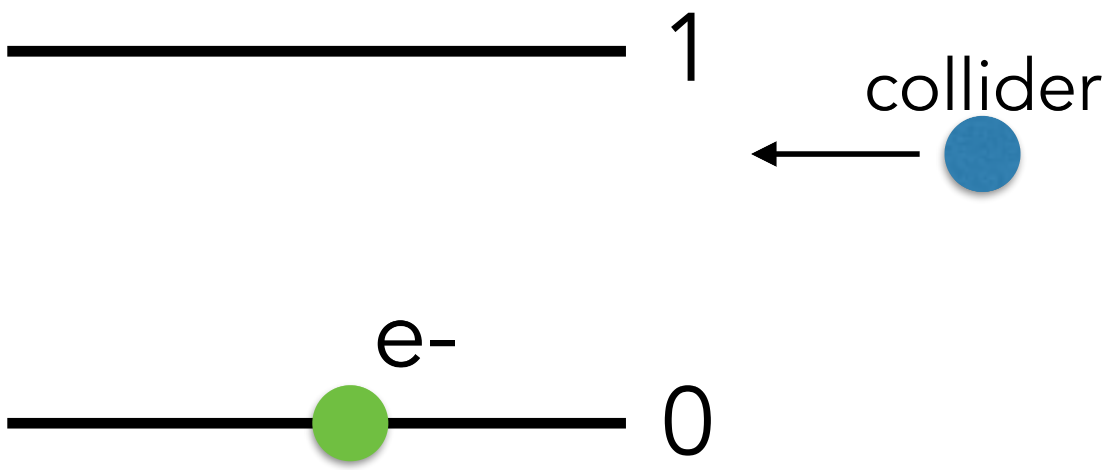
In steady state:

$dn_1/dt = 0$

$$\frac{n_1}{n_0} = \frac{n_c\, k_{01}}{n_c\, k_{10} + A_{10}}$$

Collisional Excitation

Two Level Atom



In steady state:

$dn_1/dt = 0$

$$\frac{n_1}{n_0} = \frac{n_c\, k_{01}}{n_c\, k_{10} + A_{10}}$$

from detailed balance: $k_{01} = \frac{g_1}{g_0} k_{10} e^{-E_{10}/kT_{gas}}$

Collisional Excitation

$$\frac{n_1}{n_0} = \left(\frac{1}{1 + A_{10}/(n_c k_{10})}\right) \frac{g_1}{g_0} e^{-E_{10}/kT_{\rm gas}}$$

Collisional Excitation

$$\frac{n_1}{n_0} = \left(\frac{1}{1 + A_{10}/(n_c k_{10})} \right) \frac{g_1}{g_0} e^{-E_{10}/kT_{\mathrm{gas}}}$$

define “critical density”

ratio of collisional to spontaneous rates that depopulate level 1

$$n_{\mathrm{crit}} = \frac{A_{10}}{k_{10}}$$

Collisional Excitation

$$\frac{n_1}{n_0} = \left(\frac{1}{1 + A_{10}/(n_c k_{10})} \right) \frac{g_1}{g_0} e^{-E_{10}/kT_{\rm gas}}$$

define “critical density”

ratio of collisional to spontaneous rates that depopulate level 1

$$n_{\rm crit} = \frac{A_{10}}{k_{10}}$$

$$\frac{n_1}{n_0} = \left(\frac{1}{1 + n_{\rm crit}/n_c} \right) \frac{g_1}{g_0} e^{-E_{10}/kT_{\rm gas}}$$

Collisional Excitation

$$\frac{n_1}{n_0} = \left(\frac{1}{1 + n_{\rm crit}/n_c}\right)\frac{g_1}{g_0}e^{-E_{10}/kT_{\rm gas}}$$

When $n_c >> n_{crit}$, level populations are set by the gas temperature and degeneracy - “thermalized”

When $n_c << n_{crit}$, factor in parenthesis goes to n_c/n_{crit}, population in level n_1 is “sub-thermal”

Collisional Excitation

General formulation takes into account stimulated emission and absorption too…

$$\frac{n_1}{n_0} = \frac{n_c k_{01} + \langle n_\gamma \rangle (g_1/g_0) A_{10}}{n_c k_{10} + (1 + \langle n_\gamma \rangle) A_{10}}.$$

general definition of n_{crit}:

$$n_{\mathrm{crit}} = \frac{(1 + \langle n_\gamma \rangle) A_{10}}{k_{10}}$$

where:

$$\langle n_\gamma \rangle = \frac{c^3}{8\pi h \nu^3} u_\nu$$

is the photon occupation number

Collisional Excitation

Useful to rewrite this with brightness temperature:

$$\langle n_\gamma \rangle = \frac{1}{e^{h\nu/kT_B} - 1},$$

$$\boxed{\frac{n_1}{n_0} = \left(\frac{1}{1 + n_{\rm crit}/n_c}\right)\frac{g_1}{g_0}e^{-E_{10}/kT_{\rm gas}} + \left(\frac{1}{1 + n_c/n_{\rm crit}}\right)\frac{g_1}{g_0}e^{-h\nu/kT_B}.}$$

Ratio of n_c/n_{crit} determines if level populations track gas temperature or radiation field temperature!

Critical Density

Multi-level atoms

$$n_{\mathrm{crit},u}(c) \equiv \frac{\sum_{l<u}[1+\langle n_\gamma \rangle_{ul}]A_{ul}}{\sum_{l<u} k_{ul}(c)}$$

ratio of total radiative and collisional depopulation rates to lower levels

note: only good in cases where gas is optically thin to radiation from u->l transition

Part III: Nebular Diagnostics

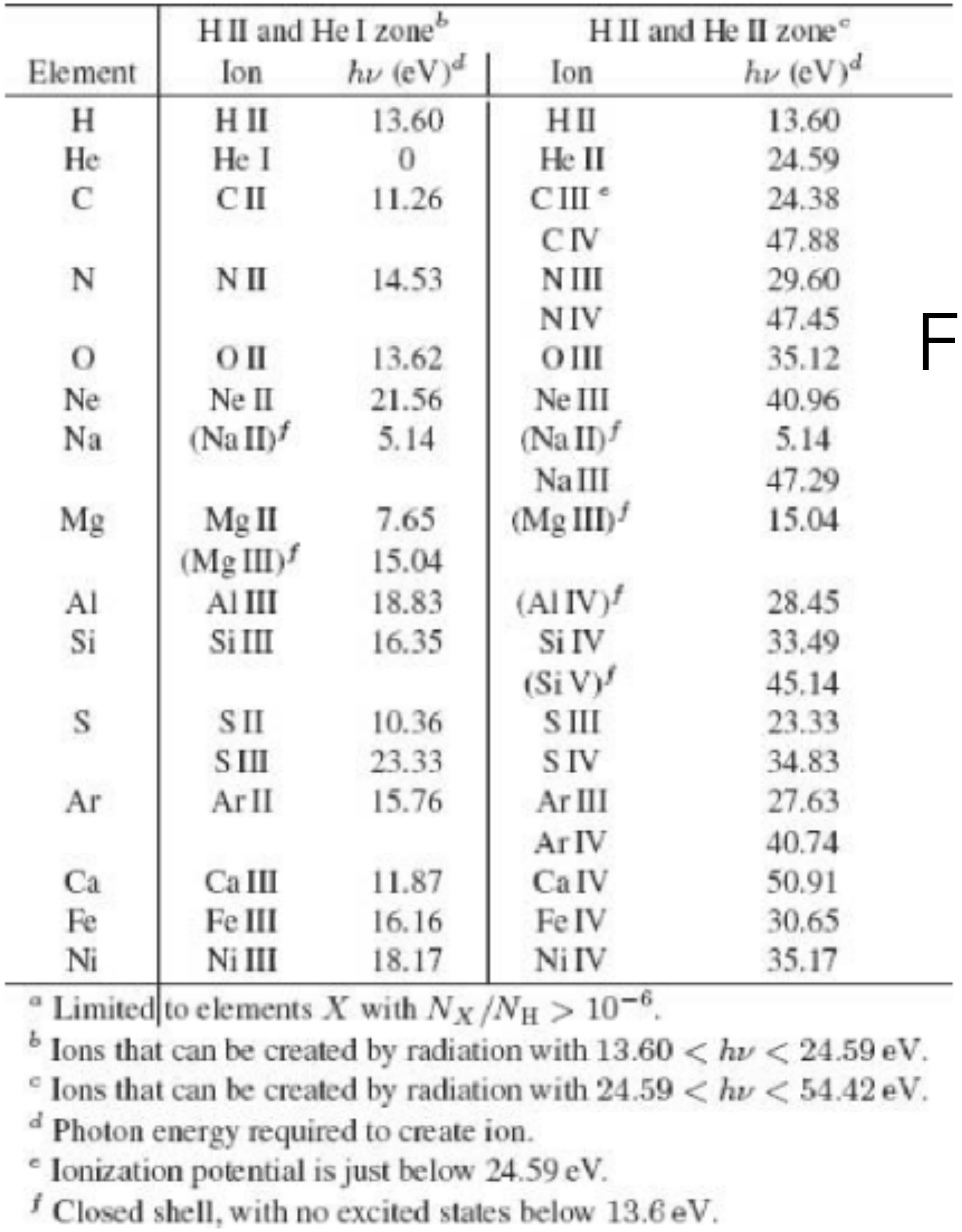
Nebular Diagnostics

Collisionally excited lines from ionized gas that give us diagnostics for density, temperature, etc.

Two types:

1) temperature sensitive
2) density sensitive

Nebular Diagnostics



Element	H II and He I zone[b]		H II and He II zone[c]	
	Ion	$h\nu$ (eV)[d]	Ion	$h\nu$ (eV)[d]
H	H II	13.60	H II	13.60
He	He I	0	He II	24.59
C	C II	11.26	C III[e]	24.38
			C IV	47.88
N	N II	14.53	N III	29.60
			N IV	47.45
O	O II	13.62	O III	35.12
Ne	Ne II	21.56	Ne III	40.96
Na	(Na II)[f]	5.14	(Na II)[f]	5.14
			Na III	47.29
Mg	Mg II	7.65	(Mg III)[f]	15.04
	(Mg III)[f]	15.04		
Al	Al III	18.83	(Al IV)[f]	28.45
Si	Si III	16.35	Si IV	33.49
			(Si V)[f]	45.14
S	S II	10.36	S III	23.33
	S III	23.33	S IV	34.83
Ar	Ar II	15.76	Ar III	27.63
			Ar IV	40.74
Ca	Ca III	11.87	Ca IV	50.91
Fe	Fe III	16.16	Fe IV	30.65
Ni	Ni III	18.17	Ni IV	35.17

[a] Limited to elements X with $N_X/N_{\rm H} > 10^{-6}$.

[b] Ions that can be created by radiation with $13.60 < h\nu < 24.59$ eV.

[c] Ions that can be created by radiation with $24.59 < h\nu < 54.42$ eV.

[d] Photon energy required to create ion.

[e] Ionization potential is just below 24.59 eV.

[f] Closed shell, with no excited states below 13.6 eV.

First good to note which atoms and ions will be abundant in HII regions.

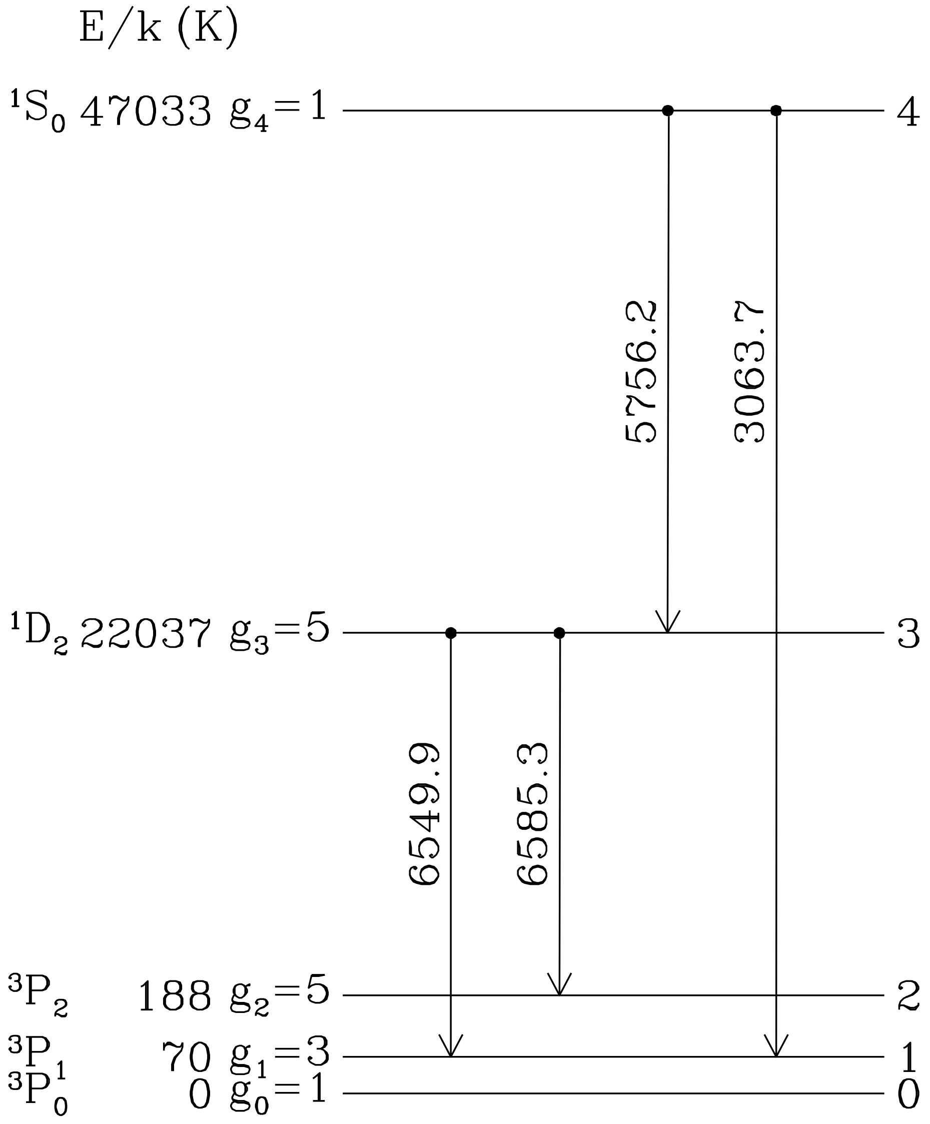
Temperature Sensitive Line Ratios

What we want:
two levels that can both be collisionally excited at typical HII region temperatures (~10^4 K) but which have different enough energies that the ratio of populations depends on temperature of the gas

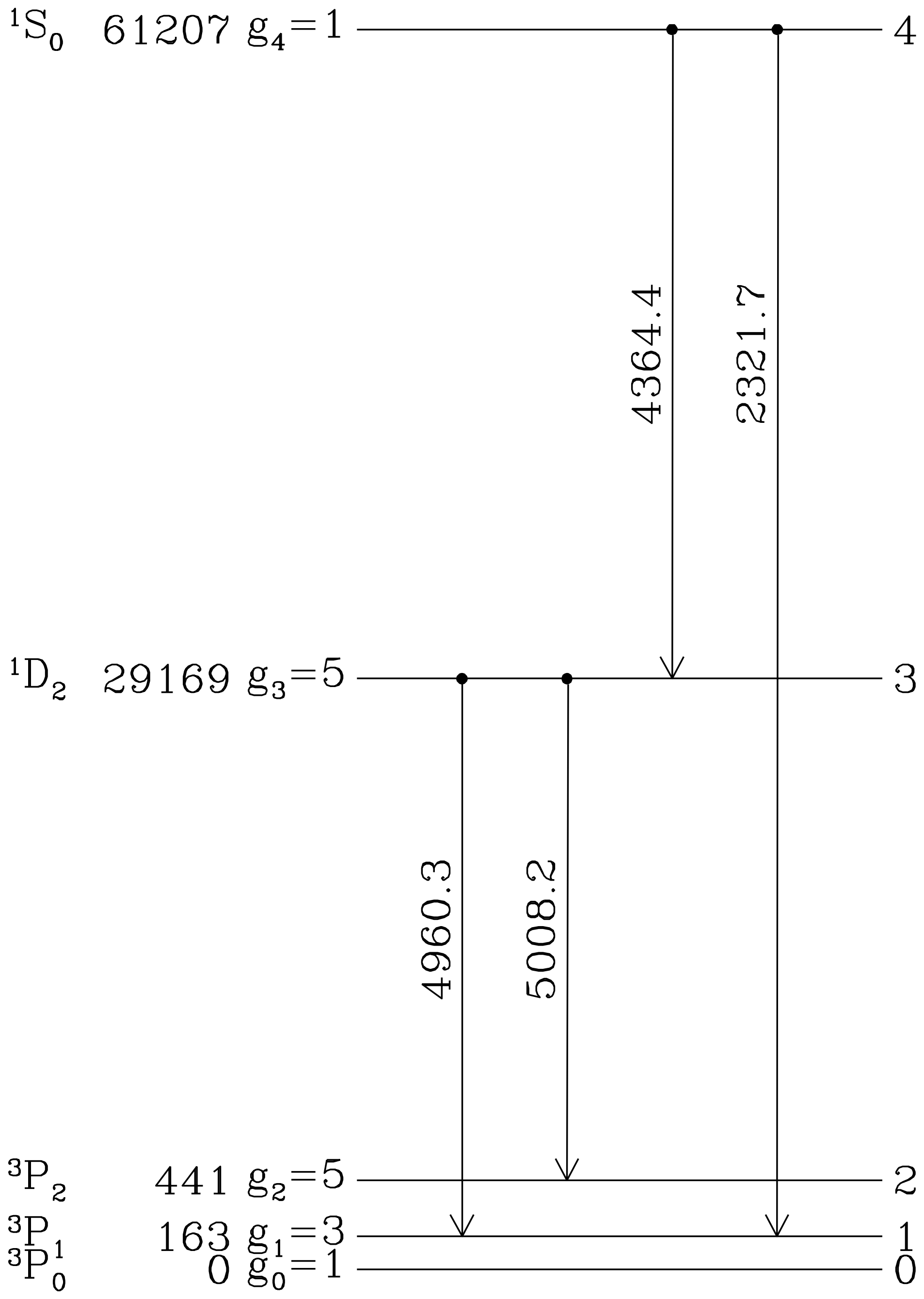
Requires two energy levels with $E/k < 70{,}000$ K

Temperature Sensitive Line Ratios

best candidates: np^2 & np^4

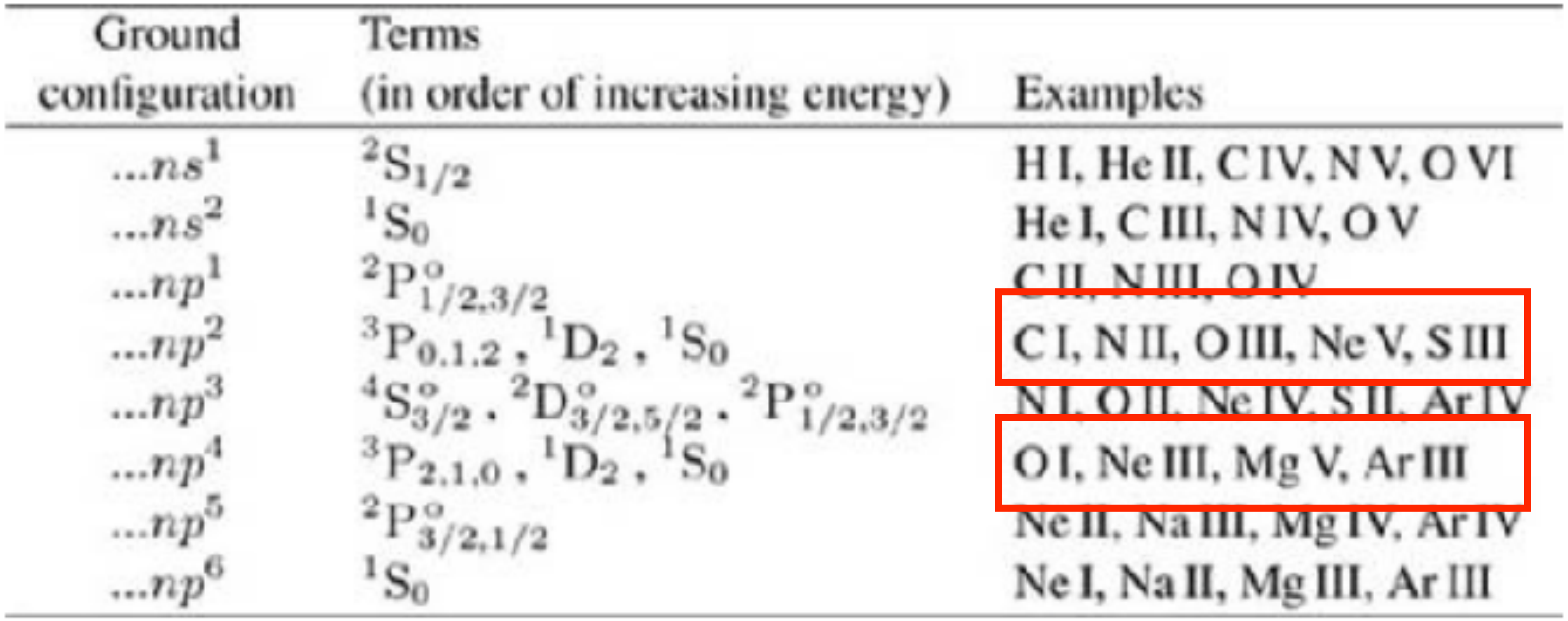


N II



O III

Temperature Sensitive Line Ratios

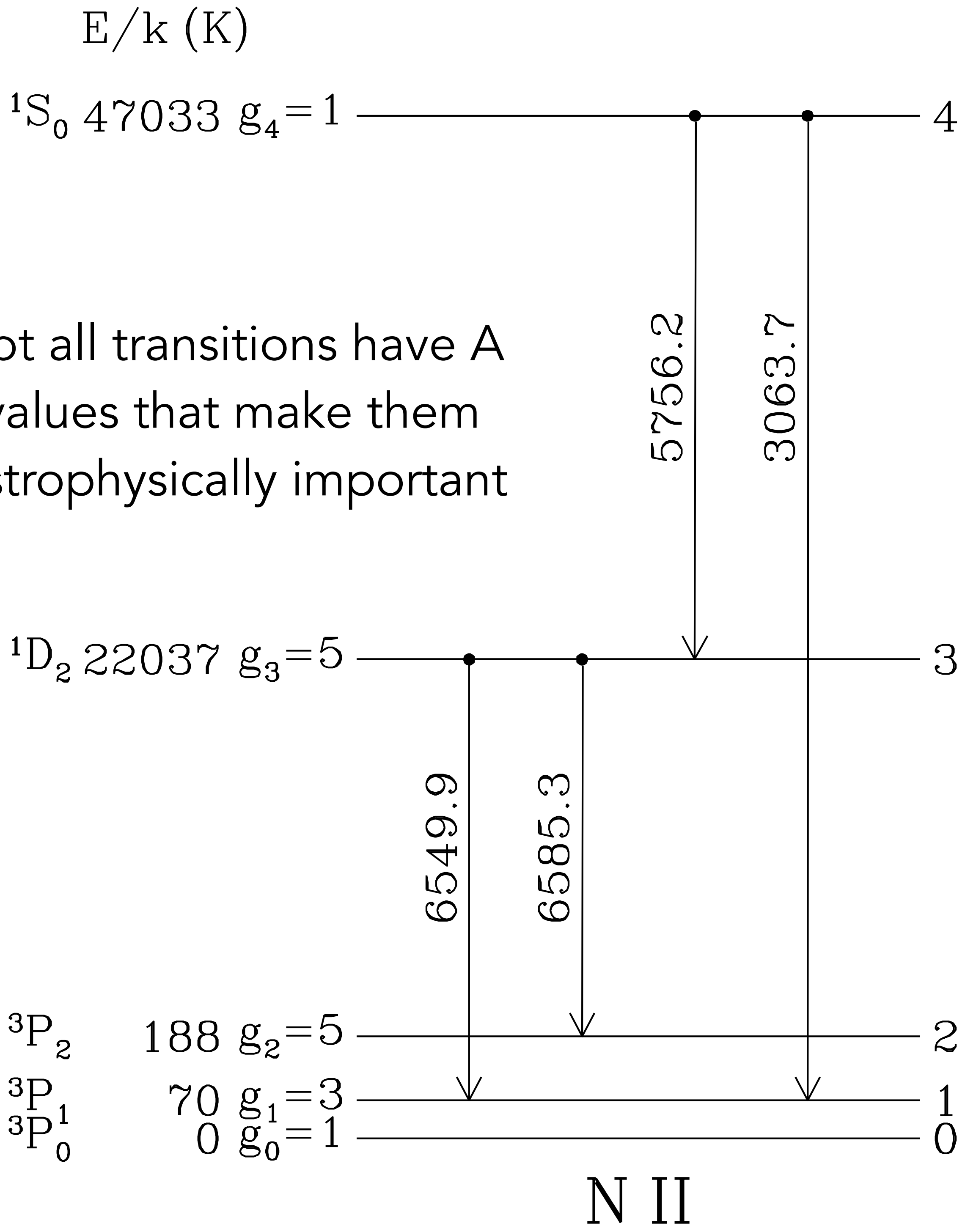


Ground configuration	Terms (in order of increasing energy)	Examples
$...ns^1$	$^2S_{1/2}$	H I, He II, C IV, N V, O VI
$...ns^2$	$^1S_0$	He I, C III, N IV, O V
$...np^1$	$^2P^o_{1/2,3/2}$	C II, N III, O IV
$...np^2$	$^3P_{0,1,2}$, $^1D_2$, $^1S_0$	C I, N II, O III, Ne V, S III
$...np^3$	$^4S^o_{3/2}$, $^2D^o_{3/2,5/2}$, $^2P^o_{1/2,3/2}$	N I, O II, Ne IV, S II, Ar IV
$...np^4$	$^3P_{2,1,0}$, $^1D_2$, $^1S_0$	O I, Ne III, Mg V, Ar III
$...np^5$	$^2P^o_{3/2,1/2}$	Ne II, Na III, Mg IV, Ar IV
$...np^6$	$^1S_0$	Ne I, Na II, Mg III, Ar III

CI,OI don’t exist in HII regions (carbon is ionized)
NeV, MgV is too highly ionized

NII, OIII and SIII are useful temperature diagnostics
(Ne III and Ar III useful as well, but req higher energy photons)

Temperature Sensitive Line Ratios



$n_{crit,4} \sim 10^7$ cm^{-3}

$n_{crit,3} \sim 7.7 \times 10^4$ cm^{-3}

not all transitions have A values that make them astrophysically important

at typical HII region densities, NII transitions from $^1S_0$ and $^1D_2$ are below critical density

means:

approximately every collision results in a radiative decay (i.e. A wins over k)