PHYS 239 Lec 1

(pls email yiz020@ucsd.edu if you find any mistake. Thank you!!)

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Three different radiative processes:

- Radiative transport: collective description treating light collectively as 'rays'.
  - important when scale is $l \gg \lambda$
- 'classical' EM waves: treating light as waves
- QM: treating light as photon

1 Radiative transport

Definition 1 (Ray)**.** Rays move in straight line in homogeneous medium (e.g. vacuum). A single ray carries essentially no energy $\Rightarrow$ define sets of rays to be differ infinitesimally.

At a given position (x, y, z) and given time t, we want to know how much energy dE moving per unit time dt per unit area $\underset{\cdot}{A} \to \vec{k} \cdot \hat{n} dA$, $\hat{n} \parallel \vec{k}$, per unit frequency $d\nu$ in a given direction $d\Omega$. Ignore polarization for now. Solid angle $d\Omega$ is given by

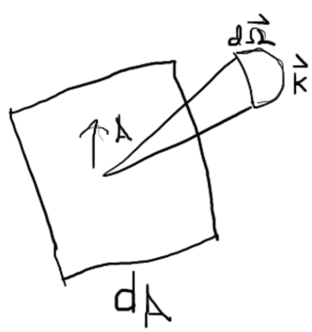


Figure 1: Pictorial description of solid angle

Solid angle of a sphere

$$4\pi \; sr$$

unit: steradian (sr)

$$d\Omega = \frac{dA}{r} = \sin\theta d\theta d\phi$$

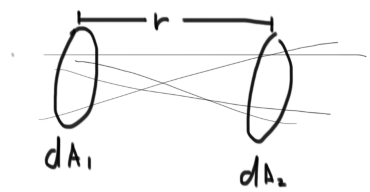
1.1 Specific Intensity

Definition 2 (Specific Intensity)**.**

$$I_\nu(\vec{k}, \vec{x}, t) = \frac{dE}{dt d\nu d\Omega \vec{k} \cdot \hat{n} dA}, \ [I_\nu] = [W/m^2/\text{Hz}/sr] \text{ or } [erg/s/cm^2/Hz/sr]$$

Consider two area dA_1, dA_2. For rays crossing between the two regions, total energy is conserved:

$$dE_1 = dE_2$$



$$d\Omega_1 = \frac{dA_2}{r^2}, \ d\Omega_2 = \frac{dA_1}{r^2}$$

$$I_{\nu_1} = \frac{dE_1}{dt d\nu d\Omega_1 dA_1} = \frac{dE_1 r^2}{dt d\nu dA_1 dA_2}$$

$$I_{\nu_2} = \frac{dE_2}{dt d\nu d\Omega_2 dA_2} = \frac{dE_2 r^2}{dt d\nu dA_1 dA_2}$$

$$\Rightarrow I_{\nu_1} = I_{\nu_2}$$

More generally

$$\boxed{\vec{k} \cdot \nabla I_\nu(\vec{k}, \vec{x}, t) = 0}$$

where ∇ is the gradient of specific intensity in the direction of propagation

1.2 Moments of specific intensity

Moments: averages of specific intensity over particular unit. Most useful:

- mean intensity

$$\int I_\nu \, d\Omega$$

- Flux density (energy/time/freq in unit area $[W/m^2/Hz]$)

$$F_\nu = \int I_\nu \cos\theta d\Omega$$

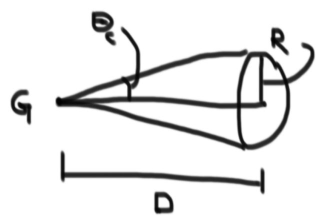
The directional dependence is integrated out in flux density. Total flux is given by

$$F = \int F_\nu \, d\nu, \ [W/m^2]$$

Note

- for isotropic field, the total flux is 0
- the specific intensity of the sun is the same everywhere, but the flux density is different (drops $\sim \frac{1}{D^2}$) .

Assume $I_\nu = B$



$$F_\nu = \int I_\nu \cos\theta d\Omega = B \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \underbrace{\int_0^{\theta_c} \sin\theta\cos\theta d\theta}_{\frac{1}{2}\sin^2\theta_c}$$

$$= \pi B \sin^2\theta_c = \pi B \left(\frac{R}{D}\right)^2$$

Other moments to consider: Radiation pressure

$$\text{Pressure} = \frac{\frac{dp}{dt}}{dA}$$

$$\text{for radiation: momentum flux/Area } \frac{dF_\nu}{c} = \frac{I_\nu \cos\theta d\Omega}{c}$$

$$\text{total momentum normal to dA } \frac{dF_\nu}{c}\cos\theta = \frac{I_\nu \cos^2\theta d\Omega}{c}$$

$$\Rightarrow P_\nu = \frac{1}{c}\int I_\nu \cos^2\theta d\Omega$$

Note Radiation pressure does not go to 0 in isotropic field