PHYS 239 Lec 3

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1 Radiative Transfer in medium, Thermal Equilibrium

Ignoring scattering. Absorption and Emission depends on the properties of the medium. Under TE:

$$\cancelto{0}{\frac{\partial I_\nu}{d\tau_\nu}} = S_\nu - I_\nu = S_\nu - B_\nu$$

$$\Rightarrow S_\nu = \boxed{B_\nu = \frac{j_\nu}{\alpha_\nu}} \quad \text{(Kirchoff's law)}$$

Introducing detail balance since now the ratio of j_ν and α_ν is fixed. This is true in all cases since j_ν and α_ν do not depend on the environment the medium is in.

Recall: $B_\nu(T) \neq B_\lambda(T) \Rightarrow B_\lambda = \left|\frac{d\nu}{d\lambda}\right| B_\nu(T)$. In regime $h\nu \ll kT$ (Rayleigh Jeans Tail):

$$e^{h\nu/kT} - 1 = 1 + \frac{h\nu}{kT} - 1 = \frac{h\nu}{kT}$$

$$B_{\nu,RT}(T) = \frac{2\nu^2}{c^2} kT$$

The peak of the blackbody spectrum:

$$\left.\frac{\partial B_\nu}{\partial \nu}\right|_{\nu=\nu_{\max}} = 0 \Rightarrow h\nu_{\max} = 2.82kT$$

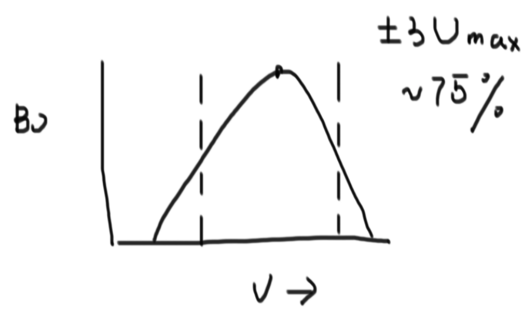

Figure 1: Peak of Blackbody spectrum

Definition 1 (Brightness Temperature)**.** T_B: a point in frequency-brightness plane unique determines a blackbody spectrum at T_B. In the rayleigh jeans tail:

$$T_B(\nu) = \frac{c^2}{2\nu^2 k} I_\nu(\nu)$$

We can express $B_{\nu,RT}$ in terms of T_B.

$$\frac{dT_B}{d\tau_\nu} = -T_B + T \Rightarrow T_B = T_B(0)e^{-\tau_\nu} + T(1 - e^{-\tau_\nu})$$

For $\tau \gg 1,\ T_B = T$.

2 Detail Balace

Consider a two level atom: g is the degeneracy or the statistical weight of the states

Figure 2: Set up of detail balance

- Spontaneous emission: rate of spontaneous emission per unit volume

$$= n_2 A_{21}$$

  where A_{21} is the Einstein A coefficient

- Absorption: rate/unit volumn

$$n_1 B_{12} \bar{J},\ \bar{J} = \frac{1}{4\pi} \int I_\nu d\Omega$$

  where B_{12} is the Einstein B coefficient for absorption

- Stimulated emission: rate/unit volumn

$$n_2 B_{21} \bar{J}$$

  where B_{21} is the Einstein B coefficient for stimulated emission

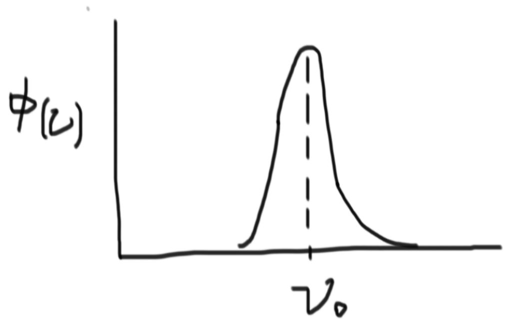

Figure 3: Definition of line profile

We can define a line profile where

$$\int_{-\infty}^{\infty} \phi(\nu) d\nu = 1$$

We then modify the definition for rates:

$$n_2 B_{21} \int \bar{J}_\nu \phi(\nu) d\nu = \frac{n_2 B_{21}}{4\pi} \int \phi(\nu) d\nu \int I_\nu d\Omega$$

Using detail balance:

$$n_1 B_{12} \bar{J} = n_2 A_{21} + n_2 B_{21} \bar{J}$$
$$\bar{J} = \frac{n_2 A_{21}}{n_1 B_{12} - n_2 B_{21}} = \frac{A_{21}/B_{21}}{\frac{n_1}{n_2}\frac{B_{12}}{B_{21}} - 1}$$

In TE

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-h\nu_0/kT}, \ \bar{J} = B_\nu$$
$$\Rightarrow \frac{A_{21}/B_{21}}{\frac{g_1}{g_2} e^{h\nu_0/kT} \frac{B_{12}}{B_{21}} - 1} = \frac{2h\nu_0^3}{c^2(e^{h\nu_0/kT} - 1)}$$
$$\Rightarrow \boxed{\frac{g_1 B_{12}}{g_2 B_{21}} = 1, \ \frac{A_{21}}{B_{21}} = \frac{2h\nu^3}{c^2}}$$

Note the two relations again do not depend on the environment, hence they work everywhere.

Putting everything together. Unit analysis:

$$j_\nu = \frac{dE_{em}}{dt dV d\nu d\Omega} \Rightarrow \boxed{j_\nu = \frac{A_{21} h\nu_0 n_2}{4\pi} \phi(\nu)}$$

$dE =$ total energy absorbed in dt and dV:

$$= n_1 B_{12} \frac{h\nu_0}{4\pi} dt dV \int d\Omega \underbrace{\int d\nu \phi(\nu) I_\nu}_{\bar{J}}$$

$$\text{(specific solid angle and freq.)} \quad = \frac{h\nu_0}{4\pi} n_1 B_{12} I_\nu dV dt d\Omega d\nu$$

Also can write dE in terms of intensity:

$$dE = I_\nu \alpha_\nu ds dt d\nu d\Omega dA \Rightarrow \boxed{\alpha_\nu = \frac{h\nu_0}{4\pi} \phi(\nu) n_1 B_{12}}$$

$$\text{correction for stimulated emission: } \alpha_\nu = \frac{h\nu_0}{4\pi} \phi(\nu)(n_1 B_{12} - n_2 B_{21})$$