Physics 224: Homework 1
Spring 2018

1. Consider a spherical dust grain of radius a and mass $M \gg m_H$, where m_H is the mass of an H atom. Suppose that the grain is initially at rest in a gas of H atoms with number density n_H and temperature T. Assume the grain is large compared to the radius of an H atom. Suppose that the H atoms "stick" to the grain when they collide with it, so that all of their momentum is transferred to the grain, and that they subsequently "evaporate" from the grain with no change in the grain velocity during the evaporation.

   (a) What is the mean velocity $\langle v_H \rangle$ of the hydrogen atoms (in terms of m_H, T and Boltzmann's constant k)?

   (b) Calculate the time τ_M for the grain to be hit by its own mass M in gas atoms. Express τ_M in terms of M, a, n_H, and $\langle v_H \rangle$.

   (c) Evaluate $\langle v_H \rangle$ and τ_M for a grain of radius $a = 10^{-5}$ cm and density $\rho = 3$ g cm^{-3}, in a gas with $n_H = 30$ cm^{-3} and T$= 10^2$ K.

   (d) If the collisions are random, the grain velocity will undergo a random walk Estimate the *initial* rate of increase $(dE/dt)_0$ of the grain kinetic energy E due to these random collisions. Express $(dE/dt)_0$ in terms of n_H, m_H, kT, a, and M. [Hint: think of the random walk that the grain momentum $\vec{p}$ undergoes, starting from the initial state $\vec{p} = 0$. What is the rate at which $\langle p^2 \rangle$ increases?]

   (e) Eventually the grain motion will be "thermalized", with time-averaged kinetic energy $\langle E \rangle = (3/2)kT$. Calculate the timescale

   $$\tau_E = \frac{(3/2)kT}{(dE/dt)_0} \tag{1}$$

   for thermalization of the grain speed. Compare to τ_M calculated in (b).

   (f) Make a plot showing the dependence of τ_E on a given ρ, n_H and T from (c).

   (g) Polycyclic aromatic hydrocarbons are large molecules or small grains which are composed of benzene rings (hexagonal carbon rings) with attached hydrogen along the periphery. These grains are planar with $a \simeq 0.9 N_C^{1/2}$ Å, where N_C is the number of carbon atoms in the PAH. Explain if and how the answers above would change for this type of grain.

2. Most interstellar CO is ^{12}C^{16}O. The rotational J=1 $\rightarrow$ 0 transition is at $\nu = 115.27$ GHz, or $\lambda = 0.261$ cm, and the vibrational $\nu = 1 \rightarrow 0$ transition is at $\lambda = 4.61 \mu$m.

   (a) Estimate the frequencies of the J=1 $\rightarrow$ 0 transitions in ^{13}C^{16}O and ^{12}C^{17}O.

   (b) Estimate the wavelengths of the $\nu = 1 \rightarrow 0$ transitions in ^{13}C^{16}O and ^{12}C^{17}O.

   (c) Suppose that the ^{13}C^{16}O J=1 $\rightarrow$ 0 line were mistaken for the ^{12}C^{16}O J=1 $\rightarrow$ 0 line. What would be the error in the inferred radial velocity of the emitting gas?