Physics 224
The Interstellar Medium

Lecture #10: Dust Optical Properties, Heating & Cooling

Outline

- Part I: Dust Optical Properties
- Part II: Dust Heating & Cooling
- Part III: Dust Composition

How we learn about dust

- Extinction: wavelength dependence of how dust attenuates (absorbs & scatters) light
- Polarization: of starlight and dust emission
- Thermal emission from grains
- Microwave emission from spinning small grains
- Depletion of elements from the gas relative to expected abundance
- Presolar grains in meteorites or ISM grains from Stardust mission (7 grains!)

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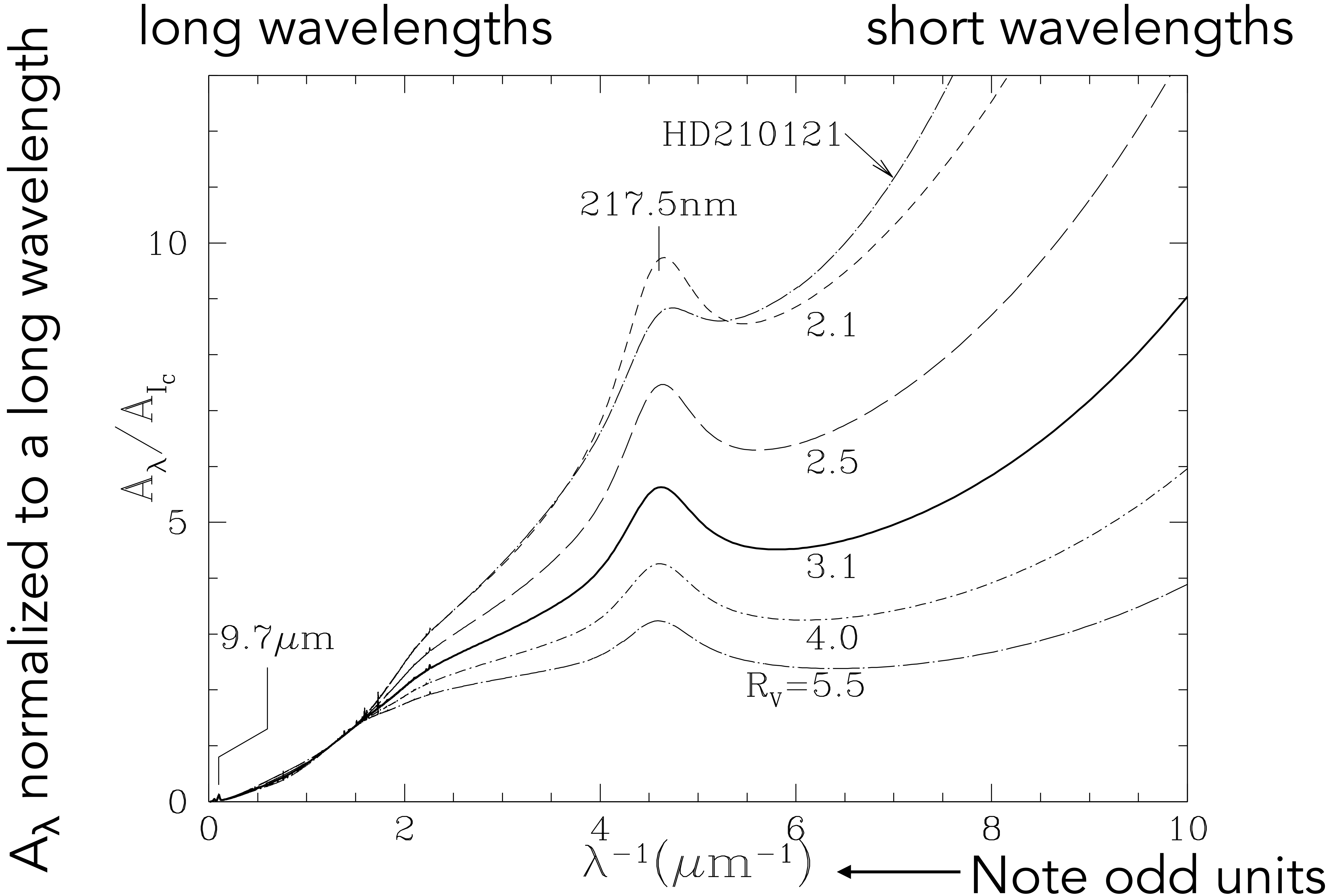
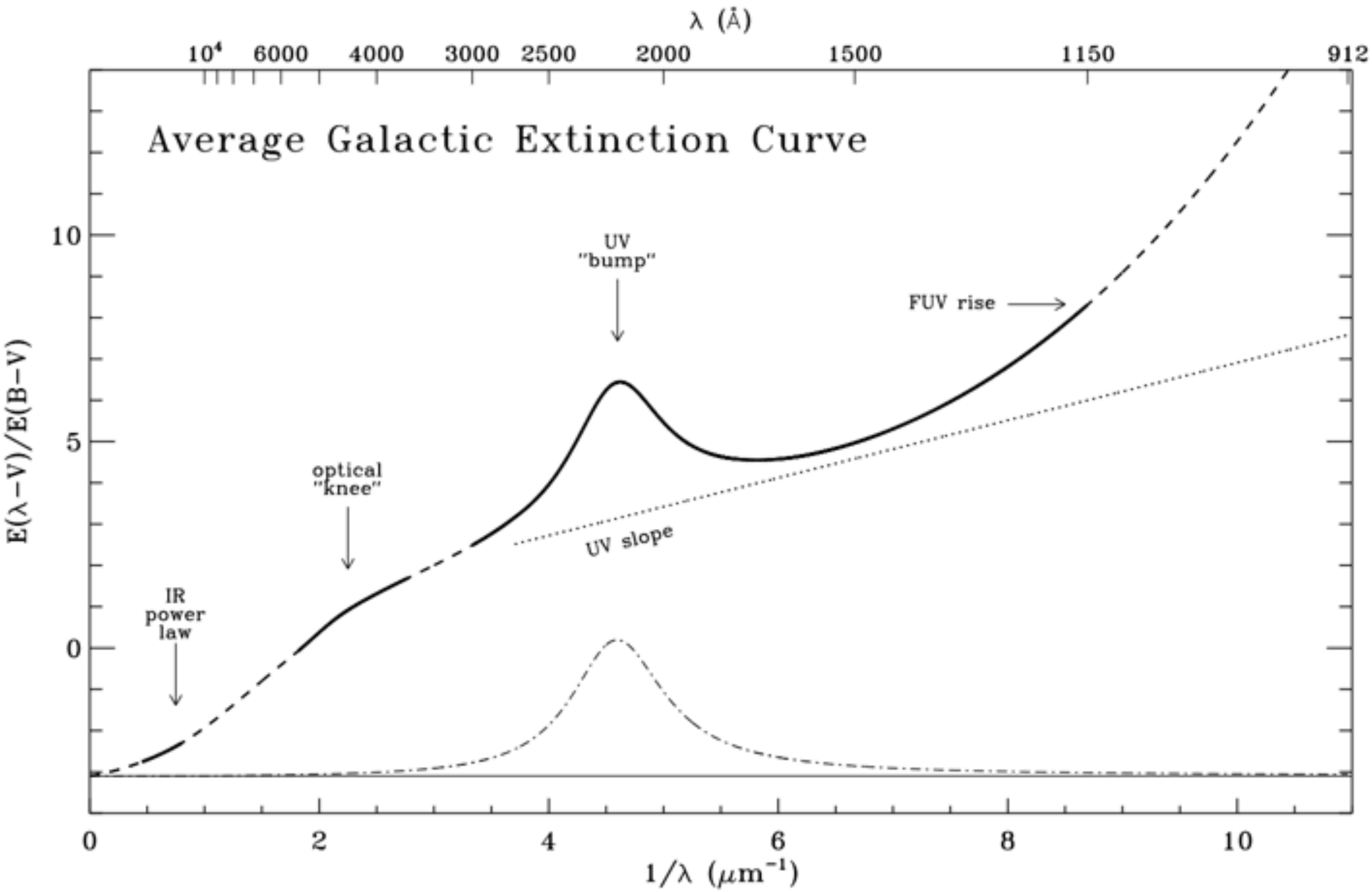
Dust/Light Interaction

Milky Way Dust Extinction Curves

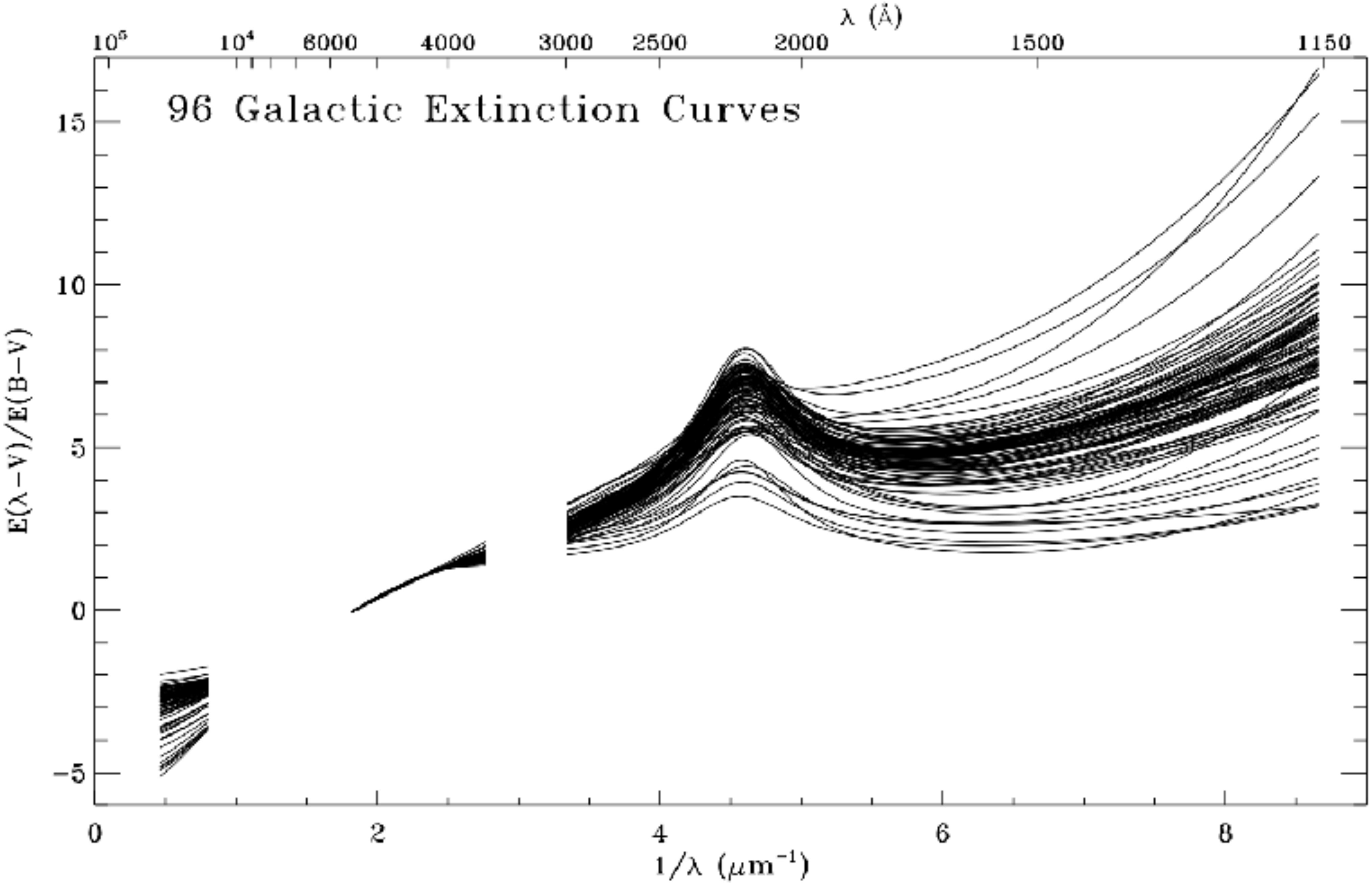


Milky Way Dust Extinction Curves



from Fitzpatrick 2004 review “Astrophysics of Dust”

Milky Way Dust Extinction Curves



from Fitzpatrick 2004 review "Astrophysics of Dust"

Reddening or "Color Excess"

Reddening or "Color Excess"

If we don't know the distance, we can still measure the change in the color of a star due to dust.

"color" = difference in magnitude at 2 wavelengths for example B band (4405 Å) and V band (5470 Å)

intrinsic $$(B-V)_0 = 2.5\log_{10}[F_B^0/F_V^0]$$

observed $$(B-V) = 2.5\log_{10}[F_B/F_V]$$

dependence on distance cancels, since it is the same at both wavelengths

Reddening or "Color Excess"

If we don't know the distance, we can still measure the change in the color of a star due to dust.

$$E(B-V) = (B-V)_0 - (B-V) = 2.5\,\log_{10}\left[\frac{F_B^0/F_V^0}{F_B/F_V}\right]$$

"color excess"
or "reddening"

Reddening or “Color Excess”

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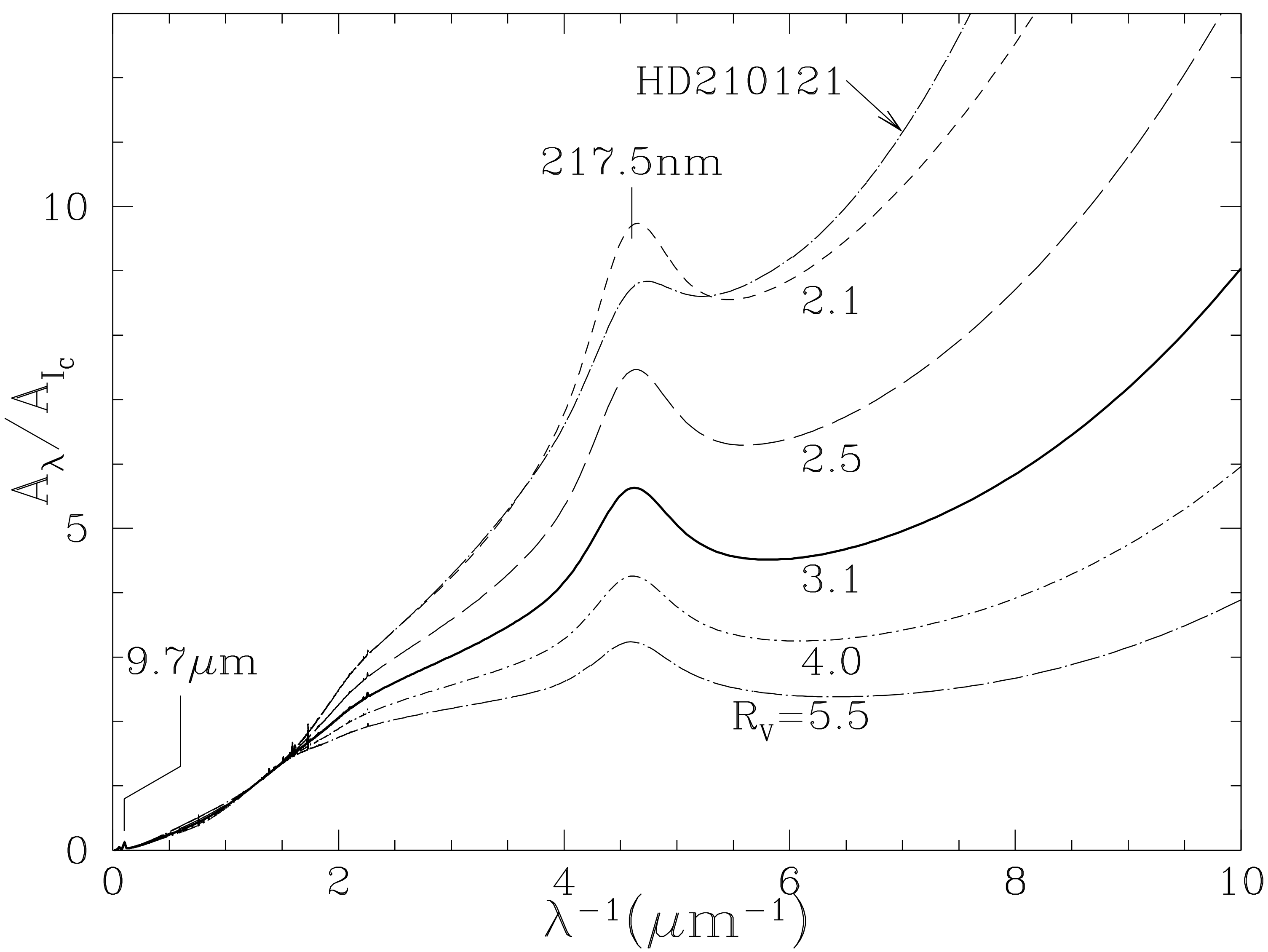
“color excess” or “reddening”

rearrange this

$$E(B-V) = 2.5\log_{10}[F_B^0/F_B] - 2.5\log_e[F_V^0/F_V] = A_B - A_V$$

Selective Extinction R_V

$$R_V \equiv \frac{A_V}{A_B - A_V} \equiv \frac{A_V}{E(B-V)}$$



R_V = slope of extinction curve in optical B & V bands

MW average R_V = 3.1 but it varies!

Selective Extinction R_V

$$R_V \equiv \frac{A_V}{A_B - A_V} \equiv \frac{A_V}{E(B-V)}$$

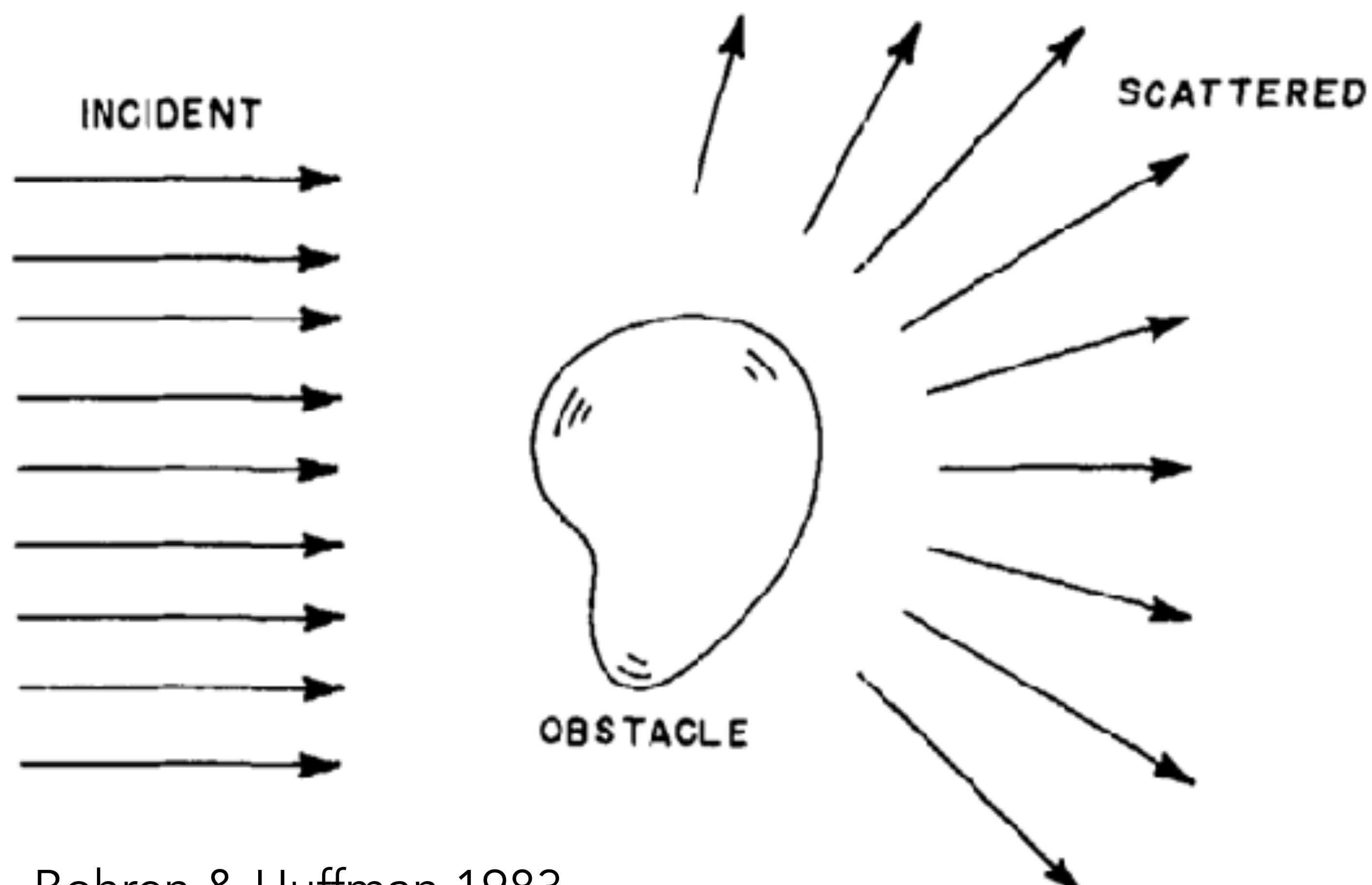
$V) - A_V/E(B-V)$. The quantity $A_V/E(B-V)$, i.e., the ratio of total extinction to color excess in the optical region, is usually denoted R_V. If its value can be determined for a line of sight, then the easily-measured normalized extinction can be converted into total extinction.

It has been noted often that $E(B-V)$ is a less-than-ideal normalization factor. Certainly a physically unambiguous quantity, such as the dust mass column density, would be preferred, or even a measure of the total extinction at some particular wavelength, such as A_V. However, the issue is simply measurability. We have no model-independent ways to assess dust mass and total extinction requires either that we have precise stellar distances or can measure the stellar SEDs in the far-IR where extinction is negligible. While IR photometry is now available for many stars through the *2MASS* survey, the determination of total extinction from these data still requires assumptions about the λ-dependence of extinction longward of 2μm and can be compromised by emission or scattering by dust grains near the stars. In this paper, all the observed extinction curves will be presented in the standard form of $E(\lambda - V)/E(B-V)$. Only in the case

- Fitzpatrick 2004 review “Astrophysics of Dust”

Scattering & Absorption of Light by Small Particles

Incoming EM wave, oscillations excited in scatterer, acceleration of charges causes re-radiation of EM waves in various directions.



Bohren & Huffman 1983

Scattering & Absorption of Light by Small Particles

define $x = 2\pi a/\lambda$ where a is the size of the object

can't treat entire grain as on dipole once $\lambda \sim a$, e.g., when $x \sim 1$ - need Mie Theory

$x \ll 1$: Rayleigh scattering

$x \sim 1$: Mie scattering

$x \gg 1$: Geometric scattering

Scattering & Absorption of Light by Small Particles

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$x \gg 1$: Geometric scattering

geometric optics, reflection, refraction, etc

Scattering & Absorption of Light by Small Particles

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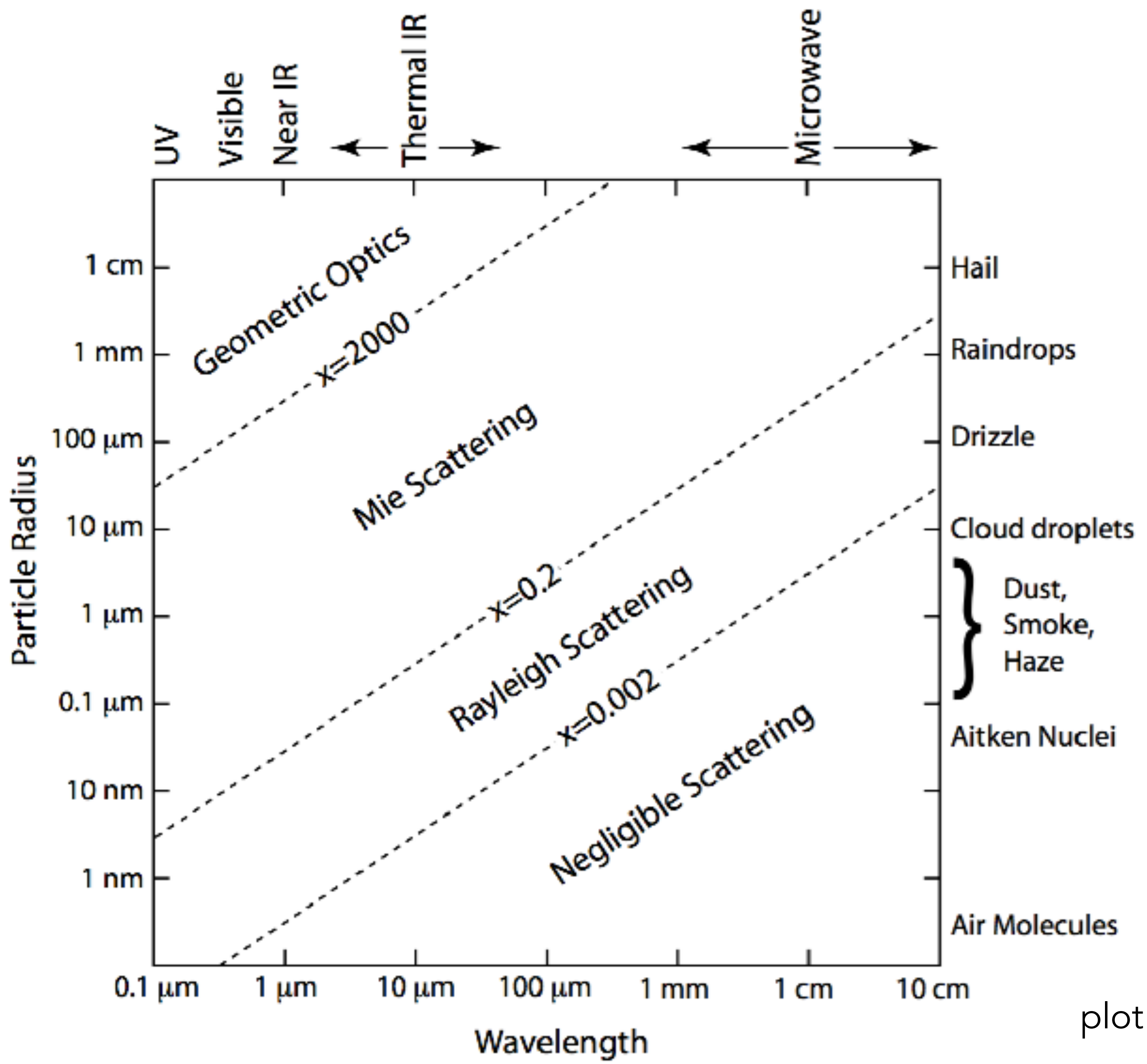
x $\ll$ **1**: Rayleigh scattering → treat scatterers as dipoles

x $\sim$ **1**: Mie scattering → the hard stuff in between

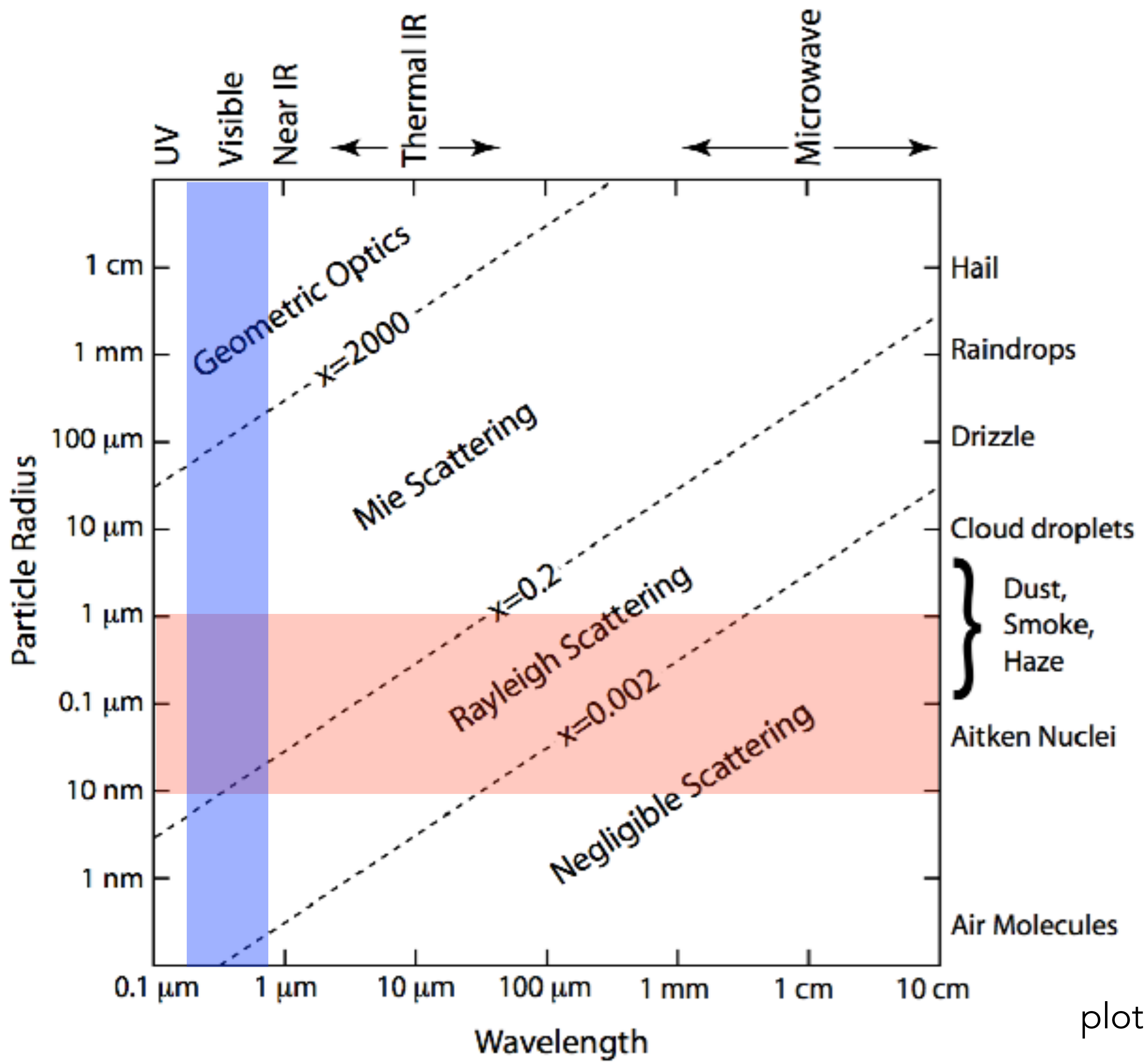
x $\gg$ **1**: Geometric scattering → geometric optics, reflection, refraction, etc

Scattering & Absorption of Light by Small Particles



plot from L. Lelli WS2014 slides

Scattering & Absorption of Light by Small Particles



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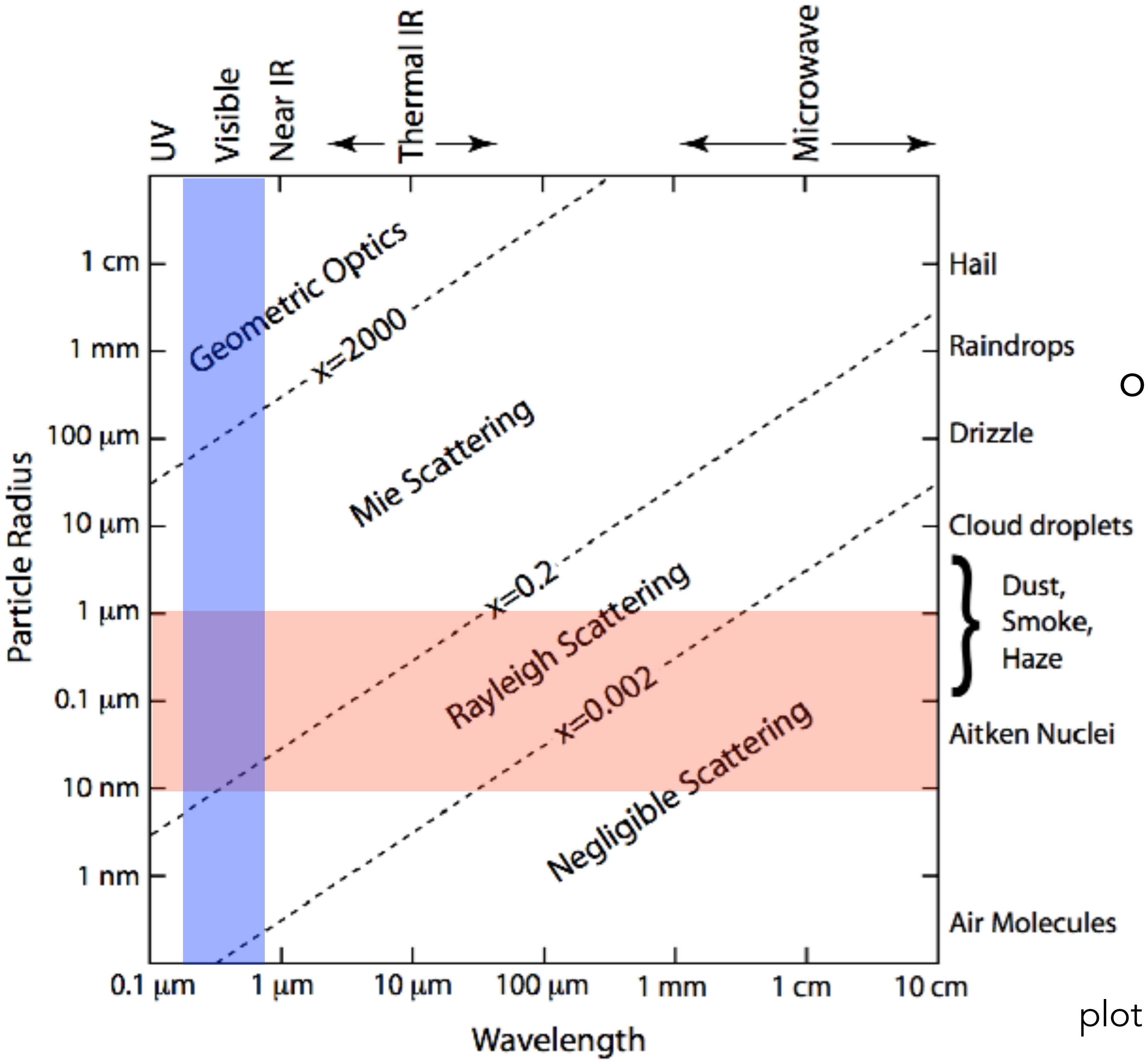
Scattering & Absorption of Light by Small Particles

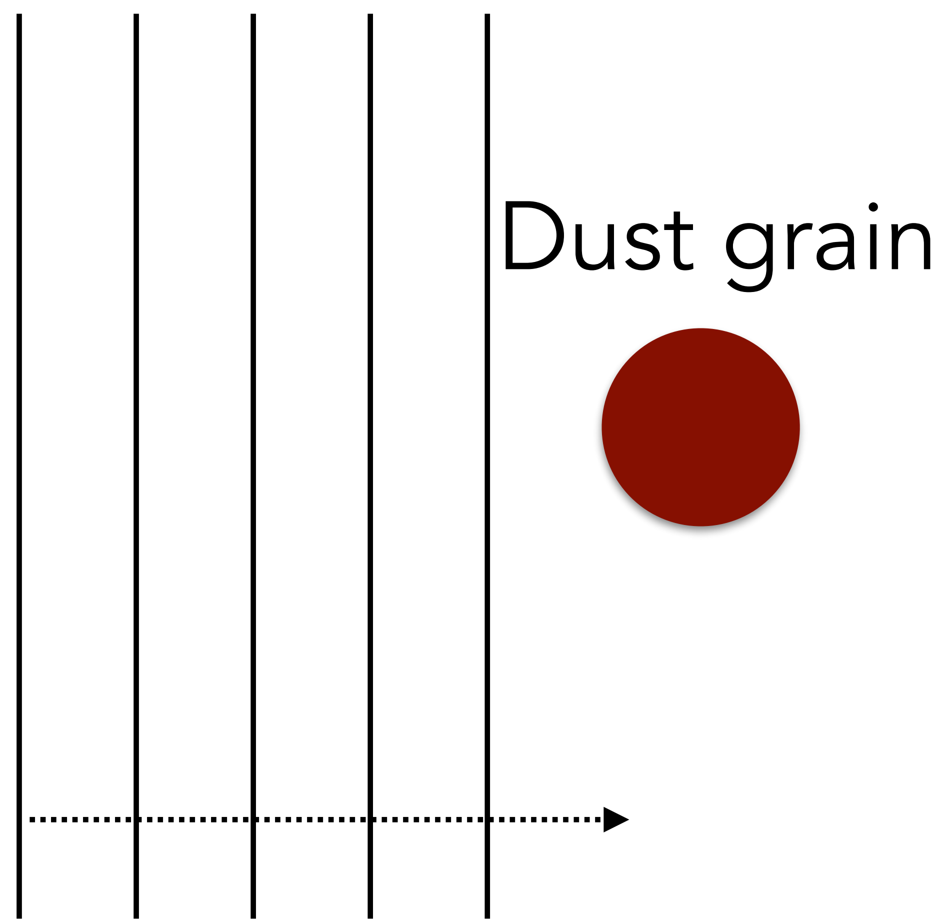


interstellar dust interaction with optical light is firmly in Mie scattering

plot from L. Lelli WS2014 slides

Scattering & Absorption of Light by Small Particles

key reference: Bohren & Huffman textbook



Scattering & absorption result from interaction of grain material with oscillating E & B field

when wavelength of light is < mm
magnetic permeability = 1
can ignore magnetic field interaction

plane EM wave $\lambda = 2\pi c/\omega$

$E = E_0 \, e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}$

Scattering & Absorption of Light by Small Particles

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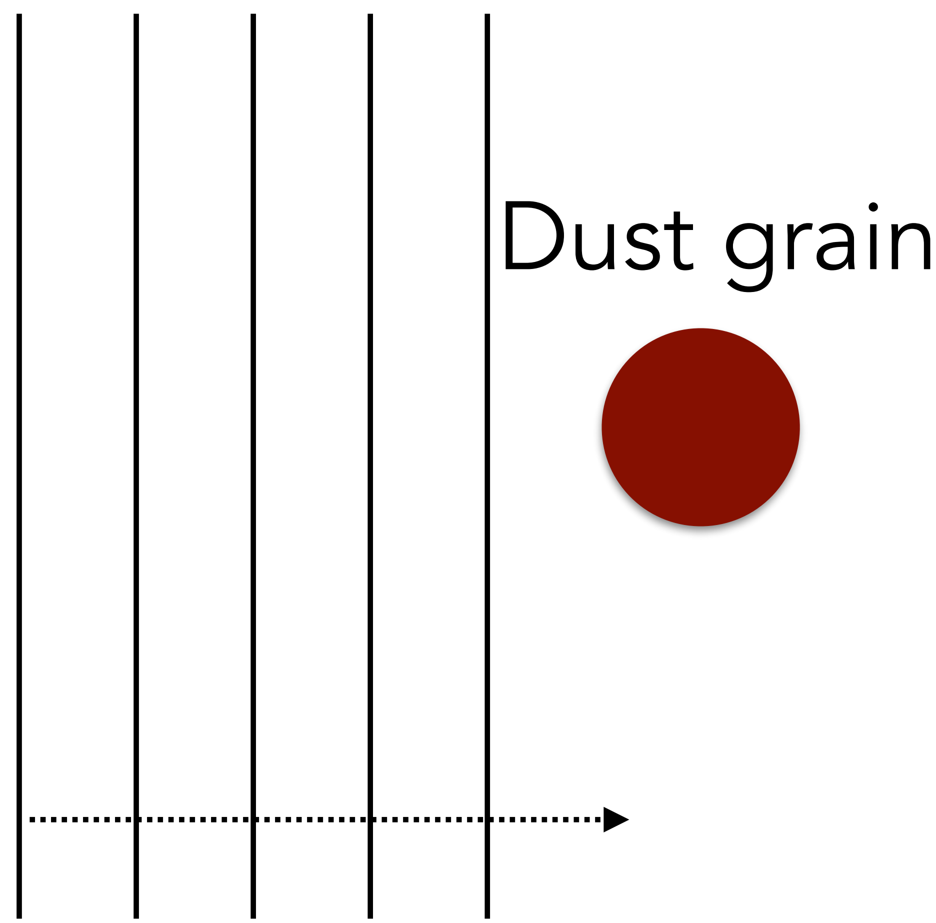


plane EM wave

$E = E_0 \, e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}$

Scattering & absorption result from interaction of grain material with oscillating E & B field

response of material to E field set by *dielectric function*

$$\epsilon(\omega) = \epsilon_1 + i\epsilon_2$$

related to *refractive index*

$$m = \sqrt{\epsilon}$$

Scattering & Absorption of Light by Small Particles

Define:

Geometrical Cross Section: πa^2

Absorption Cross Section: $C_{abs}(\lambda)$

Scattering Cross Section: $C_{sca}(\lambda)$

Extinction Cross Section: $C_{ext}(\lambda) = C_{abs}(\lambda) + C_{sca}(\lambda)$

Scattering & Absorption of Light by Small Particles

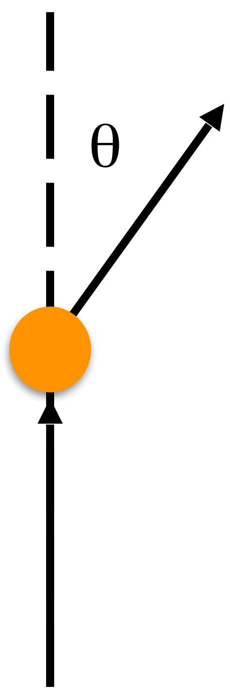
Define:

Geometrical Cross Section: πa^2

Scattering & Absorption Efficiency Factors

$Q_{abs} = C_{abs}/\pi a^2$ $\qquad$ $Q_{sca} = C_{sca}/\pi a^2$

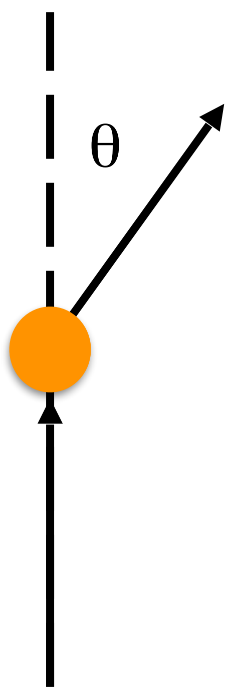
Scattering & Absorption of Light by Small Particles



Scattering Definitions:

Albedo = C_{sca}/C_{ext}

Scattering & Absorption of Light by Small Particles



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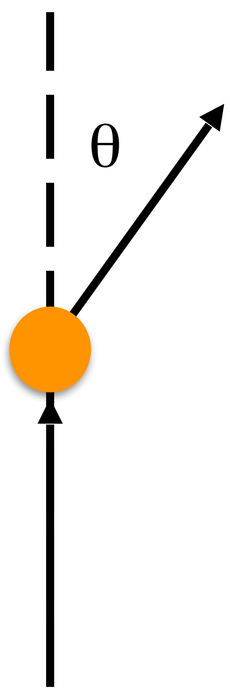
Differential scattering angle $$\frac{dC_{sca}(\theta)}{d\Omega}$$

Scattering & Absorption of Light by Small Particles



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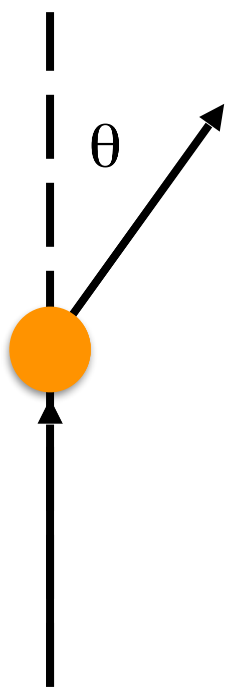
Scattering asymmetry factor $\langle\cos\theta\rangle = \frac{1}{C_{sca}}\int_0^{\pi}\cos\theta\frac{dC_{sca}(\theta)}{d\Omega}2\pi\sin\theta d\theta$

Scattering & Absorption of Light by Small Particles



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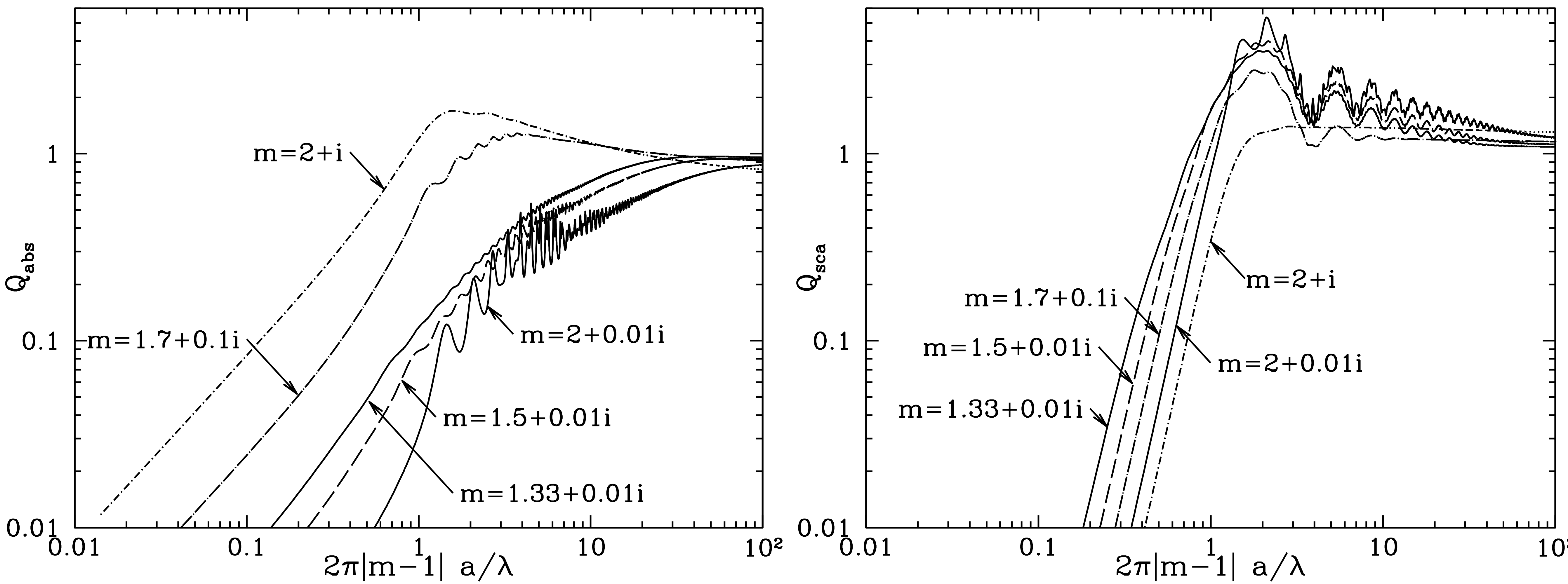
- Isotropic scattering $\langle\cos\theta\rangle = 0$
- Forward scattering $\langle\cos\theta\rangle = 1$
- Back scattering $\langle\cos\theta\rangle = -1$

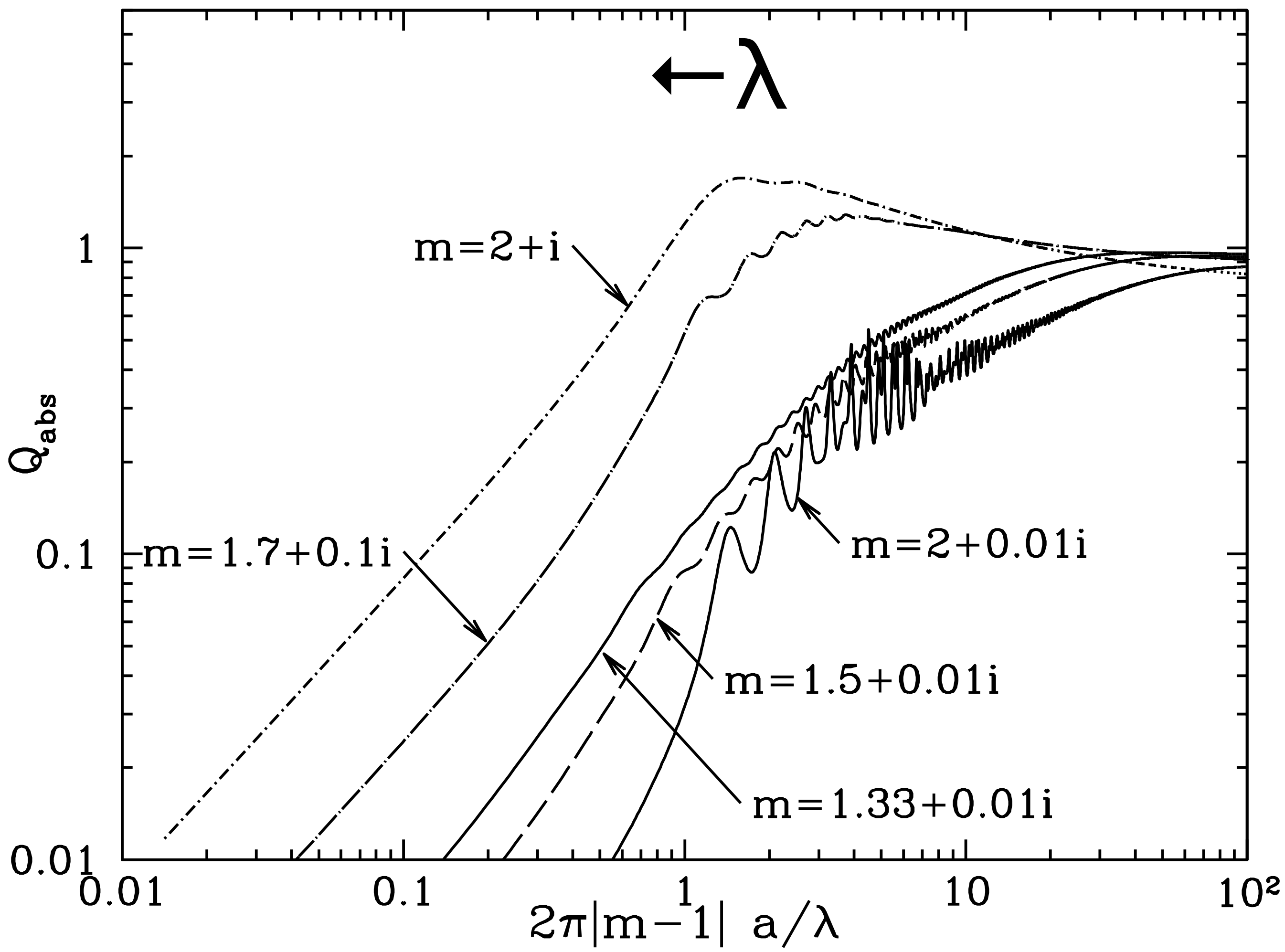
Scattering & Absorption of Light by Small Particles



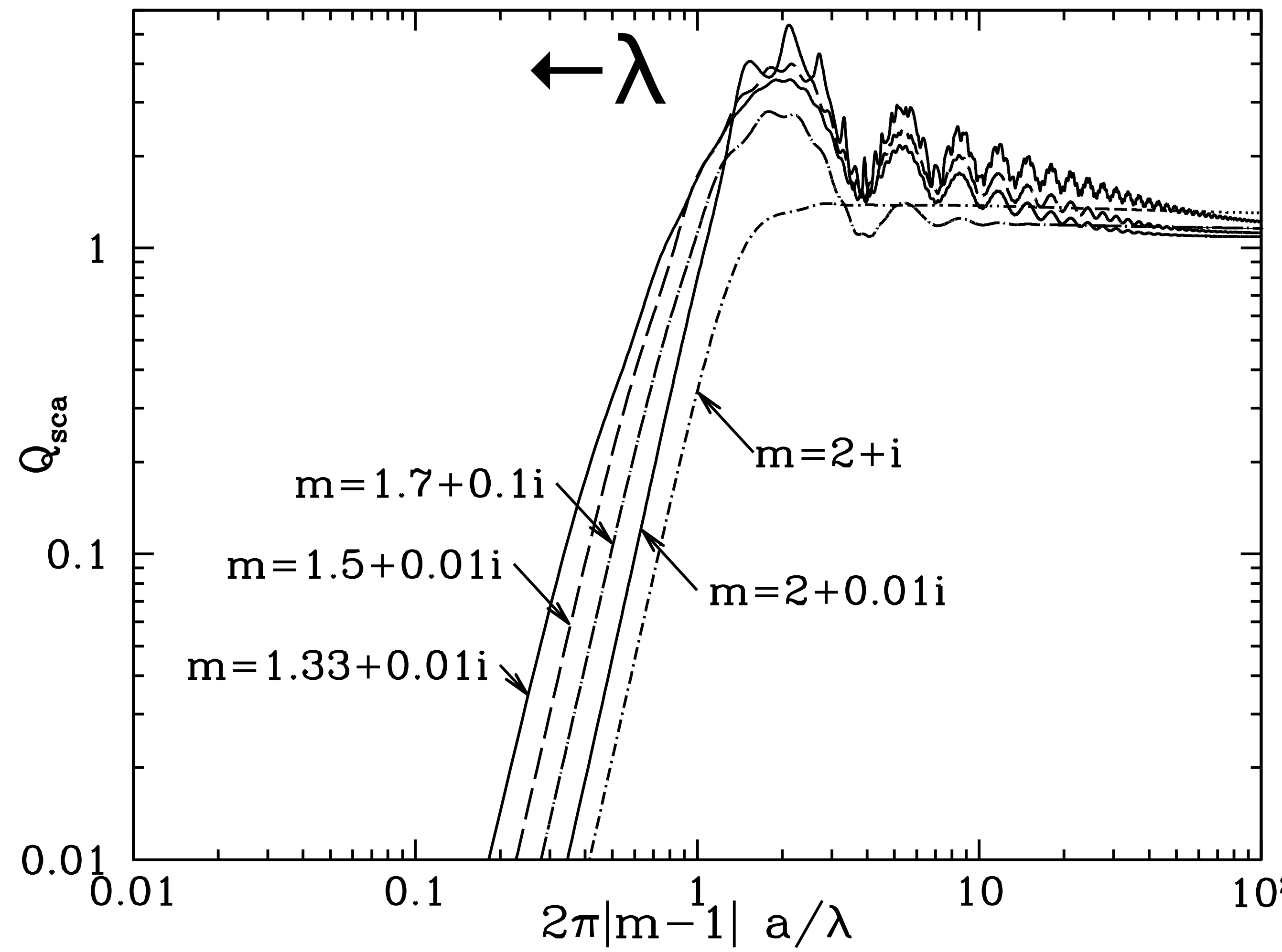
a/λ - grain size relative to wavelength of light defines different regimes

Scattering & Absorption of Light by Small Particles

for fixed a



for fixed a



$a/\lambda << 1$ grain much smaller than wavelength - analytic soln's

$$Q_{\rm abs} = 4\frac{2\pi a}{\lambda}{\rm Im}\left(\frac{\epsilon-1}{\epsilon+2}\right)$$

$$Q_{\rm sca} = \frac{8}{3}\left(\frac{2\pi a}{\lambda}\right)^4\left|\frac{\epsilon-1}{\epsilon+2}\right|^2$$

Scattering & Absorption of Light by Small Particles

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In long wavelength limit, general behavior is:

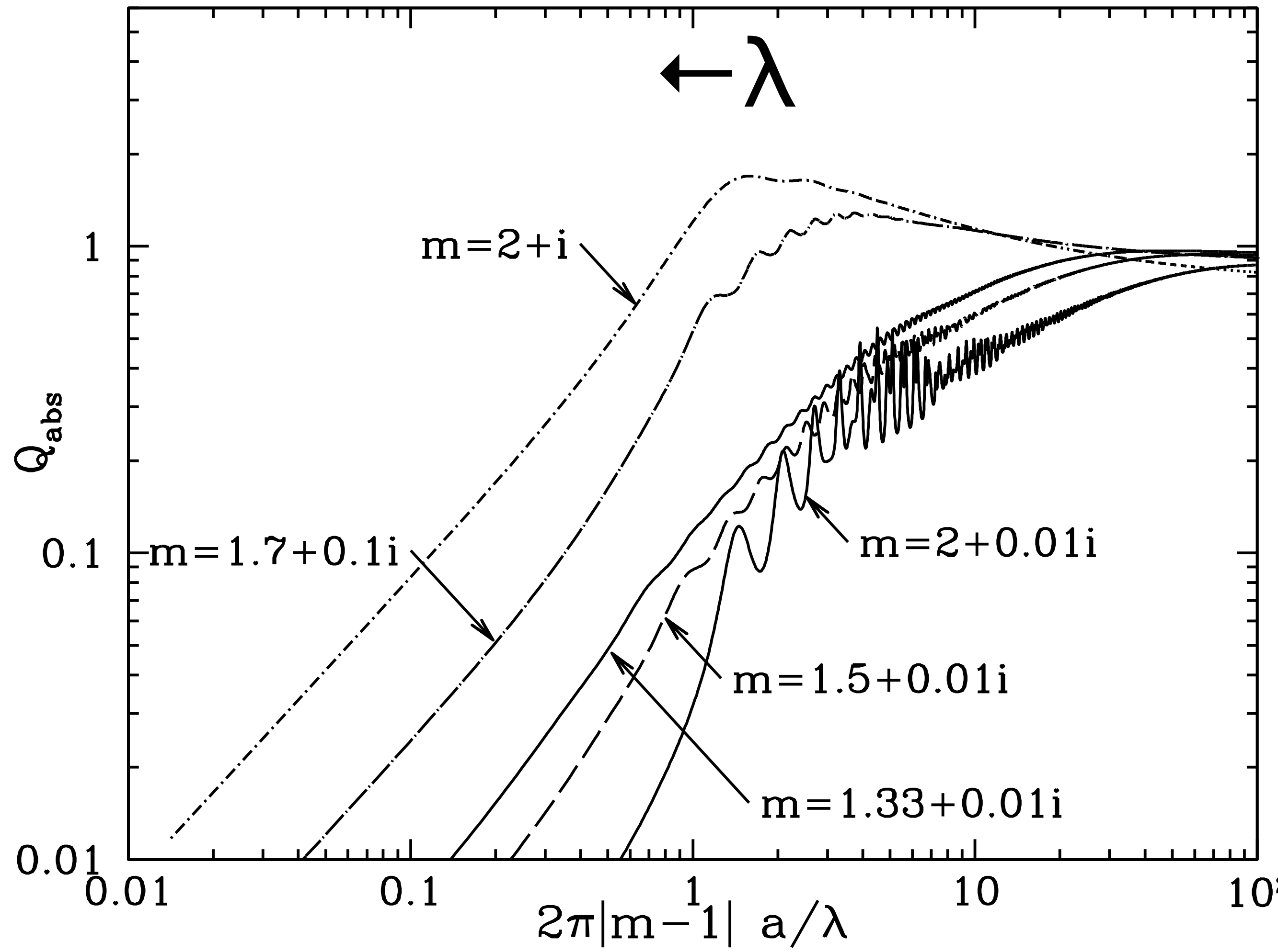
$Q_{abs} \sim V/\lambda^2$

$Q_{sca} \sim V^2/\lambda^4$

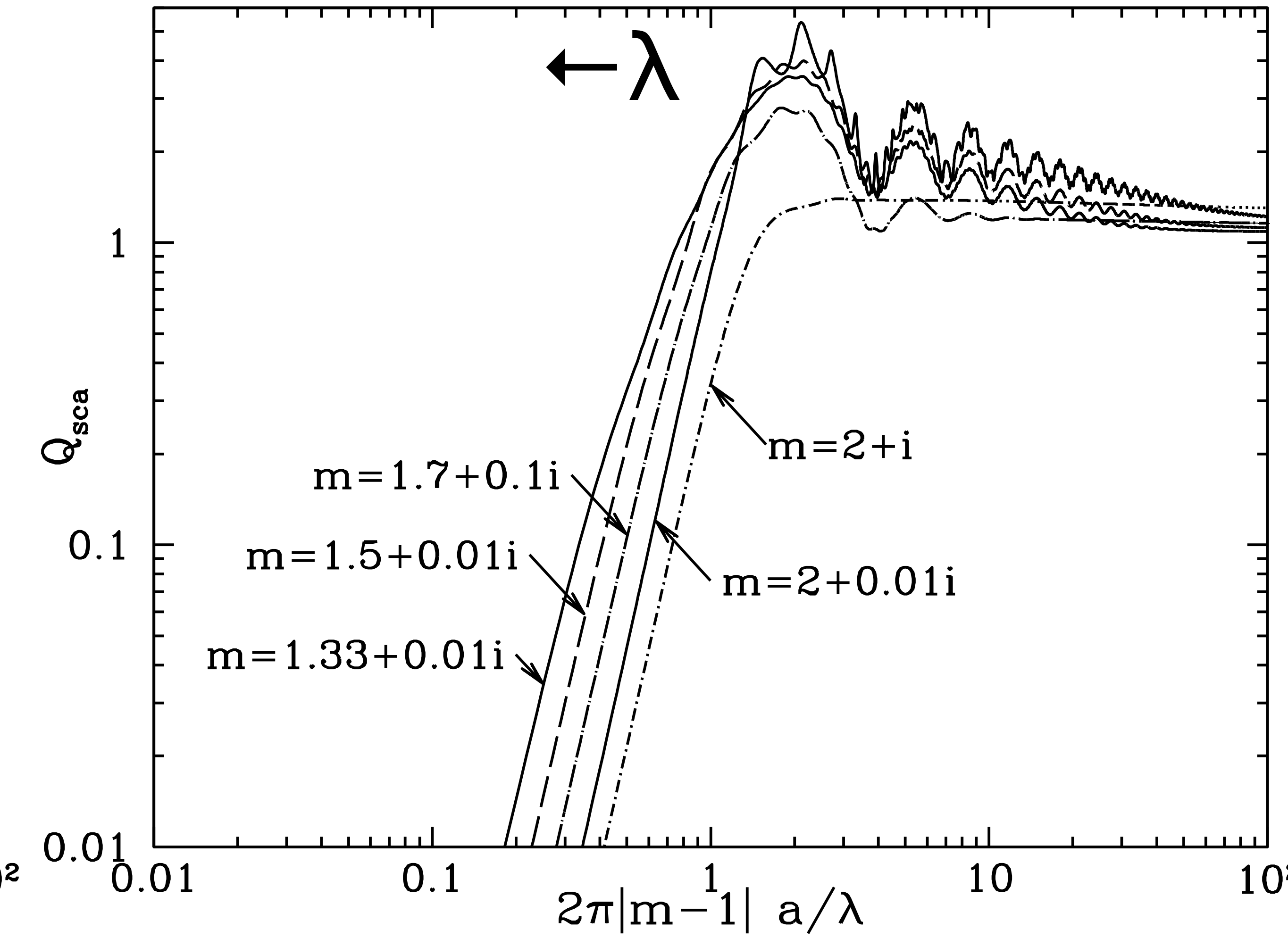
where V = grain volume

Scattering & Absorption of Light by Small Particles

for fixed a



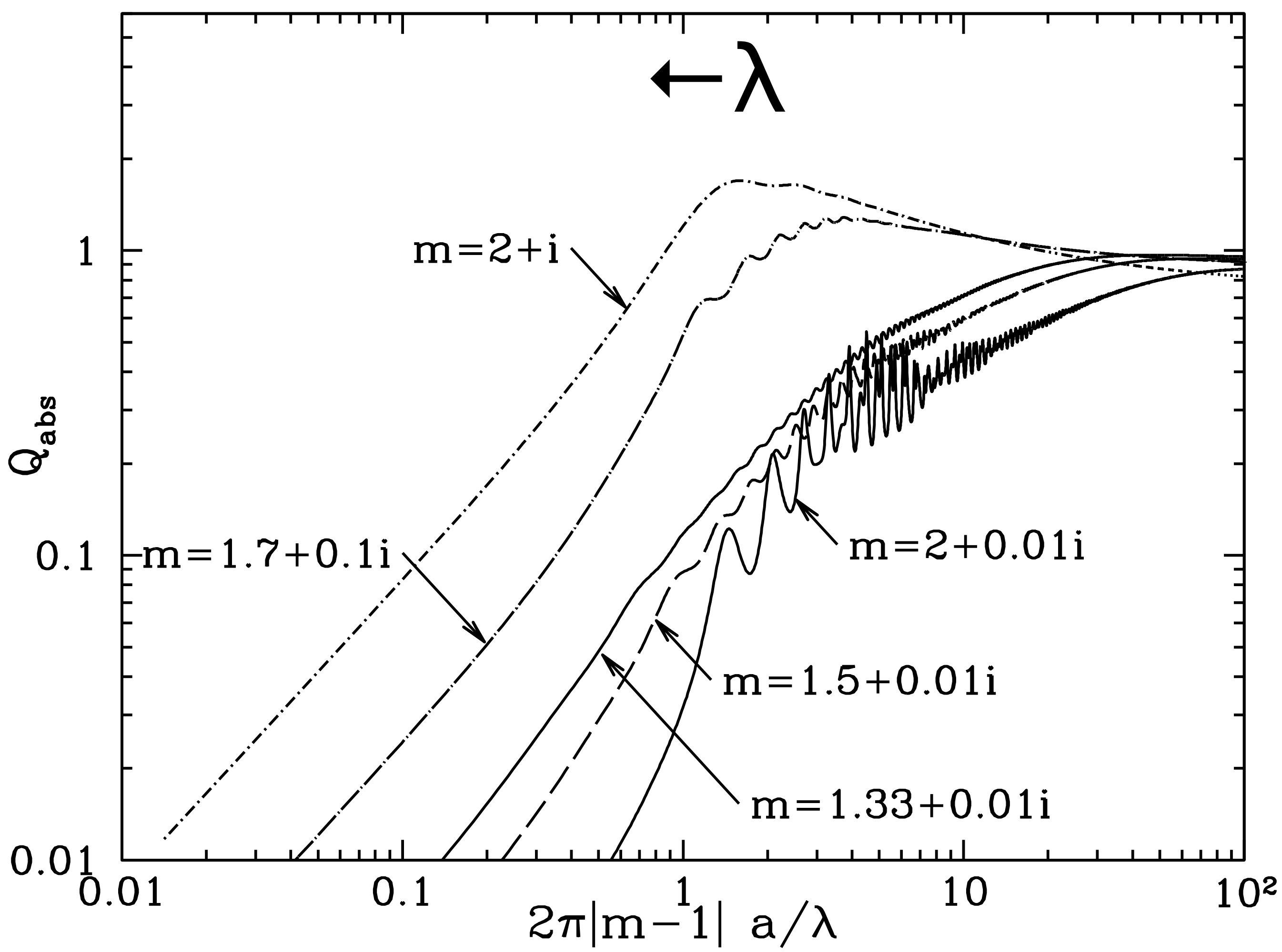
for fixed a



a/λ ~ 1 use Mie theory

Scattering & Absorption of Light by Small Particles

for fixed a



$$Q_{\mathrm{abs}} = 4\frac{2\pi a}{\lambda}\mathrm{Im}\left(\frac{\epsilon - 1}{\epsilon + 2}\right)$$

<u>Absorption</u>

note that
at long wavelength:

$C_{abs} = Q_{abs}\pi a^2 \propto a^3 \propto m_{dust}$

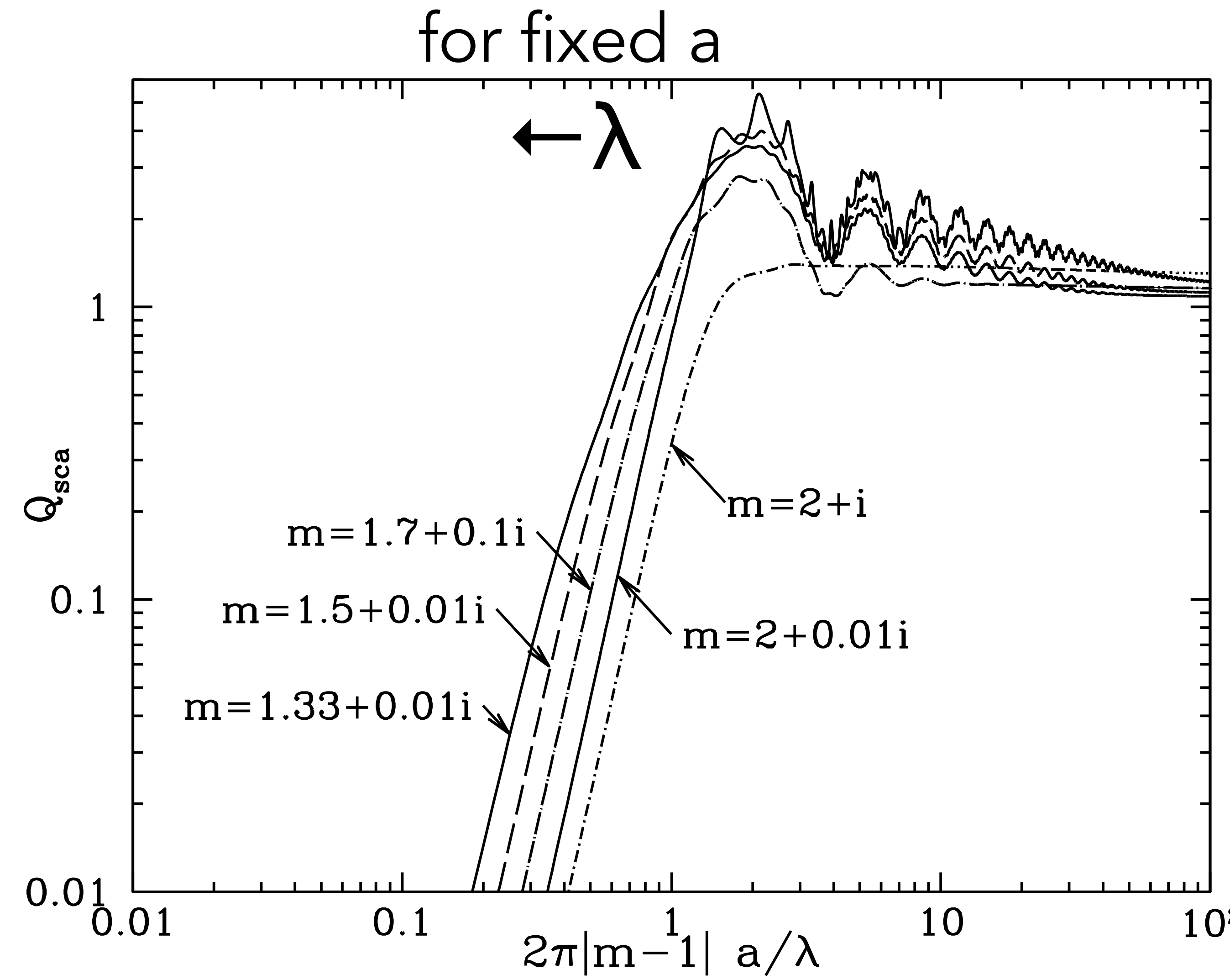
absorption efficiency when
a >> λ
levels off to 1, $C_{abs} = \pi a^2$

Scattering & Absorption of Light by Small Particles

<u>Scattering</u>

scattering efficiency drops steeply with wavelength when $a/\lambda << 1$

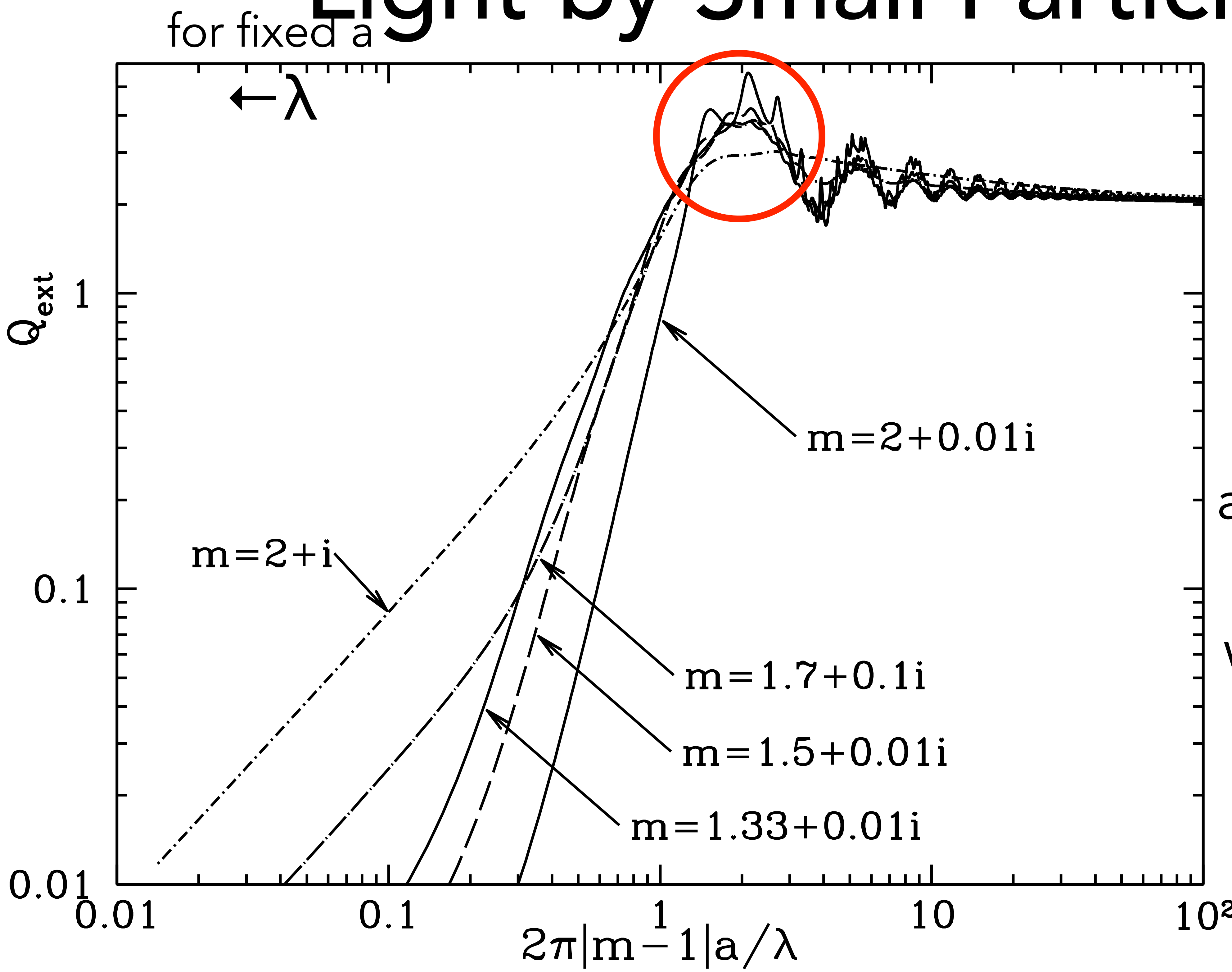
Rayleigh scattering λ^{-4}



$$Q_{\rm sca} = \frac{8}{3}\left(\frac{2\pi a}{\lambda}\right)^4 \left|\frac{\epsilon - 1}{\epsilon + 2}\right|^2$$

Reflection Nebula vdB1
Image Credit & Copyright: Adam Block,
Mt. Lemmon SkyCenter, University of Arizona

Scattering & Absorption of Light by Small Particles

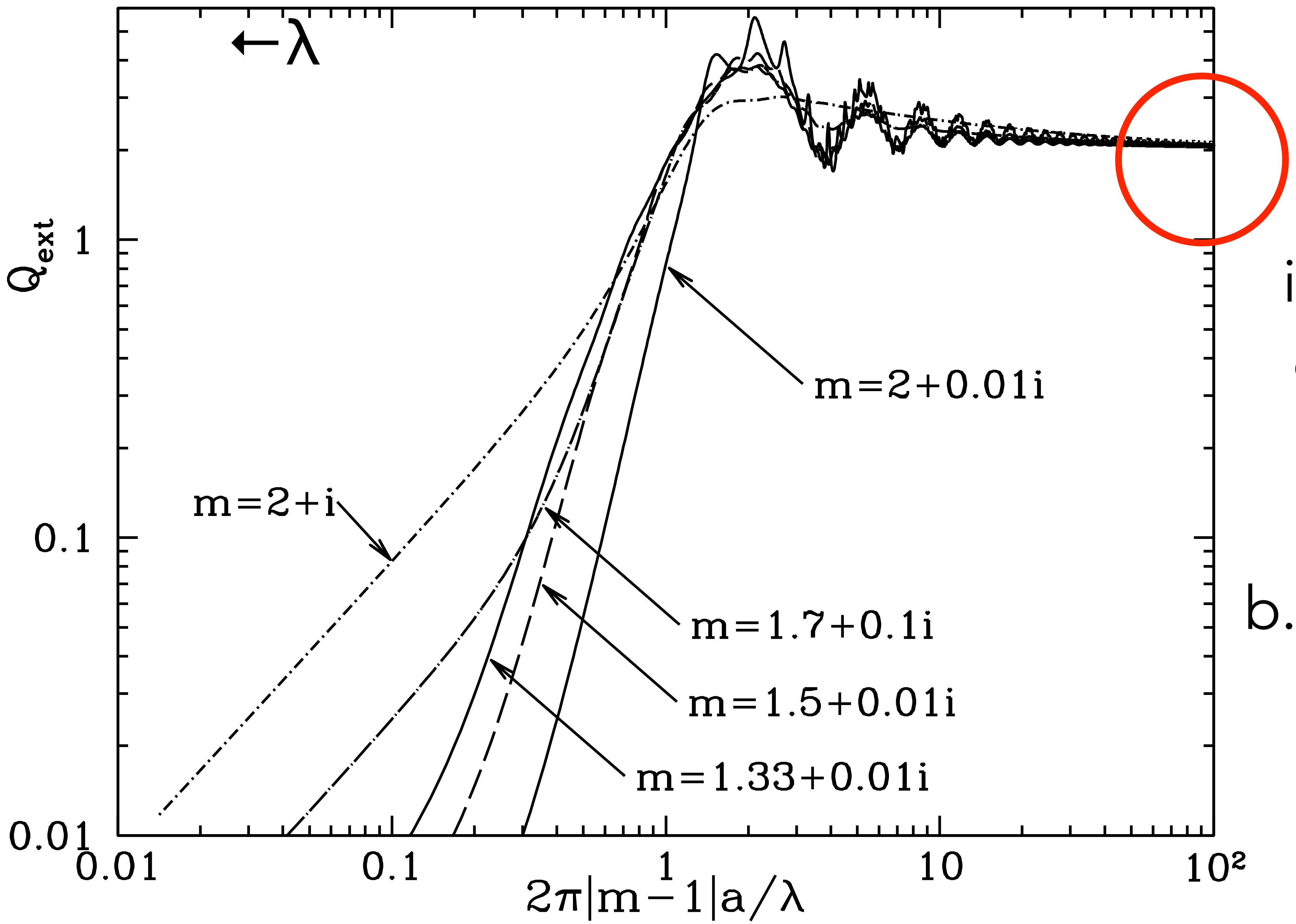


Maximum Q_{ext} occurs where $\lambda \sim a$

i.e. dust grains are most effective at blocking light with wavelengths close to their sizes

Scattering & Absorption of Light by Small Particles

for fixed a



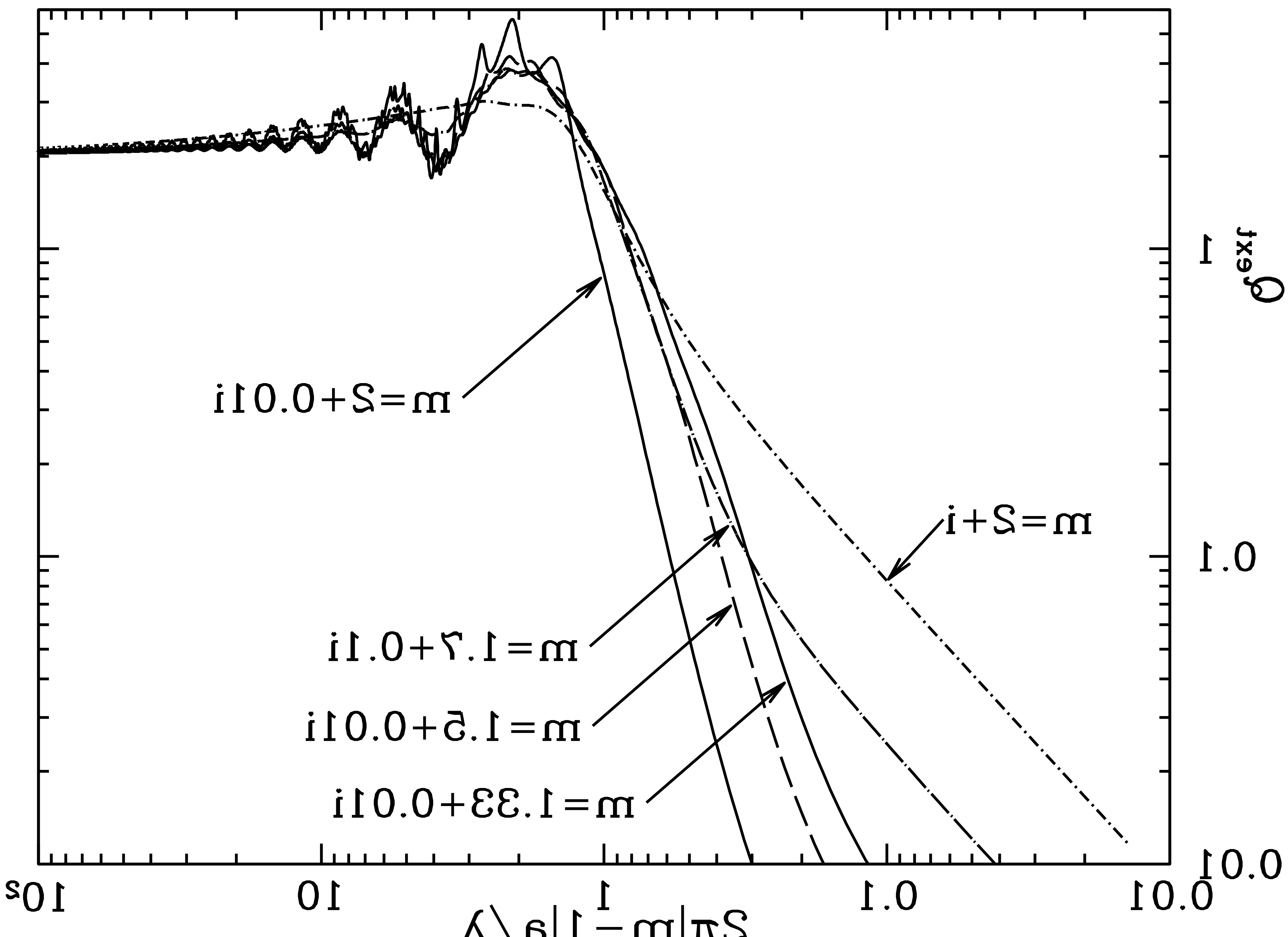
when a>>λ

Q_{ext} ~ 2

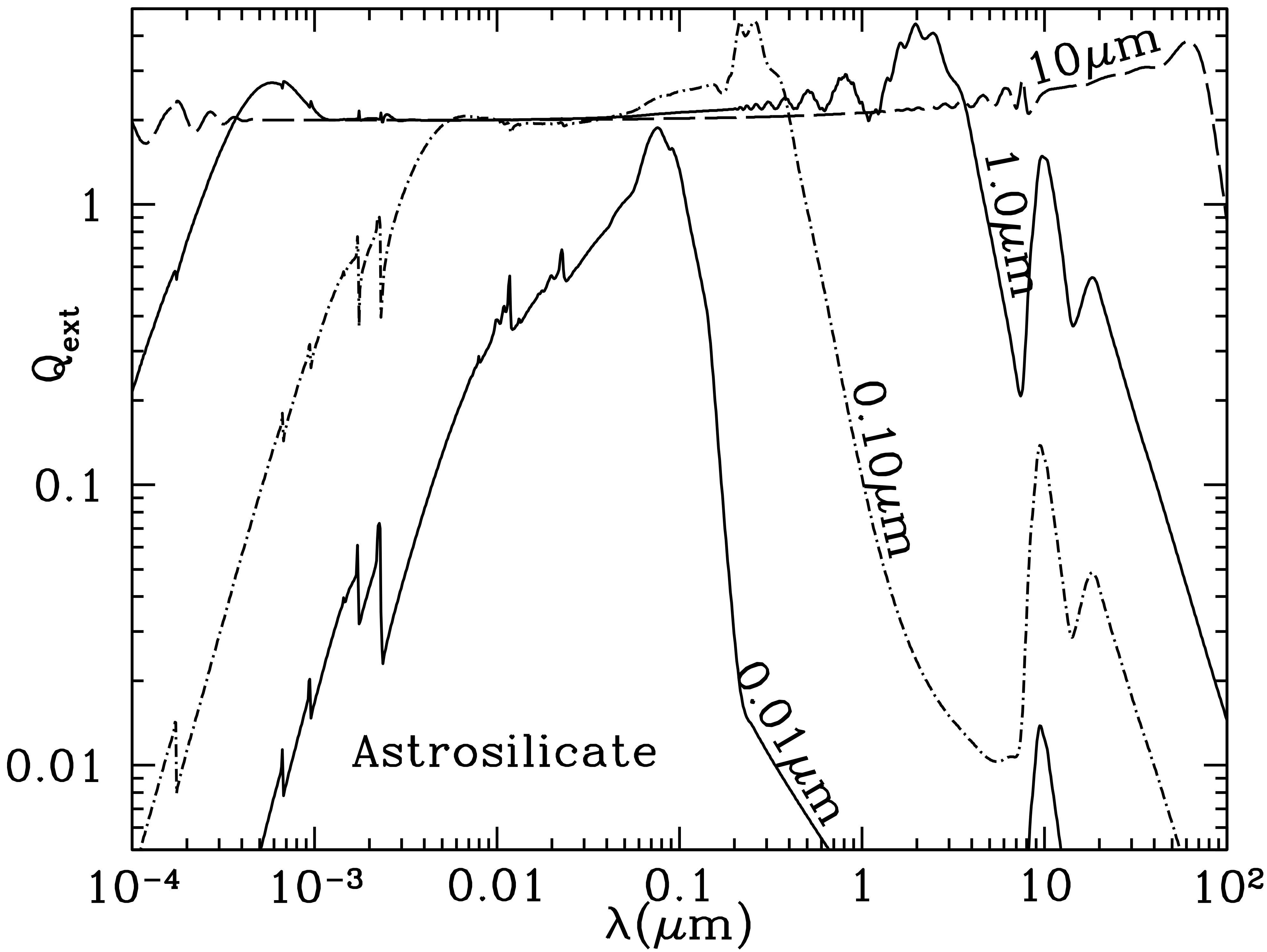
i.e. twice the geometrical cross sec

b.c. diffraction!!

Scattering & Absorption of Light by Small Particles

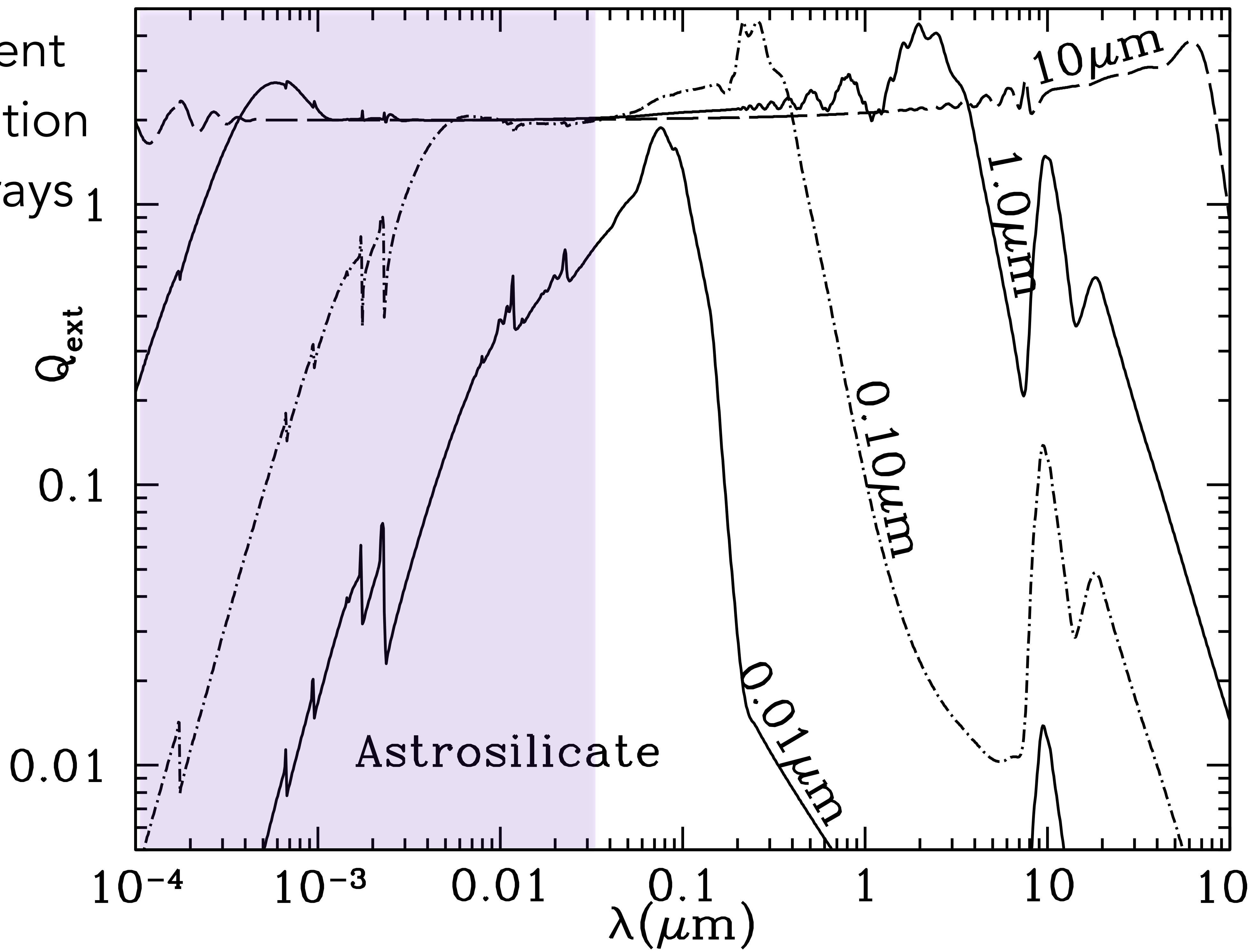


Astronomical Dust

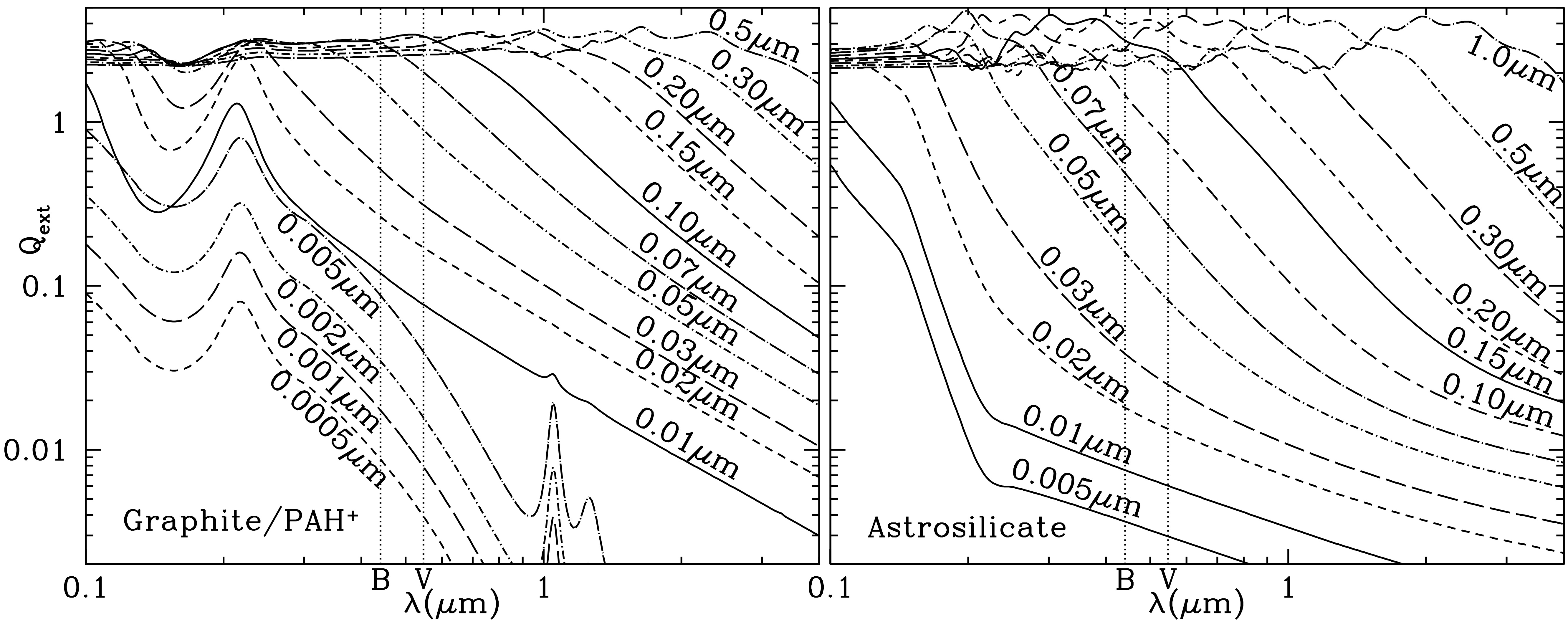


Astronomical Dust

Need different calculation for x-rays

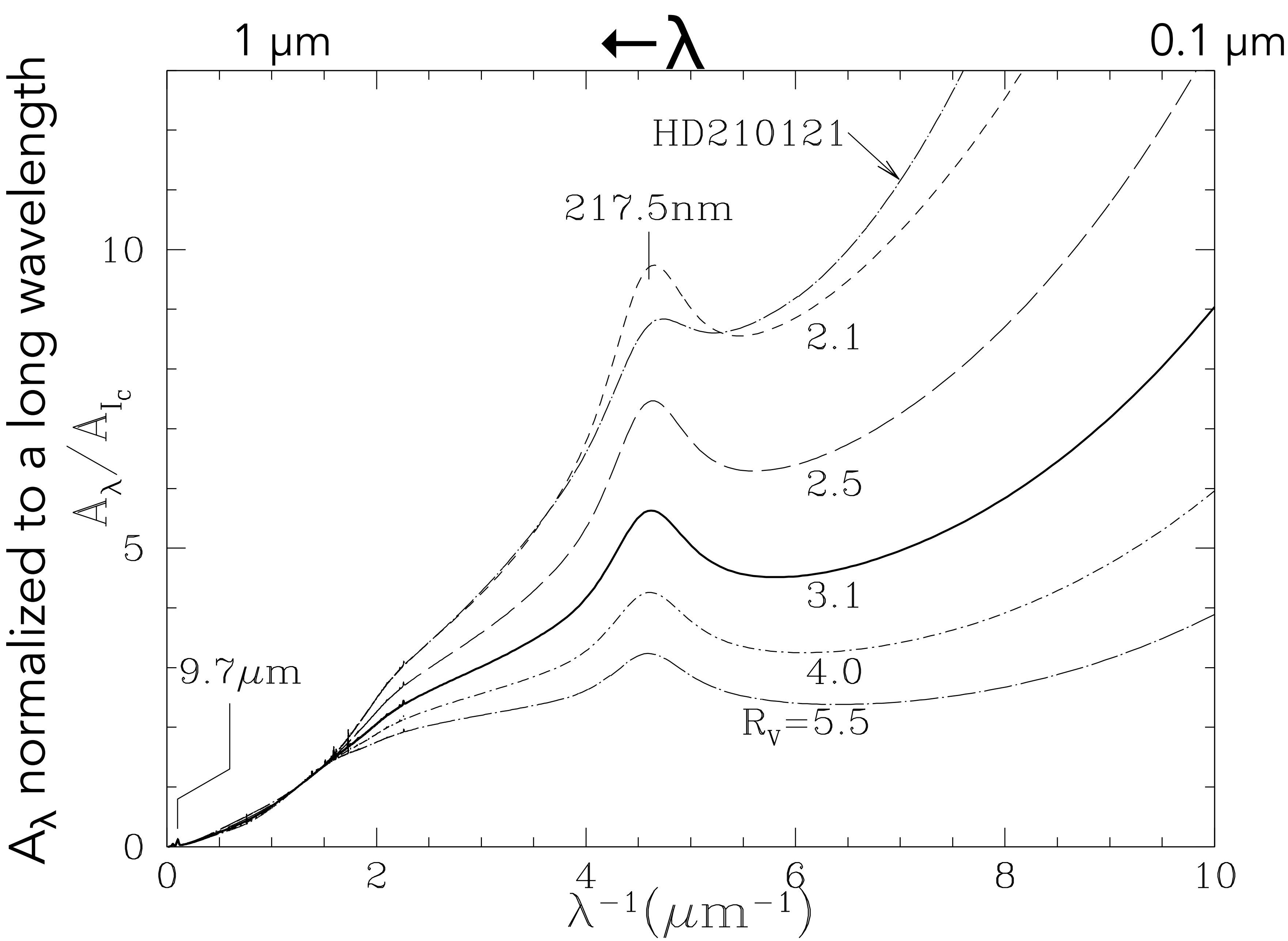


Astronomical Dust



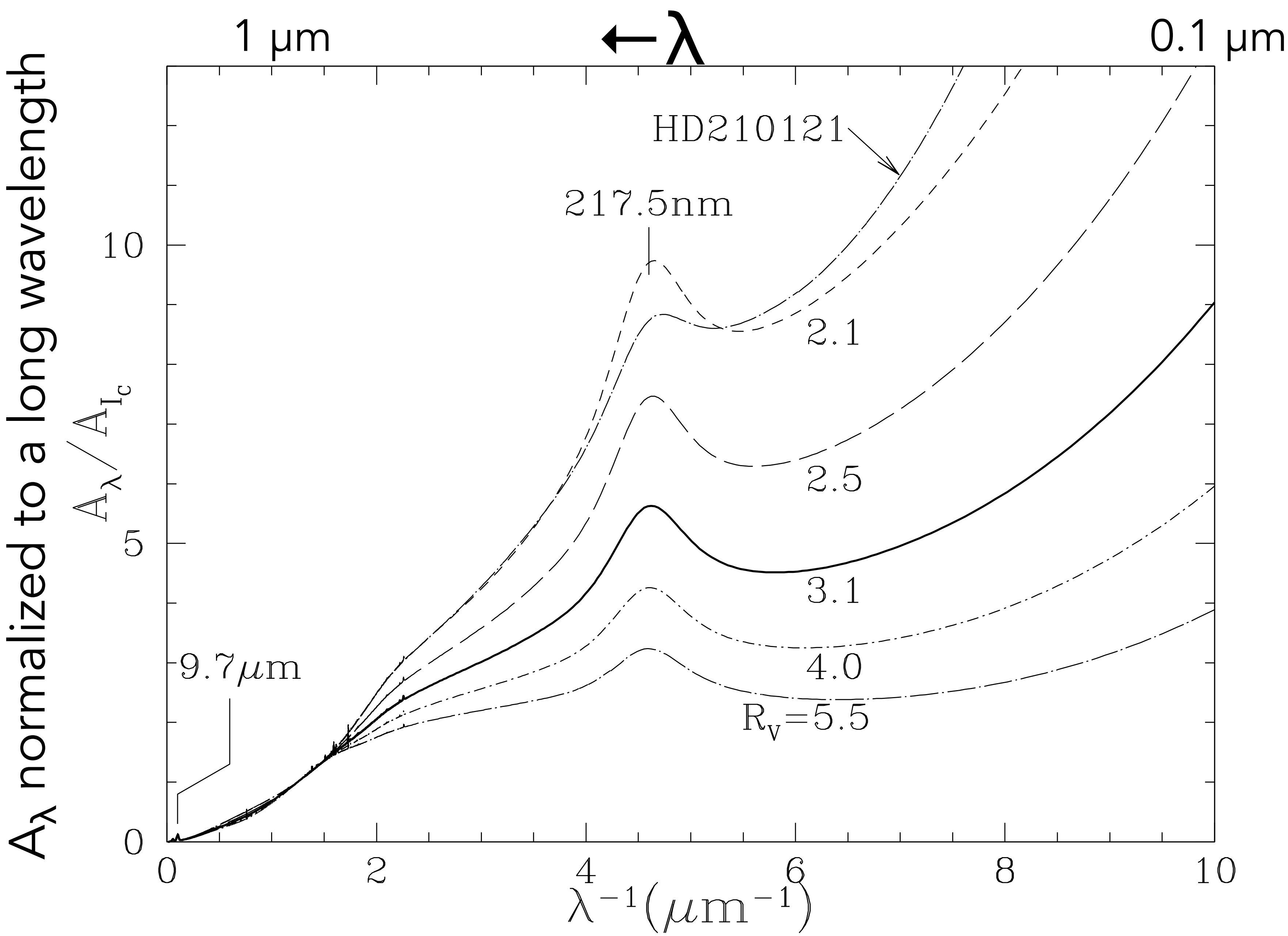
Q_{ext} for astronomical dust analogs

Extinction Curve



This does not look like the Q_{ext} plots from before - why?

Extinction Curve



This does not look like the Q_{ext} plots from before - why?

There is a range of grain sizes!

Extinction Curve

$$\frac{A_\lambda}{\mathrm{mag}} = 2.5 \log \left[e^{\tau_\lambda}\right] = 1.086 \tau_\lambda$$

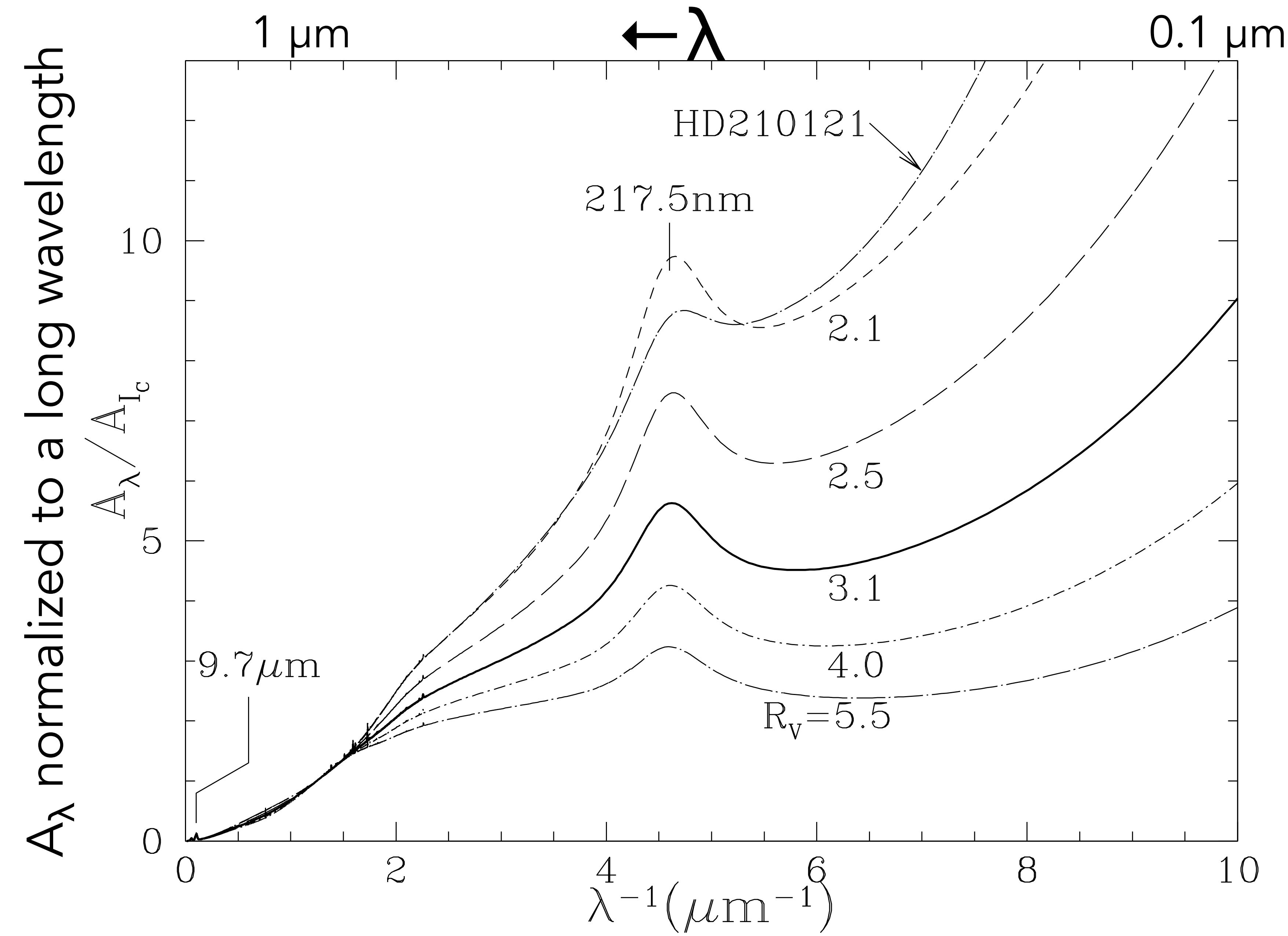
For a given grain size:

$$\tau_\nu(a) = N_d(a)\ Q_{ext}(a)\ \pi a^2$$

Rearrange units to get Weingartner & Draine 2001 eq 7:

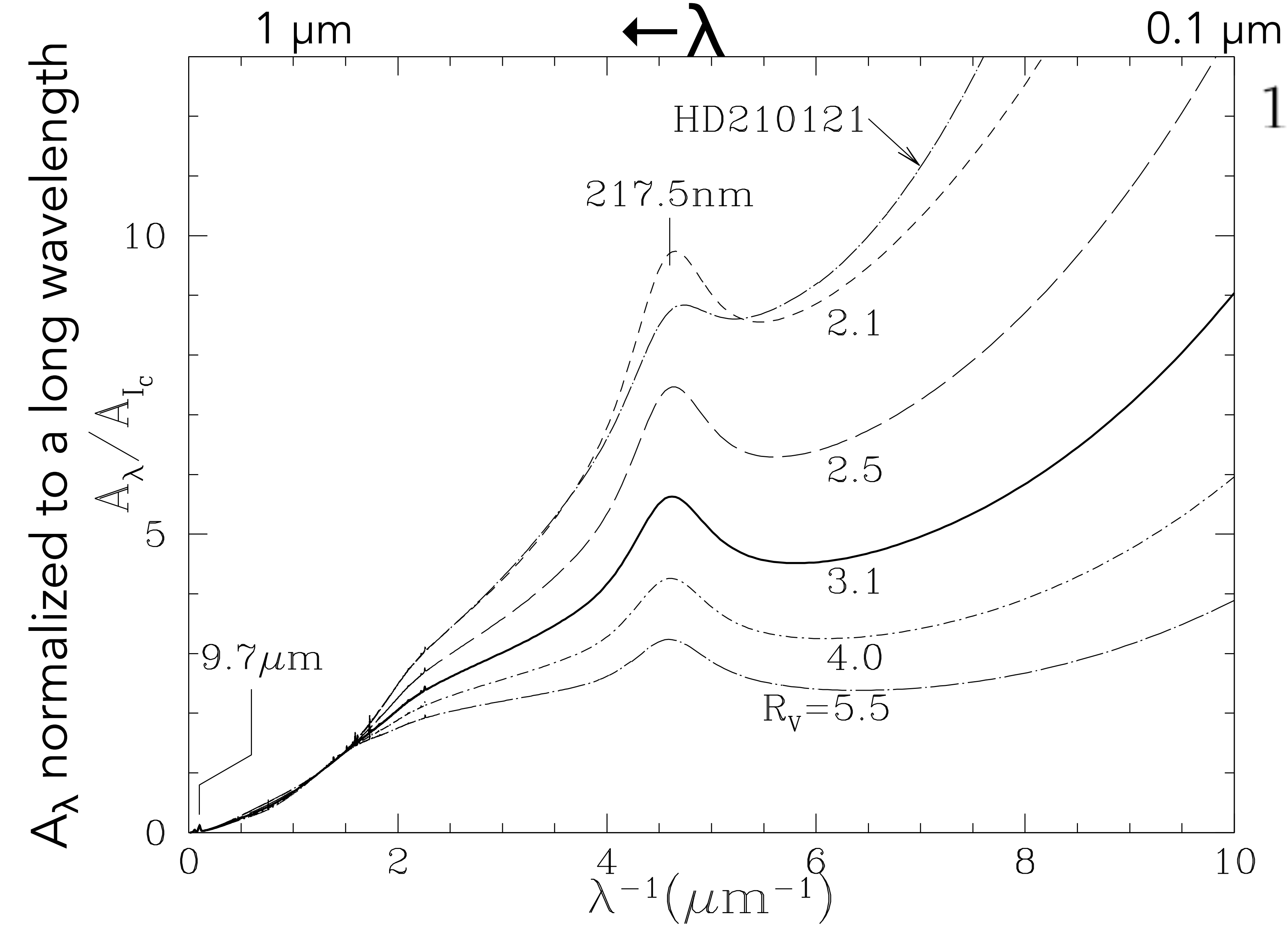
$$A(\lambda) = (2.5\pi \log e) \int d \ln a \, \frac{dN_{\mathrm{gr}}(a)}{da} \, a^3 Q_{\mathrm{ext}}(a, \lambda)$$

Extinction Curve



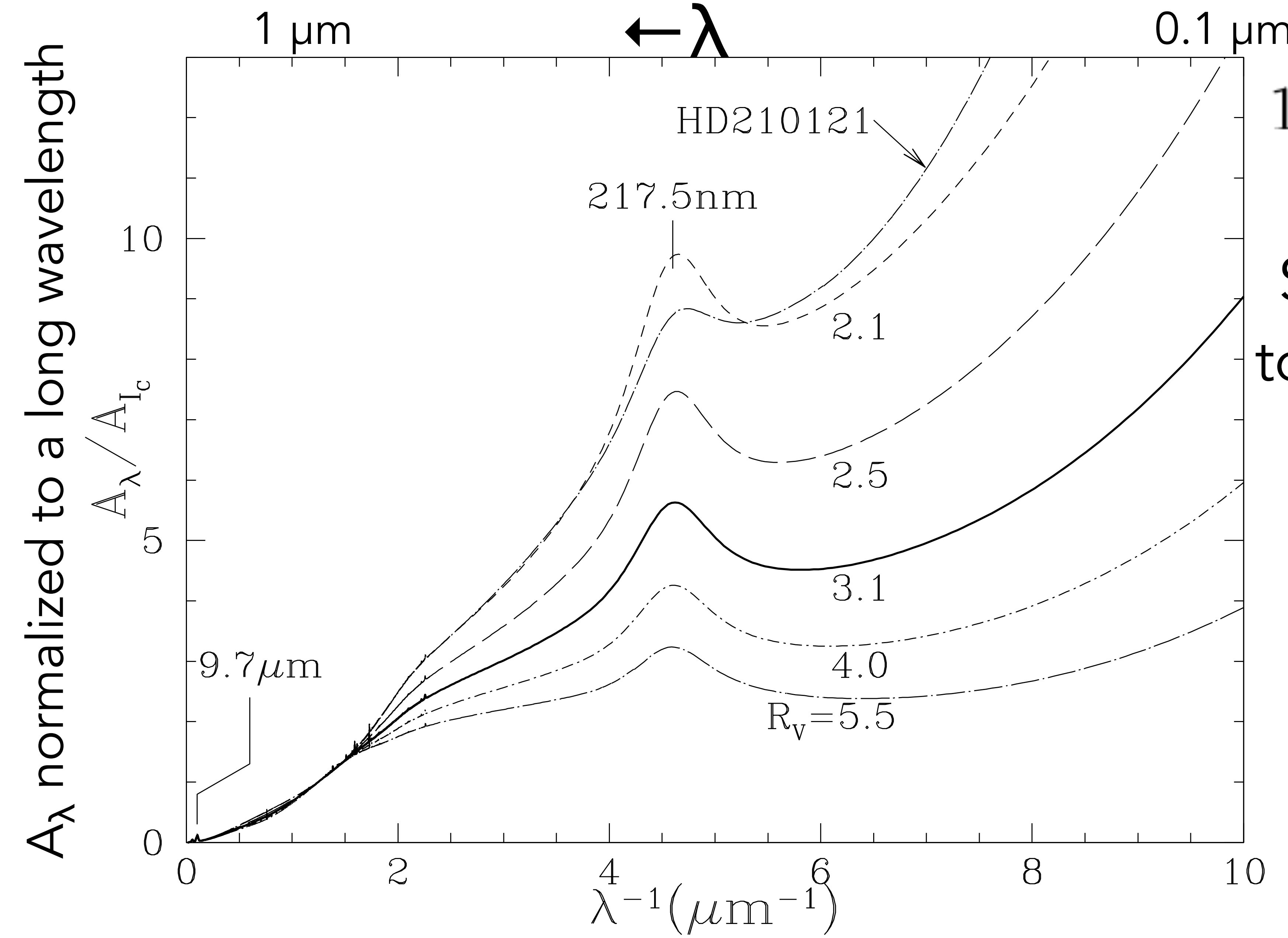
Continual rise to far-UV means there are more small grains than large grains.

Extinction Curve



$$A_\lambda(a) = 1.086\ N_d(a)\ Q_{ext}(a)\ \pi a^2$$

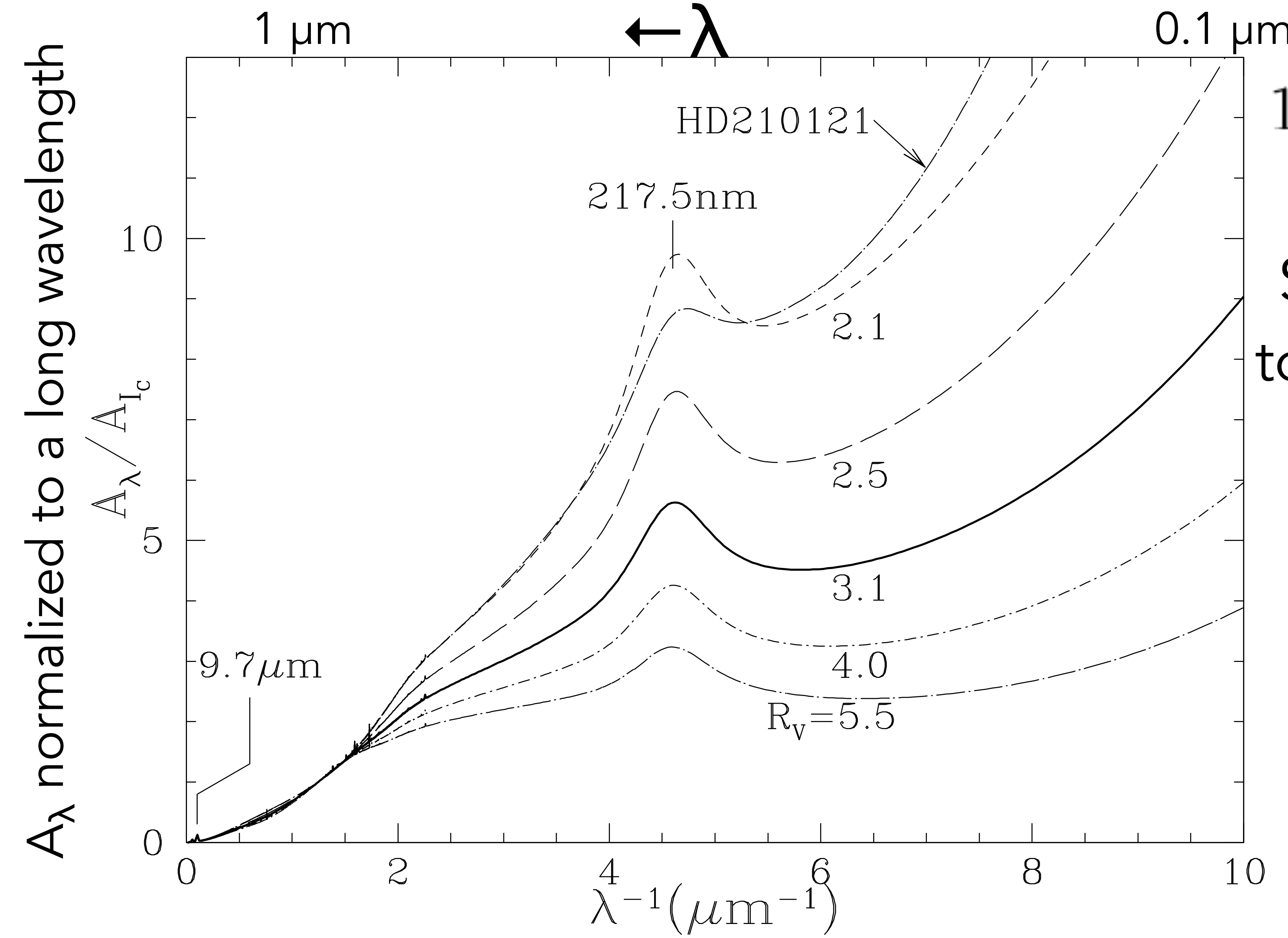
Extinction Curve



$$A_\lambda(a) = 1.086 \; N_d(a) \; Q_{ext}(a) \; \pi a^2$$

Say grains only contribute to A_λ at their maximum Q_{ext}

Extinction Curve

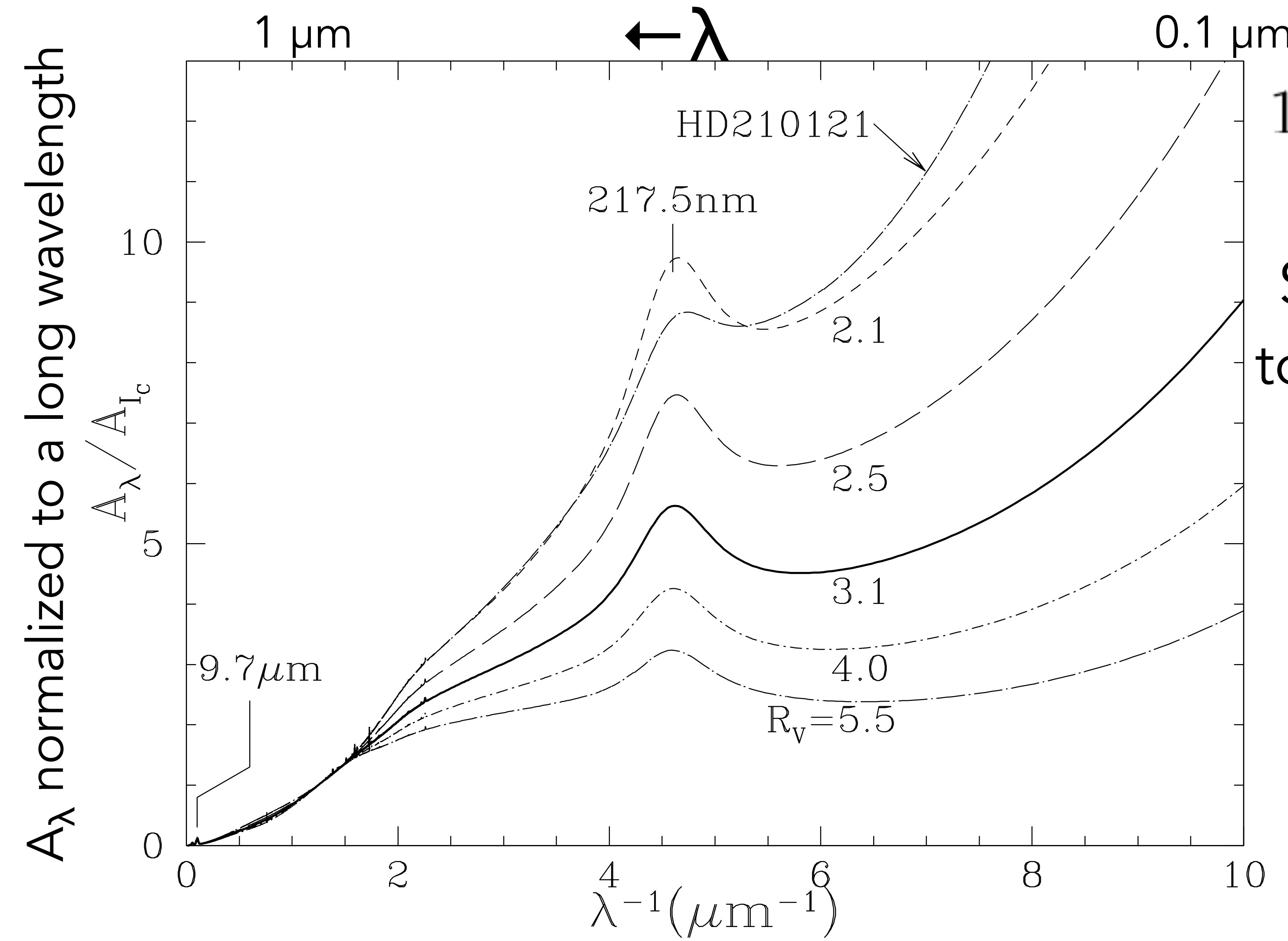


$$A_\lambda(a) = 1.086\ N_d(a)\ Q_{ext}(a)\ \pi a^2$$

Say grains only contribute to A_λ at their maximum Q_{ext}

$$\frac{A_{0.1\mu m}}{A_{0.5\mu m}} = \frac{N_{0.1\mu m}}{N_{0.5\mu m}} \left(\frac{0.1}{0.5}\right)^2$$

Extinction Curve



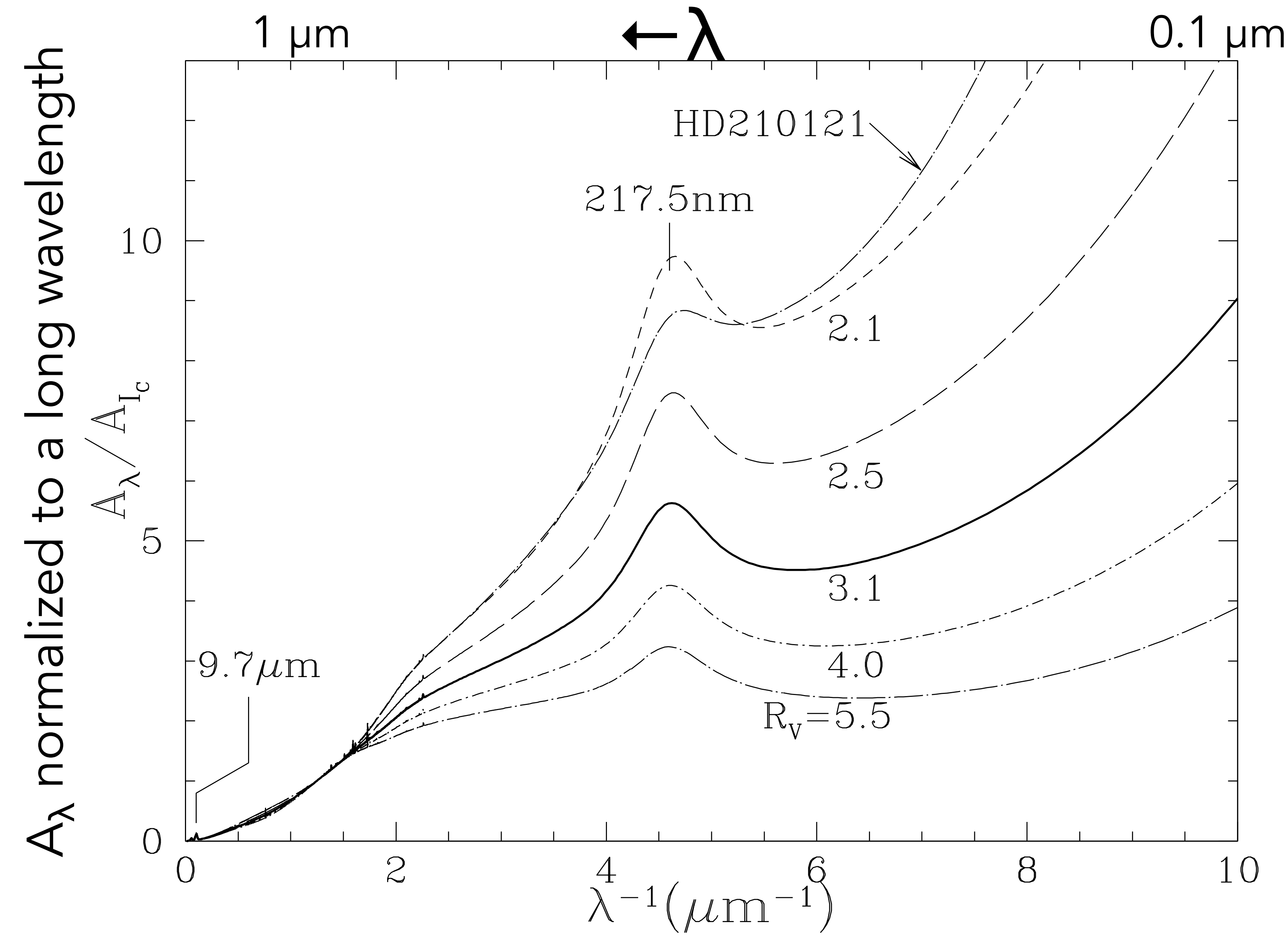
$$A_\lambda(a) = 1.086\ N_d(a)\ Q_{ext}(a)\ \pi a^2$$

Say grains only contribute to A_λ at their maximum Q_{ext}

$$\frac{A_{0.1\mu m}}{A_{0.5\mu m}} = \frac{N_{0.1\mu m}}{N_{0.5\mu m}} \left(\frac{0.1}{0.5}\right)^2$$

$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} \sim 150$$

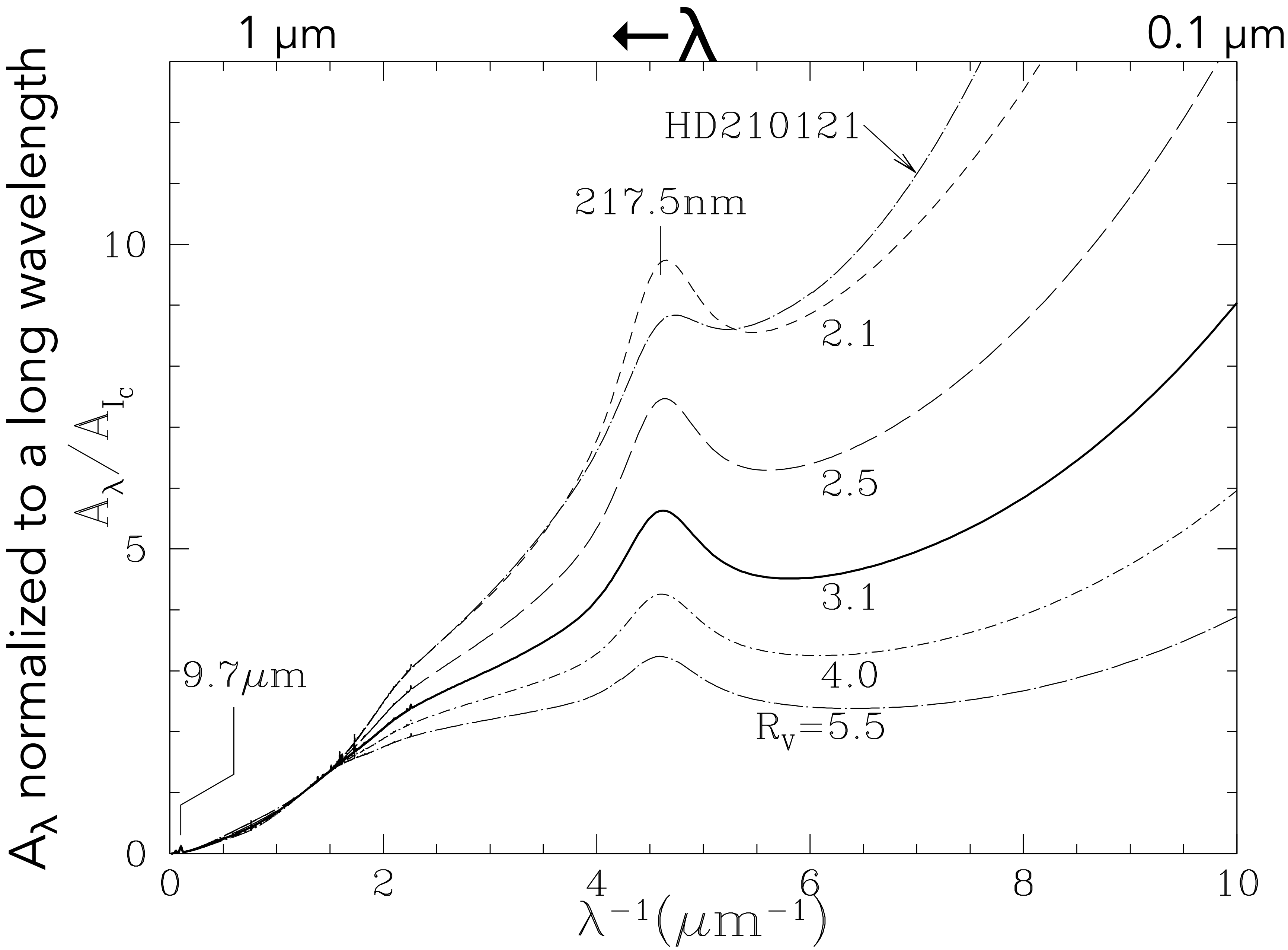
Extinction Curve



$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} \sim \quad 150$$

Assume power-law slope to size distribution $N(a) \sim a^{-\beta}$

Extinction Curve

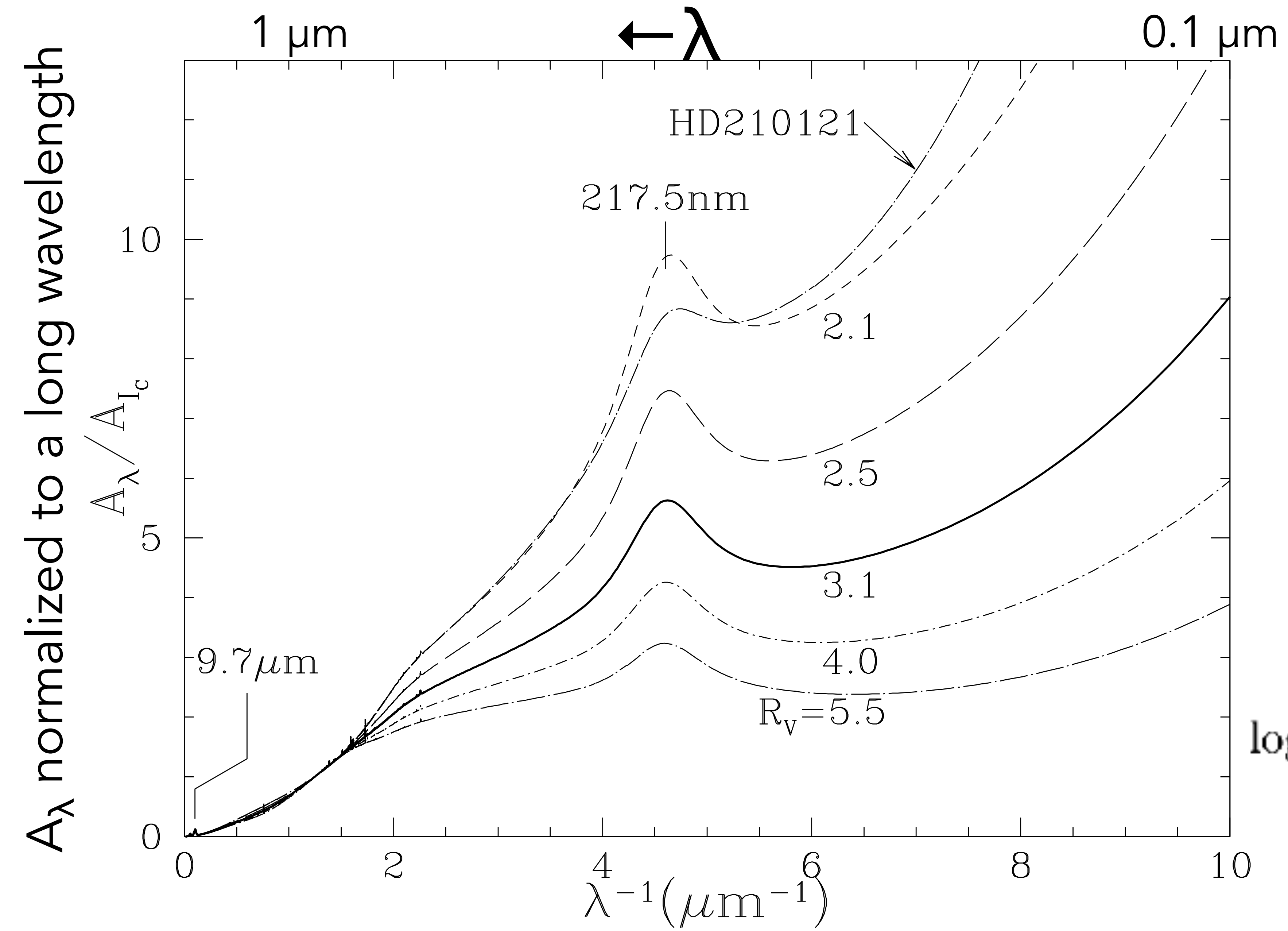


$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} \sim \quad 150$$

Assume power-law slope to size distribution $N(a) \sim a^{-\beta}$

$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} = \left(\frac{0.1}{0.5}\right)^{-\beta}$$

Extinction Curve



$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} \sim \quad 150$$

Assume power-law slope to size distribution

$N(a) \sim a^{-\beta}$

$$\frac{N_{0.1\mu m}}{N_{0.5\mu m}} = \left(\frac{0.1}{0.5}\right)^{-\beta}$$

$$\log_{10}\left(\frac{N_{0.1\mu m}}{N_{0.5\mu m}}\right) = -\beta \log_{10}\left(\frac{0.1}{0.5}\right)$$

β ~ 3

THE SIZE DISTRIBUTION OF INTERSTELLAR GRAINS

John S. Mathis, William Rumpl, and Kenneth H. Nordsieck
Washburn Observatory, University of Wisconsin–Madison
Received 1977 January 24; accepted 1977 April 11

ABSTRACT

The observed interstellar extinction over the wavelength range $0.11\ \mu m < \lambda < 1\ \mu m$ was fitted with a very general particle size distribution of uncoated graphite, enstatite, olivine, silicon carbide, iron, and magnetite. Combinations of these materials, up to three at a time, were considered. The cosmic abundances of the various constituents were taken into account as constraints on the possible distributions of particle sizes.

Excellent fits to the interstellar extinction, including the narrowness of the λ2160 feature, proved possible. Graphite was a necessary component of any good mixture, but it could be used with any of the other materials. The particle size distributions are roughly power law in nature, with an exponent of about -3.3 to -3.6. The size range for graphite is about $0.005\ \mu m$ to about $1\ \mu m$. The size distribution for the other materials is also approximately power law in nature, with the same exponent, but there is a narrower range of sizes: about 0.025–0.25 μm, depending on the material. The number of large particles is not well determined, because they are gray. Similarly, the number of small particles is not well determined because they are in the Rayleigh limit. This power-law distribution is drastically different from an Oort–van de Hulst distribution, which is much more slowly varying for small particles but drops much faster for particles larger than average.

$$\frac{dn}{da} \propto a^{-3.5}$$

THE SIZE DISTRIBUTION OF INTERSTELLAR GRAINS

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Received 1977 January 24; accepted 1977 April 11

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$$\frac{dn}{da} \propto a^{-3.5}$$

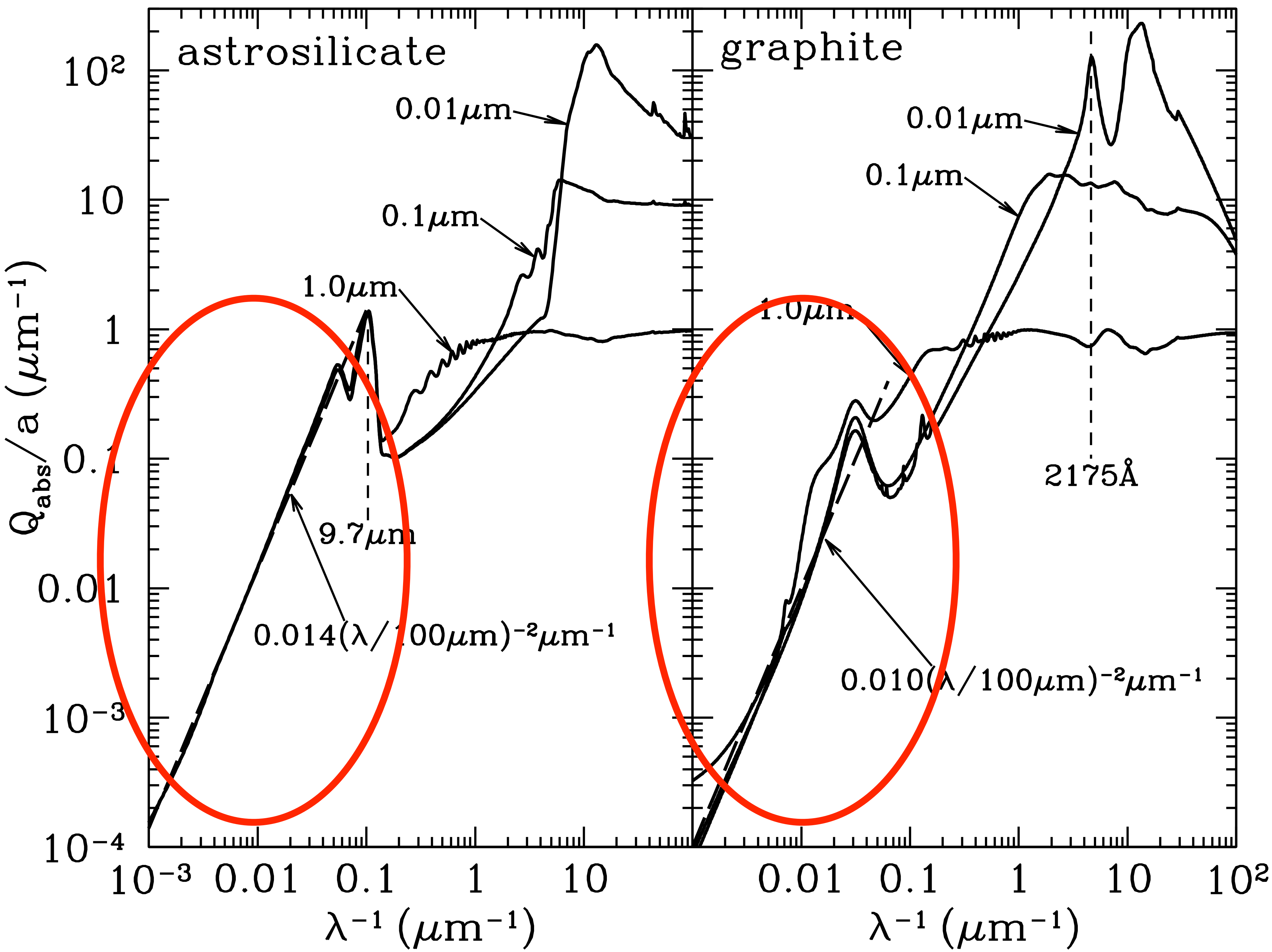
$$\mathrm{Mass}(a) \propto \int a^3 \frac{dn}{da} da \propto a^{0.5}$$

most mass in large grains

$$\mathrm{Area}(a) \propto \int a^2 \frac{dn}{da} da \propto a^{-0.5}$$

most area in small grains

←λ



At long wavelengths $Q_{abs}/a \propto \lambda^{-2}$

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \; \frac{u_\nu}{h\nu} \; c \; Q_{\rm abs}(\nu) \pi a^2 d\nu$$

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\text{abs}} = \int h\nu \boxed{\frac{u_\nu}{h\nu}\; c\; Q_{\text{abs}}(\nu)\pi a^2} d\nu$$

n_{ph} v σ

Dust Thermal Balance

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$n_{ph}\ v\ \sigma$

energy per
absorbed photon

Dust Thermal Balance

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n_{ph} v σ

energy per absorbed photon

rate a dust grain of size a emits energy

$$\left(\frac{dE}{dt}\right)_{\rm emit} = \int 4\pi B_\nu(T)\; Q_{\rm em}(\nu)\; \pi a^2 d\nu$$

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \boxed{\frac{u_\nu}{h\nu}\, c\, Q_{\rm abs}(\nu)\pi a^2} d\nu$$

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blackbody emitting over 4π str with efficiency Q_{em}

Dust Thermal Balance

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$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \; \frac{u_\nu}{h\nu} \; c \; Q_{\rm abs}(\nu) \pi a^2 d\nu$$

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \, \frac{u_\nu}{h\nu} \, c \, Q_{\rm abs}(\nu)\pi a^2 d\nu$$

<u>in LTE</u>

$$\boxed{\begin{gathered} u_\nu = \frac{4\pi}{c} B_\nu(T) \\ \left(\frac{dE}{dt}\right)_{\rm abs} = \left(\frac{dE}{dt}\right)_{\rm emit} \end{gathered}}$$

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu\ \frac{u_\nu}{h\nu}\ c\ Q_{\rm abs}(\nu)\pi a^2 d\nu$$

in LTE

$$\int \frac{4\pi}{c} B_\nu(T)\ c\ Q_{\rm abs}(\nu)\pi a^2 d\nu = \int 4\pi B_\nu(T)\ Q_{\rm em}(\nu)\pi a^2 d\nu$$

Dust Thermal Balance

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$$\left(\frac{dE}{dt}\right)_{\text{abs}} = \int h\nu \; \frac{u_\nu}{h\nu} \; c \; Q_{\text{abs}}(\nu)\pi a^2 d\nu$$

<u>in LTE</u>

$$\int \frac{4\pi}{c} B_\nu(T) \; c \; Q_{\text{abs}}(\nu)\pi a^2 d\nu = \int 4\pi B_\nu(T) \; Q_{\text{em}}(\nu)\pi a^2 d\nu$$

Therefore: $Q_{\text{abs}} = Q_{\text{em}}$

Dust Thermal Balance

rate a dust grain of size a absorbs energy

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \; \frac{u_\nu}{h\nu} \; c \; Q_{\rm abs}(\nu)\pi a^2 d\nu$$

Dust Thermal Balance

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$$\left(\frac{dE}{dt}\right)_{\rm abs} = \int h\nu \ \frac{u_\nu}{h\nu} \ c \ Q_{\rm abs}(\nu) \pi a^2 d\nu$$

Define "spectrum averaged absorption cross section"

$$\langle Q_{\rm abs} \rangle_* \equiv \frac{\int u_{*\nu} Q_{\rm abs}(\nu) d\nu}{\int u_{*\nu} d\nu}$$

Dust Thermal Balance

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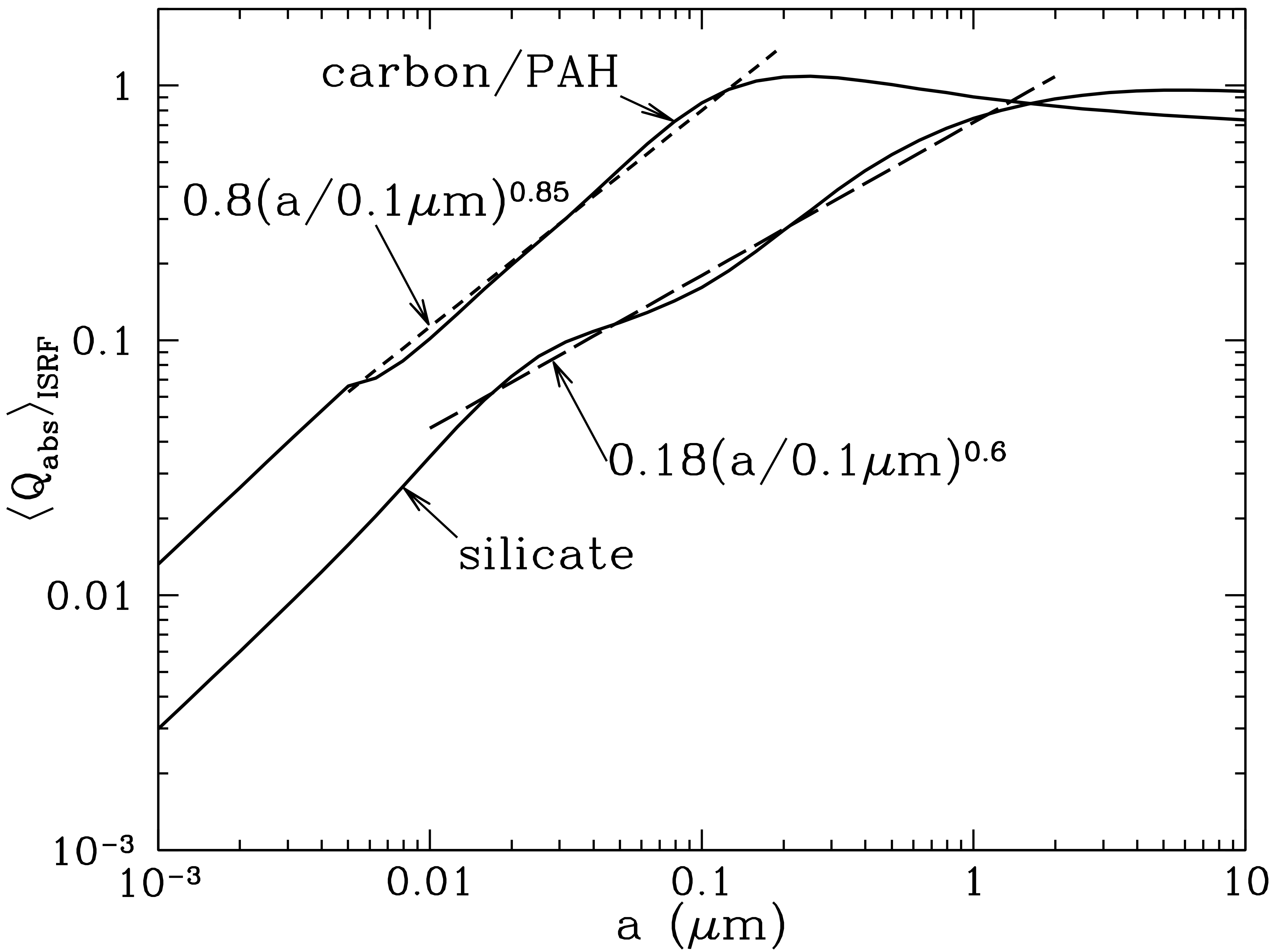
$$\langle Q_{\rm abs} \rangle_* \equiv \frac{\int u_{*\nu} Q_{\rm abs}(\nu) d\nu}{\int u_{*\nu} d\nu}$$

so that: $$\left(\frac{dE}{dt}\right)_{\rm abs} = \langle Q_{\rm abs} \rangle_* \, \pi a^2 \; u_* \; c$$

Dust Thermal Balance



$<Q_{abs}>_*$ for the average interstellar radiation field in the MW, and two astronomical dust analogs.

Dust Thermal Balance

rate a dust grain of size a emits energy

$$\left(\frac{dE}{dt}\right)_{\rm emit} = \int 4\pi B_\nu(T)\ Q_{\rm em}(\nu)\ \pi a^2 d\nu$$

$$Q_{\rm abs} = Q_{\rm em}$$

Dust Thermal Balance

rate a dust grain
of size a
emits energy

$$\left(\frac{dE}{dt}\right)_{\rm emit} = \int 4\pi B_\nu(T)\ Q_{\rm em}(\nu)\ \pi a^2 d\nu$$

$$Q_{\rm abs} = Q_{\rm em}$$

Define "Planck averaged emission efficiency"

$$\langle Q_{\rm abs}\rangle_{\rm T} \equiv \frac{\int B_\nu(T) Q_{\rm abs} d\nu}{\int B_\nu(T) d\nu}$$

Dust Thermal Balance

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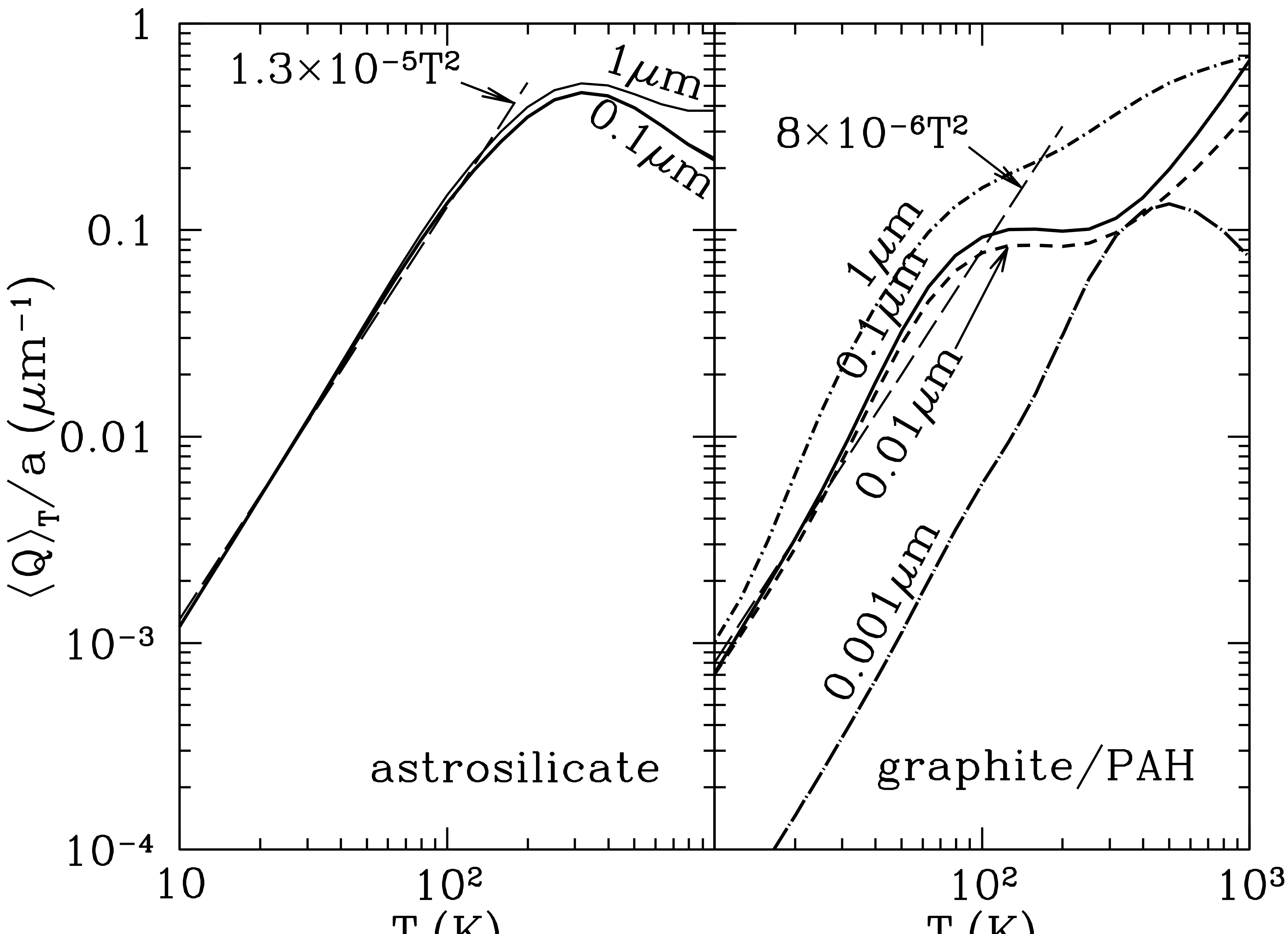
so that: $$\left(\frac{dE}{dt}\right)_{\rm em} = 4\pi a^2\ \langle Q_{\rm abs}\rangle_{\rm T}\, \sigma T^4$$

Dust Thermal Balance



$<Q_{abs}>_T/a$ for the a range of blackbody temperatures

Below ~100 K $<Q_{abs}>_T \sim T^2$

Dust Thermal Balance

In steady state, emission = absorption.

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \langle Q_{\rm abs} \rangle_* \, \pi a^2 \; u_* \; c$$

$$\left(\frac{dE}{dt}\right)_{\rm em} = 4\pi a^2 \, \langle Q_{\rm abs} \rangle_{\rm T} \, \sigma T^4$$

Dust Thermal Balance

In steady state, emission = absorption.

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \langle Q_{\rm abs}\rangle_* \, \pi a^2 \; u_* \; c$$

$$\left(\frac{dE}{dt}\right)_{\rm em} = 4\pi a^2 \, \langle Q_{\rm abs}\rangle_{\rm T} \, \sigma T^4$$

for MW interstellar radiation field and dust properties we found:

$$\langle Q_{\rm abs}\rangle_* \sim 0.8(a/0.1\mu m)^{0.85} \quad \text{carbon}$$

$$\langle Q_{\rm abs}\rangle_* \sim 0.18(a/0.1\mu m)^{0.6} \quad \text{silicate}$$

Dust Thermal Balance

In steady state, emission = absorption.

$$\left(\frac{dE}{dt}\right)_{\rm abs} = \langle Q_{\rm abs}\rangle_* \, \pi a^2 \; u_* \; c$$

$$\left(\frac{dE}{dt}\right)_{\rm em} = 4\pi a^2 \, \langle Q_{\rm abs}\rangle_{\rm T} \, \sigma T^4$$

for MW interstellar radiation field and dust properties we found:

$\langle Q_{\rm abs}\rangle_T \sim 8 \times 10^{-6} T^2$ carbon

$\langle Q_{\rm abs}\rangle_T \sim 1.3 \times 10^{-5} T^2$ silicate

Dust Thermal Balance

Very weak dependence of equilibrium temperature on grain size.

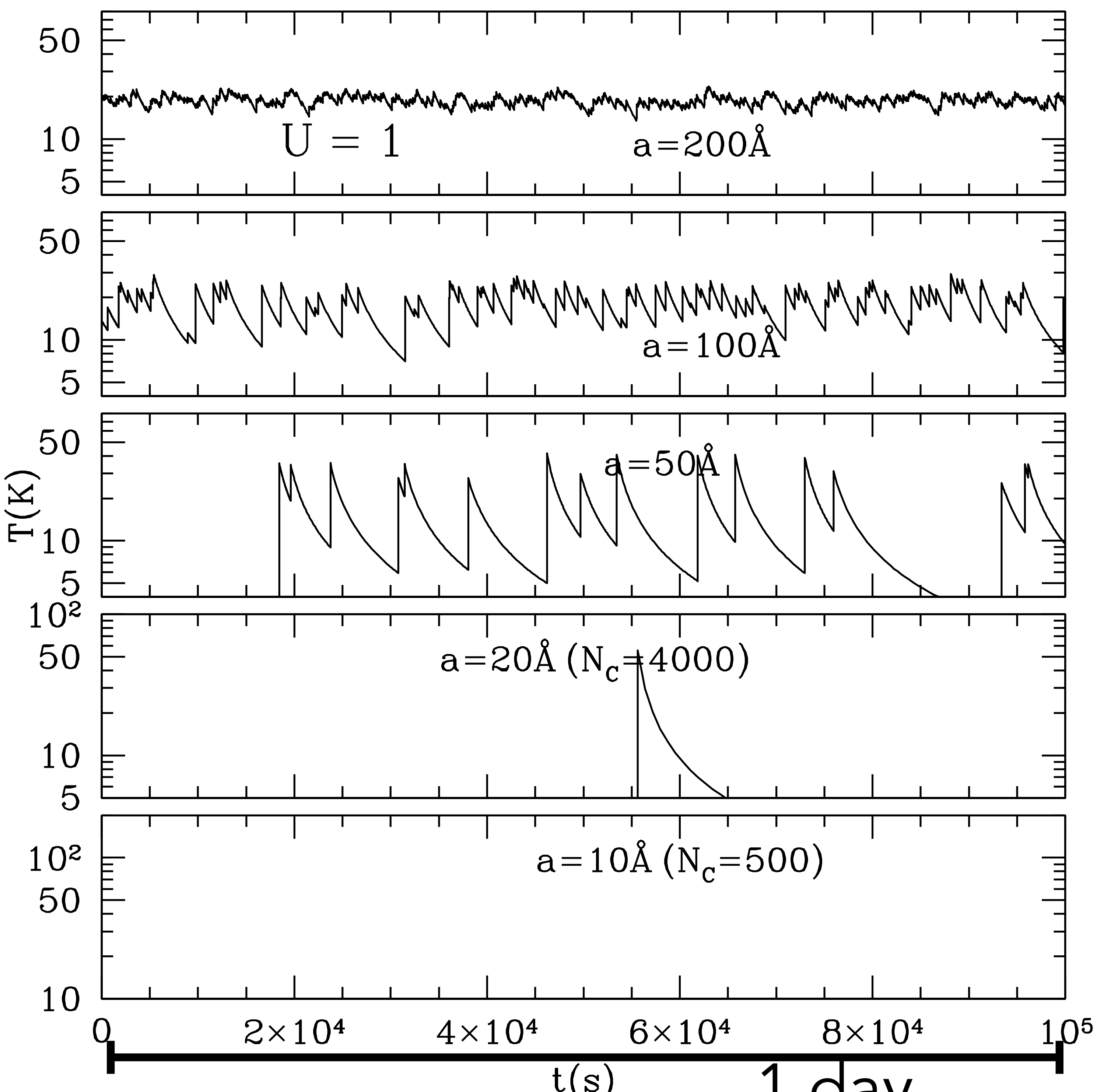
$$T \approx 22.3(a/0.1\mu m)^{-1/40} U^{1/6} K$$

carbon

$$T \approx 16.4(a/0.1\mu m)^{1/15} U^{1/6} K$$

silicate

Dust Thermal Balance



Not all grains are in steady state…

When:

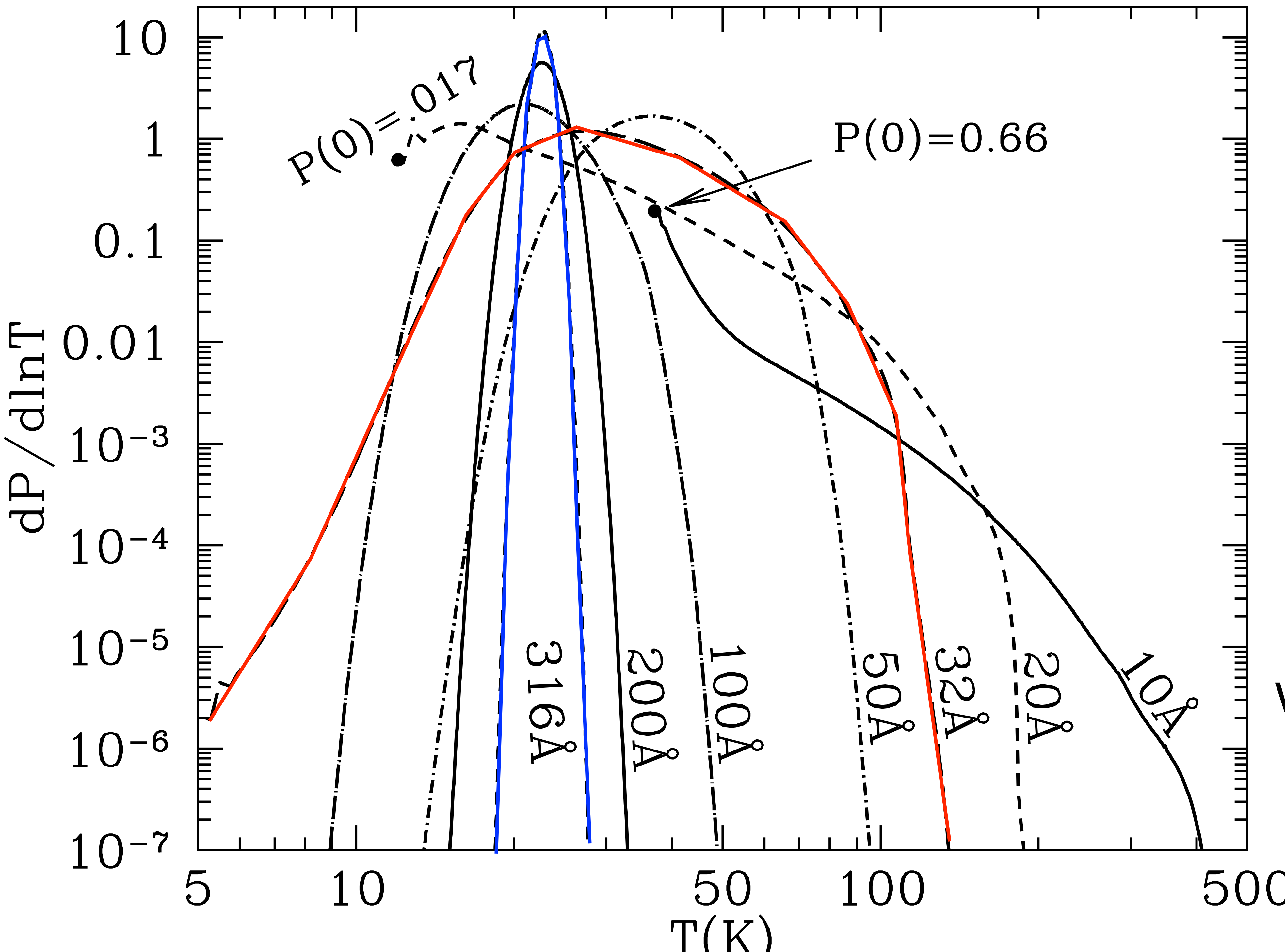
$(dE/dt)_{cool}$ << photon absorption rate

and/or

$h\nu >> E_{ss}$

Need to consider non-steady state

Dust Thermal Balance



Probability of finding grain with temp T in average MW ISRF.

PDF narrows with increasing size.

Dust Thermal Balance

While it is unlikely to find a small grain at
very high temperatures, most energy is emitted there!

$$\left(\frac{dE}{dt}\right)_{\rm em} = 4\pi a^2 \left\langle Q_{\rm abs}\right\rangle_{\rm T} \sigma T^4$$

$$\langle Q_{\rm abs}\rangle_T \sim 1.3 \times 10^{-5} T^2$$ silicate

dE/dt ~ T^6

Dust Thermal Balance

Is collisional heating important?

absorption $$\left(\frac{dE}{dt}\right)_{\rm abs} = \langle Q_{\rm abs}\rangle_* \pi a^2 \; u_* \; c$$

collisions $$\left(\frac{dE}{dt}\right)_0 = \quad n_{\rm H}\pi a^2 \langle v_{\rm H}\rangle 2kT\,\alpha$$

factor ~unity
for energy transfer from
collider to grain

Dust Thermal Balance

Is collisional heating important?

$$\frac{(dE/dt)_{\rm col}}{(dE/dt)_{\rm abs}} = \frac{3.8 \times 10^{-6}}{U} \frac{\alpha}{\langle Q_{\rm abs} \rangle_*} \left(\frac{n_H}{30cm^{-3}}\right) \left(\frac{T}{10^2 K}\right)^{3/2}$$

radiation field strength
normalized to MW average ISRF

collisional heating important in dense and/or hot gas