Physics 224: Lecture 3 notes
Spring 2018

"Charged-Charged Collisions" Continued:

In the previous lecture we started to describe collisions between charged particles where long-range interactions are not negligible. We will first look into how quickly a charged particle moving through a field of other charged particles shares its energy (important example being a photoelectron heating up gas). The set up involves a particle with charge $Z_1 e$ moving through a field of particles with $Z_2 e$. The particle undergoes many individual momentum kicks resulting in a random walk in momentum. We worked out before that:

$$\Delta p_\perp = 2\frac{Z_1 Z_2 e^2}{b v_1}. \quad (1)$$

The change in rms momentum for random collisions can be found by multiplying the average change in momentum squared by the collision rate and integrating over the impact parameters:

$$\left\langle \frac{d}{dt}\left[\Delta p_\perp\right]^2 \right\rangle = \int_{b_{min}}^{b_{max}} 2\pi b db\ n_2 v_1 \times \left[\frac{2Z_1 Z_2 d^2}{b v_1}\right]^2 = \frac{8\pi n_2 Z_1^2 Z_2^2 e^4}{v_1}\int_{b_{min}}^{b_{max}} \frac{db}{b} \quad (2)$$

We can define:

$$\ln\Lambda \equiv \int_{b_{min}}^{b_{max}} \frac{db}{b} \quad (3)$$

Next we need to determine what b_{max} and b_{min} are (there must be bounds otherwise the integral would go to infinity). For b_{min} we can take the impact parameter where the assumption of weak interactions breaks down, i.e. where energy in the center of mass frame, $E_{cm} \sim Z_1 Z_2 e^2/b_{min}$. For b_{max} we can use the Debye length $L_D = (kT/4\pi n_e e^2)^{1/2}$, which is the length where the plasma will shield excess charge. In the ISM, typical values for $\ln\Lambda \approx 20 - 35$.

Given the information above we can work out the time it takes for a charged particle projectile to lose its energy via interactions with other charged particles:

$$t_{loss} = \frac{E}{\langle (dE/dt)_{loss}\rangle} = \frac{m_1 v_1^2}{\left\langle \left(\frac{d}{dt}\left[\Delta p_\perp^2\right]\right)\right\rangle / m_2} = \frac{m_1 m_2 v_1^3}{8\pi n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda}. \quad (4)$$

As an example, for an electron moving through a field of protons:

$$t_{loss} = 1.4\times 10^7 \left(\frac{T_e}{10^4\mathrm{K}}\right)^{3/2}\left(\frac{n_e}{1\mathrm{cm}^{-3}}\right)^{-1}\left(\frac{\ln\Lambda}{25}\right)^{-1}. \quad (5)$$

The time to lose energy is less than one year. This tells us that on astronomical timescales there are more than enough collisions to maintain a Maxwellian velocity distribution in most situations in the ISM.

Local Thermodynamic Equilibrium:

Local thermodynamic equilibrium is the situation where the large scale properties of a region (n, T, radiation field) can vary in space and time, but slowly enough that for any given point one can assume thermodynamic equilibrium in the vicinity of that point at a given time. *The ISM is general not in LTE.* Why:

- The radiation field is often not generated locally (think of a radiation field dominated by starlight heating a cloud of gas).
- Radiative de-excitation of energy levels can happen quickly compared to collisions (low density). So the $T_{\rm kinetic}$ (from the Maxwellian velocity distribution) and $T_{\rm exc}$ (from the energy level populations of an atom) are generally not the same.

Despite the lack of LTE, there are still many tools from statistical mechanics that let us understand ISM processes. Specifically, the principle of detailed balance is an extremely useful tool for understanding various rates coefficients for ISM processes. In many situations the rate coefficients for various processes are not dependent on assuming LTE. For example, the collisional coefficients we've discussed earlier in the class only require a Maxwellian velocity distribution to be maintained and we demonstrated that this is likely to be the case independent of whether the system is in LTE. The principle of detailed balance lets us determine the relationship between forward and backward rate coefficients by figuring out what they are in LTE. Lets go through the process to illustrate. The general set up involves some process with rate coefficients forward (k_f) and backward (k_b):

$$R_1 + R_2 + R_3 \cdots \leftrightarrow P_1 + P_2 + P_3 \cdots \tag{6}$$

This can be any process that involves exchange of energy—collisions, populating energy levels in atoms, chemical reactions, etc.

Lets simplify to $A + B \to C$. In LTE, we can write the following:

$$\frac{n_{LTE}(C)}{n_{LTE}(A)n_{LTE}(B)} = \frac{f(C)}{f(A)f(B)} \tag{7}$$

where $f(X)$ is the partition function per volume:

$$f(X;T) \equiv \frac{Z}{V} = \left[\frac{(2\pi M_x kT)^{3/2}}{h^3}\right] \times \sum g_i e^{-E_i/kT} \tag{8}$$

. The first part of $f(X)$ in the brackets is the translational part of the partition function, the second part is the internal part - these determine how energy is distributed among the energy levels. In LTE,

$$\frac{\text{reactions forward}}{\text{cm}^3} = k_f n_A n_B \tag{9}$$

$$\frac{\text{reactions backward}}{\text{cm}^3} = k_b n_C \tag{10}$$

Therefore, $k_f n_A n_B = k_b n_C$ and we can write:

$$\frac{k_f}{k_b} = \frac{n_C}{n_A n_B} = \frac{f(C)}{f(A)f(B)} \tag{11}$$

Lets use the example of collisional excitation of some energy level within an atom $X(l) + Y \leftrightarrow X(u) + Y$ where the energy levels in X are E_u and E_l.

$$\frac{k_{ul}}{k_{lu}} = \frac{n_{X_l} n_Y}{n_{X_u} n_Y} = \frac{n_{X_l}}{n_{X_u}} = \frac{f(X(l))}{f(X(u))} \tag{12}$$

When we write out the partition function, the translational part cancels since M_X and T are the same. All that is left is the internal part:

$$\frac{k_{ul}}{k_{lu}} = \frac{g_l}{g_u} e^{E_{ul}/kT} \tag{13}$$

where $E_{ul} = E_u - E_l$. With detailed balance we have used LTE to get the relationship between k_l and k_u, but this relation applies generally because the rate coefficients only involve the cross section (a property of quantum mechanics or the physical cross section or similar) and the velocity (which requires a Maxwellian but not LTE). We will use this sort of trick many times in the coming classes!