# PHYS 224 Lec 4

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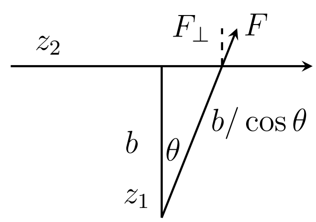
## 1 Collision

- only short distance collision: well defined cross section
- Interaction of forces become strong only within a certain region: can still define cross section
- long range forces dominate in collision: can no longer define cross section. Need to integrate over whole space

### 1.1 Collision integral




$$F_\perp = \frac{Z_1 Z_2 e^2}{(b/\cos\theta)^2}\cos\theta \tag{1}$$

$$\Delta P_\perp = \int_0^\infty F_\perp dt,\ dt \to d\theta \tag{2}$$

$$\Delta P_\perp = \frac{2Z_1 Z_2 e^2}{b v_2} \tag{3}$$

When particle moves through fields of other particles, it experiences spikes of momentum from collision with other particles. These particles are randomly distributed: can be modeled by random walk (b: impact parameter)

## 1.2 random walk and energy distribution

$$\sqrt{\langle \vec{R} \cdot \vec{R} \rangle} = \sqrt{NL^2} \tag{4}$$

$$\frac{d}{dt}\left\langle \Delta P_\perp^2 \right\rangle = \int_{b_{\min}}^{b_{\max}} \underbrace{2\pi b db n_2 v_1}_{\text{collision rate}} \times \underbrace{(\Delta P_\perp)^2}_{\text{step } \Delta P_\perp} \tag{5}$$

$$\frac{d}{dt}\left\langle \Delta P_\perp^2 \right\rangle = \frac{8\pi n_2 Z_1^2 Z_2^2 e^2}{v_1} \underbrace{\int_{b_{\min}}^{b_{\max}} \frac{db}{b}}_{l_n \Lambda} \tag{6}$$

Ignoring the collision in short range. $b_{\min}$: small defection. $b_{\max}$ at large distance, particle shield itself: distance where plasma is neutral (deby length?)

$$E_{cm} \approx \frac{Z_1 Z_2 E^2}{B_{\min}} \tag{7}$$

$$L_D = 690cm \left(\frac{T}{10^4 K}\right)^{1/2} \left(\frac{1cm^{-3}}{n_e}\right)^{1/2} \tag{8}$$

$$\Lambda \equiv \frac{b_{\max}}{b_{\min}} \approx 20 - 35 \text{ for typical ISM} \tag{9}$$

How long does it take for energy from incoming particle to be equally distributed among randomly distributed particles:

$$t_{\text{loss}} = \frac{E}{\left\langle \frac{dE}{dt} \right\rangle} = \frac{\frac{1}{2} m_1 v_1^2}{\left\langle \frac{d}{dt}(\Delta P_\perp^2) \right\rangle / m_2} \tag{10}$$

$$= \frac{m_1 m_2 v_2^2}{8\pi n_2 Z_1^2 Z_2^2 e^4 l_n \Lambda} \tag{11}$$

For electron moving through protons:

$$t_{\text{loss}} = 1.4 \times 10^7 s \left(\frac{T}{10^4 K}\right)^{3/2} \left(\frac{1cm^{-3}}{n_e}\right) \left(\frac{l_n \Lambda}{25}\right)^{-1} \tag{12}$$

On very short time scale, the energy is shared. Although the ISM is not usually equilibrium, can still assume Maxwell distribution of velocity due to fast distribution of energy

## 1.3 Thermal equilibrium

Characterized by one temperature. More useful to think about local TE (LTE): time and length scale on which temperature varies are long.

ISM not in LTE, WHY?

- Radiation from stars (nonlocal, temperature not representative of material)

- The rate of radiative de-excitation becomes quicker than collision (not dependent on local collision)

$$\Rightarrow T_{rad} \neq T_{kin} \neq T_{ex}$$

## 1.4 Relationship among temperature through detailed balance

In TE, the followings are true:

- forward and backward rates balance

$$A + B \xrightarrow[\leftarrow k_b]{k_f \rightarrow} C$$

  - reaction forward: $n_A n_B k_f, k_f \sim \langle \sigma v \rangle_{AB}$
  - reaction backward: $n_C k_b$
  - reaction forward = backward
  -
$$\frac{k_f}{k_b} = \frac{n_C}{n_A n_B}$$

- from stat mech
$$\left. \frac{n_C}{n_A n_B} \right|_E = \frac{f(C,T)}{f(A,T)f(B,T)} \text{ detailed balance}$$
  - $f(x,T) = \frac{Z}{V} = \frac{Z_{\text{trans}} \times Z_{\text{internal}}}{V} = \frac{(2\pi m_x kT)^{3/2}}{h^3} \times \sum_i g_i e^{-E_i/kt}$
  - translational times degenerancy of level
  - partition on function per volume

E.x.

$$X(l) + Y \to X(u) + Y,\ E_n, E_l; E_{ul} = E_u - E_l \tag{13}$$
$$\frac{k_u l}{k_l u} = \frac{n_{x_l} n_y}{n_{x_u} n_y} = \frac{f(X(l),T)}{f(X(u),T)} \tag{14}$$
$$Z_{int}(X(l),T) = g_l e^{-E_l/kT} \tag{15}$$
$$\frac{k_u l}{k_l u} = \frac{g_l}{g_u} e^{E_{ul}/kT} \tag{16}$$

Doesnt have to be in equilibrium as long as we know the energy distribution

E.x.

$$\Delta E = E_u - E_l \tag{17}$$
$$X_L + h\nu \to X_u \tag{18}$$

upward transition, only two levels:

$$\left(\frac{dn_u}{dt}\right)_{l\to u} = -\left(\frac{dn_l}{dt}\right)_{l\to u} = n_l u_\nu B_{lu} \sim n_A n_B \langle \sigma v \rangle_{AB}$$

$u_\nu$: energy density of radiation at $\nu$, $B_{lu}$: Einstein B value 'absorption'

Downward transition

- spontaneous transition, Einstein A: spontaneous emission

$$X_u \to X_l + h\nu$$

- stimulated emission, Einstein B stimulated emission

$$X_u + h\nu \to X_l + Zh\nu$$

$$\text{Spontaneous transtition} \left(\frac{dn_l}{dt}\right)_{u\to l} = -\left(\frac{dn_u}{dt}\right)_{u\to l} = n_u A_{ul} + n_u u_\nu B_{ul} \tag{19}$$

In LTE

- $$u_{\nu,LTE} = \frac{4\pi}{C} B_\nu = \frac{8\pi h\nu^2}{c^3} \frac{1}{e^{h\nu}kT - 1}$$

- $$\frac{n_u}{n_l} = \frac{g_l}{g_u} e^{\Delta E/kT}$$

- $$\frac{dn_u}{dt} = 0 = \left(\frac{dn_u}{dt}\right)_{l\to u} + \left(\frac{dn_l}{dt}\right)_{u\to l}$$

- $$T_{rad} = T_{ex}$$

Now solve:

$$n_l B_{lu} n_\nu = n_\nu (A_{ul} + B_{ul} u_\nu) \tag{20}$$

$$n_\nu = \frac{A_{ul}/B_{ul}}{\left(\frac{n_l}{n_u}\right)\left(\frac{B_{lu}}{B_{ul}} - 1\right)} = \frac{A_{ul}/B_{ul}}{\frac{g_l B_{lu}}{g_u B_{ul}} e^{h\nu/kT} - 1} = \frac{8\pi h\nu^2}{c^3} \frac{1}{e^{h\nu}kT - 1} \tag{21}$$

$$\boxed{\frac{g_l B_{lu}}{g_u B_{ul}} = 1} \tag{22}$$

$$\boxed{B_{ul} = \frac{c^3}{8\pi h\nu^3} A_{ul}} \tag{23}$$