Lecture 6

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1 Order of Magnitude Energy Level

Classic non-relativistic atom

1.1 'Allowed Transitions'

1.1.1 Coulomb interactions between e- and nucleus

$p^2 \to$ term $\to$ fine structure $\to$ hyperfine

$$\frac{Ze^2}{r^2} = \frac{m_e v^2}{r} \tag{1}$$

$$\text{also quantized angular momentum: } m_e v r = n\hbar \tag{2}$$

$$v = \frac{n\hbar}{mer} \Rightarrow \frac{Ze^2}{r^2} = \frac{m_e}{r}\left(\frac{n^2\hbar^2}{m_e^2 r^2}\right) \tag{3}$$

$$r = \frac{n^2\hbar^2}{Zm_e e^2},\ a_0 = \frac{\hbar^2}{m_e e^2} \text{ Bohr radius } \sim 0.5\text{Å} \tag{4}$$

$$r = \frac{n^2}{Z} a_0 \tag{5}$$

$$\frac{v}{c} = \frac{n\hbar}{m_e r c} = \frac{n\hbar}{m_e c}\left(\frac{Z}{n^2}\frac{m_e e^2}{\hbar^2}\right) = \frac{Z}{n}\alpha,\ \alpha \sim \frac{1}{137} \tag{6}$$

$$E = \frac{1}{m_e v^2} - \frac{Ze^2}{r} = -\underbrace{\frac{e^2}{2a_0}}_{\text{Rydberg}\sim 13.6eV}\left(\frac{Z^2}{n^2}\right) \tag{7}$$

Spin magnetic moment of e- interacts with orbit B-field. From e- point of view orbiting proton generates B field

1.1.2 Fine structure transition

Interaction between spin and angular momentum of e-

$$E \sim \vec{\mu} \cdot \vec{B} \tag{8}$$

$$\vec{B} = \frac{-\vec{v}}{c} \times \vec{E} \tag{9}$$

$$\vec{E} = \frac{Ze}{r^2}\hat{r} \tag{10}$$

$$\vec{v} \perp \vec{E} \tag{11}$$

$$\Rightarrow \vec{B} = -\left(\frac{Z}{n}\alpha\hat{\phi}\right) \times \left(\frac{Ze}{r^2}\hat{r}\right) = \frac{Z^2 e\alpha}{nr^2}\hat{z} \tag{12}$$

$$\mu_s = \frac{e\hbar}{m_e c} \Rightarrow \boxed{E \sim \frac{\alpha^2 Z^2}{n^5} Ryd} \tag{13}$$

E.X. for a particle with charge of 1:

$$E_{\text{fine structure}} \sim 1meV,\ \lambda \sim 1mm$$

1.1.3 Hyperfine transition

Interaction between magnetic moments of nucleus and e-

$$E_{\text{hyper}} \sim m\vec{u}_e \cdot \vec{B}_n \tag{14}$$

$$\vec{B}_n = \frac{\mu_N}{c^3},\ \mu_N = \frac{Ze\hbar}{2m_n c} \tag{15}$$

$$\Rightarrow E_{\text{hyper}} \sim \frac{Z^4\alpha^2}{n^5}\left(\frac{m_e}{m_n}\right) Ryd \tag{16}$$

E.x. Hydrogen:

$$E_{\text{hyper}} \sim 1\mu eV,\ \lambda \sim 100cm$$

1.2 Diatomic molecules

$$E \sim \underbrace{E_{rot}}_{10^{-3}-10^{-2}eV} + \underbrace{E_{vib}}_{10^{-2}-10^{-1}eV} + \underbrace{E_{el}}_{1-10eV}$$

Born -Oppenheimer approximation