Physics 224
The Interstellar Medium

Lecture #6: Radiative Transfer

- Part I: Basics of Radiative Transfer
- Part II: HI 21-cm Radiative Transfer
- Part III: Absorption Lines

Radiative Transfer

Motions of individual particles

On scales ≫ mean free path for collisions

↓

Fluid dynamics

Propagation of individual photons

On scales ≫ λ

↓

Radiative Transfer

Transport Phenomena: https://en.wikipedia.org/wiki/Transport_phenomena

Radiative Transfer

Description of the radiation field in full detail:

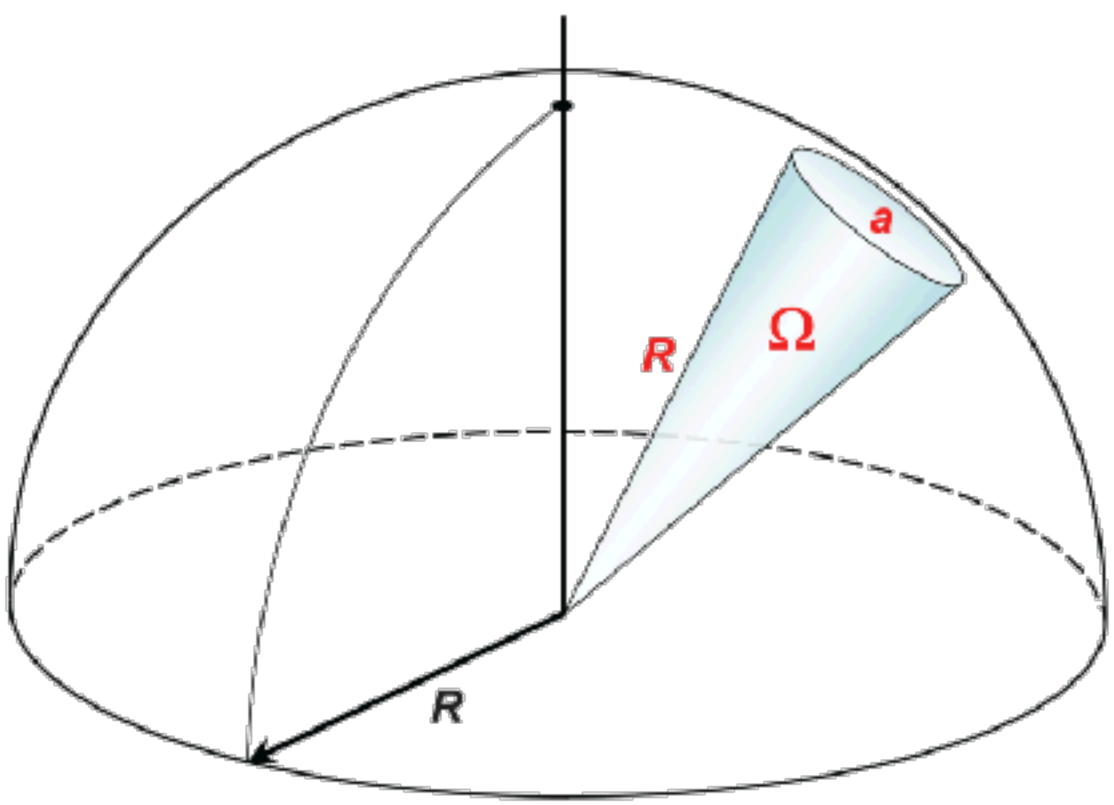
$$I_\nu = \frac{dE}{dA\ dt\ d\Omega\ d\nu}$$

specific intensity

energy per unit time (dt)
per unit frequency (dν)
passing through area (dA)
from solid angle (dΩ)

Radiative Transfer

Solid Angle



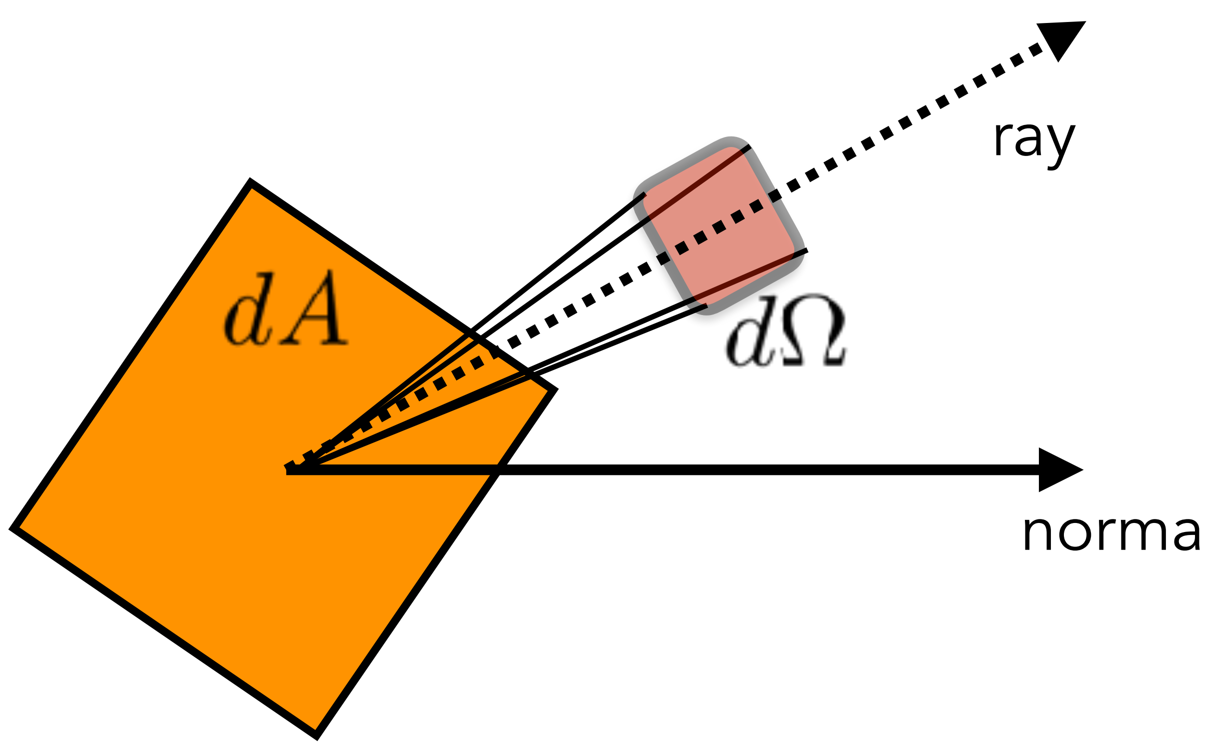
Two-dimensional angle subtended by an object.

Unit = steradian (sr)

4π sr in a sphere

Radiative Transfer

Solid angle specifies direction



Radiative Transfer

Units of Specific Intensity

$[I_\nu]$ = energy (time)$^{-1}$ (area)$^{-1}$ (solid angle)$^{-1}$ (frequency)$^{-1}$

$[I_\nu]$ = ergs s^{-1} cm^{-2} sr^{-1} Hz^{-1}

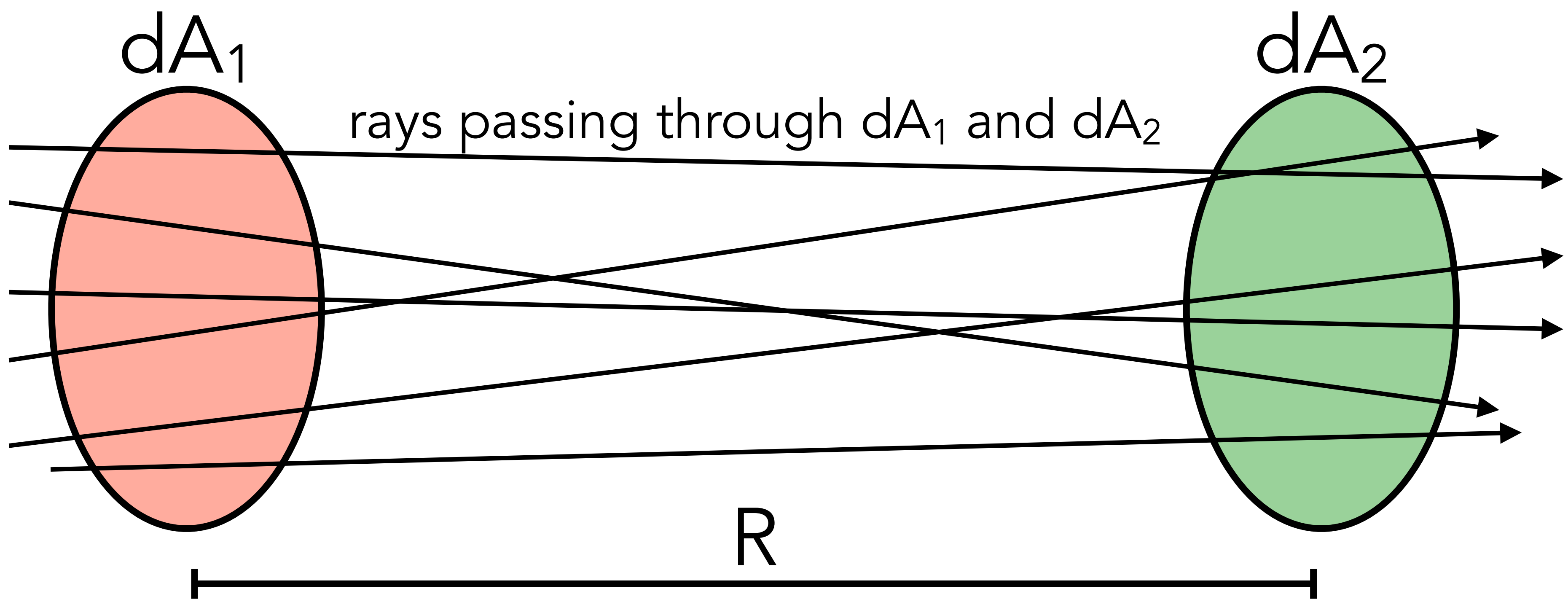
Some other common units: Jy/sr or Jy/arcsec^2

1 Jansky (Jy) = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}

Jansky is a unit of flux density

Radiative Transfer

When there is no emission/absorption, intensity along a ray is constant.



$$dE_1 = dE_2$$

$$I_{\nu,1}\, dA_1\, dt\, d\Omega_1\, d\nu = I_{\nu,2}\, dA_2\, dt\, d\Omega_2\, d\nu$$

$$I_{\nu,1} = I_{\nu,2}$$

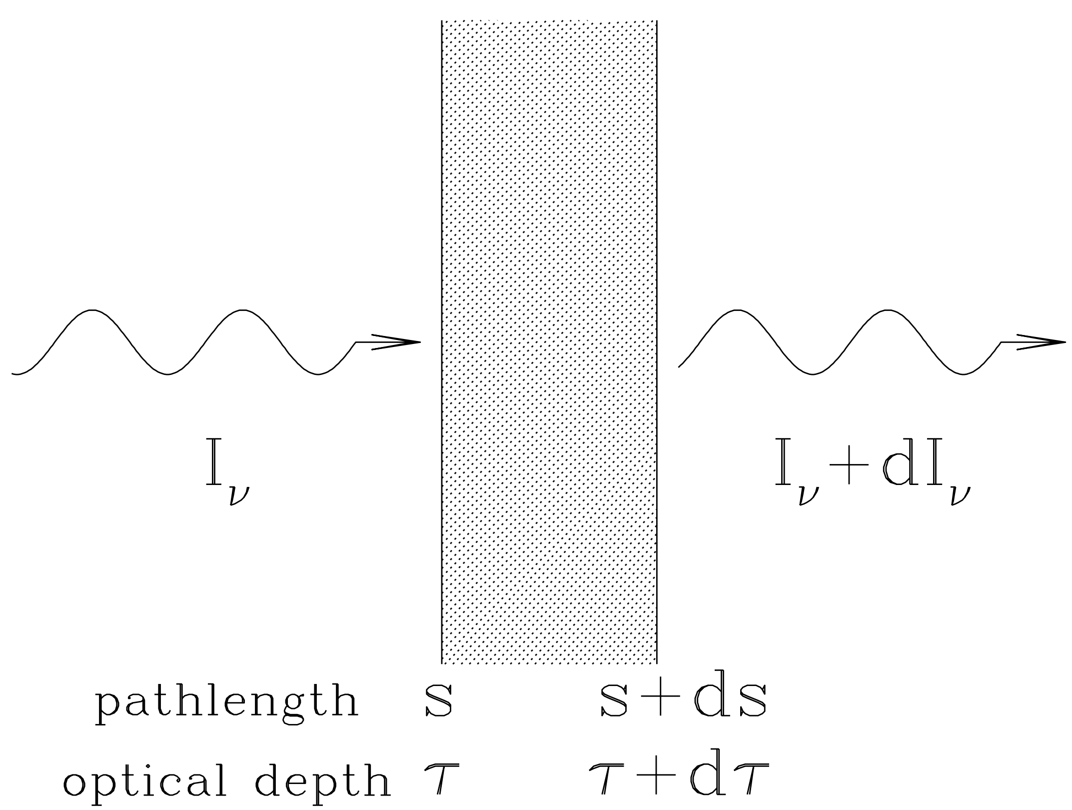
$d\Omega_1$ = solid angle subtended by dA_2 at $dA_1 = dA_2/R^2$

$d\Omega_2$ = solid angle subtended by dA_1 at $dA_2 = dA_1/R^2$

$$I_\nu = \text{constant}$$

Radiative Transfer

Equation of Radiative Transfer ignoring scattering



"emissivity"
net change in I_ν from
spontaneous emission

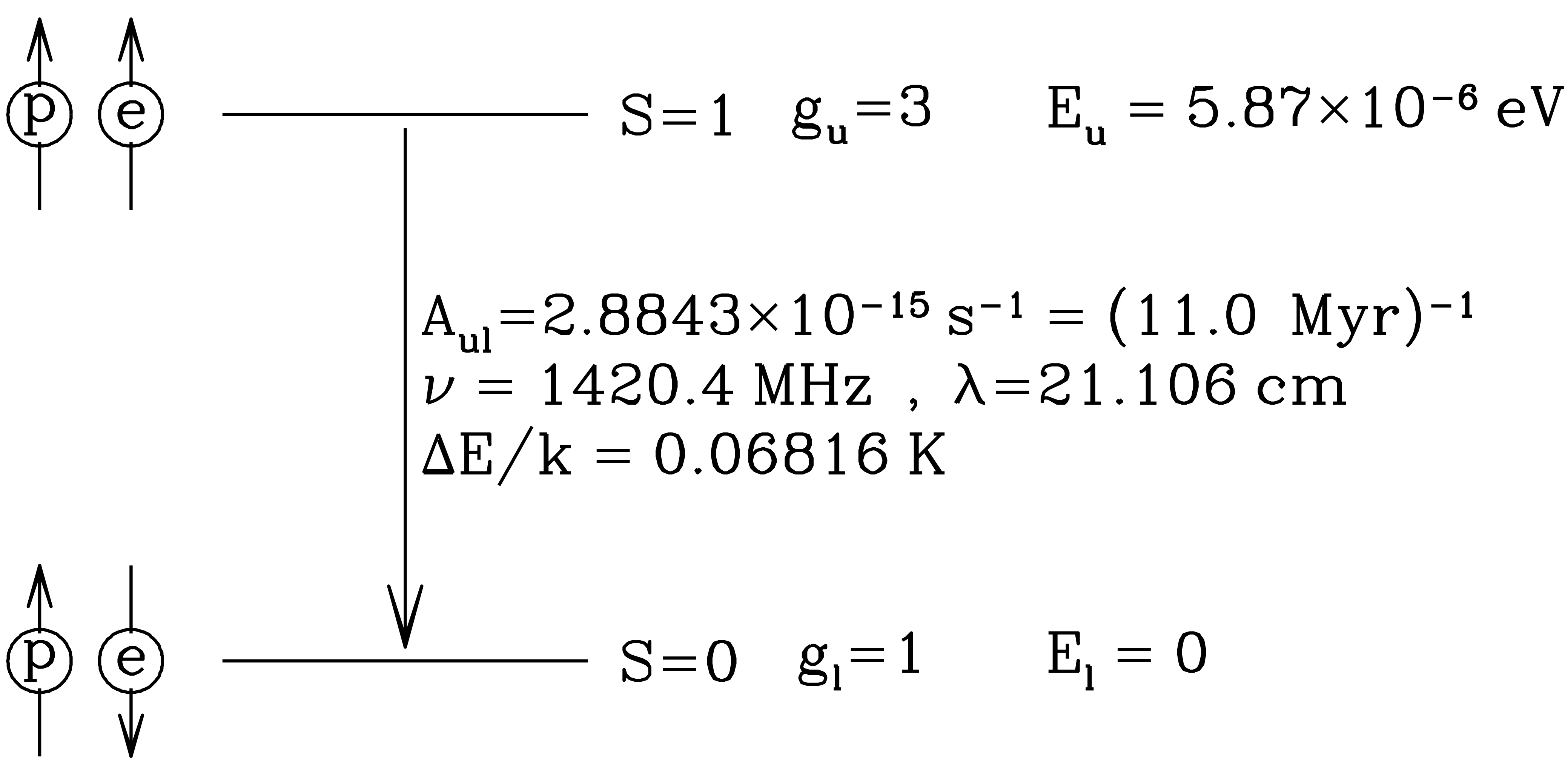
$$dI_\nu = -I_\nu \kappa_\nu ds + j_\nu ds$$

"attenuation coefficient"
net change in I_ν from absorption
& stimulated emission

Part II: HI 21-cm Radiative Transfer

HI 21-cm Radiative Transfer

Hyperfine splitting due to interaction of electron spin and nuclear spin.



*remember: degeneracy = 2(S+1) = 3 for upper

HI 21-cm Radiative Transfer

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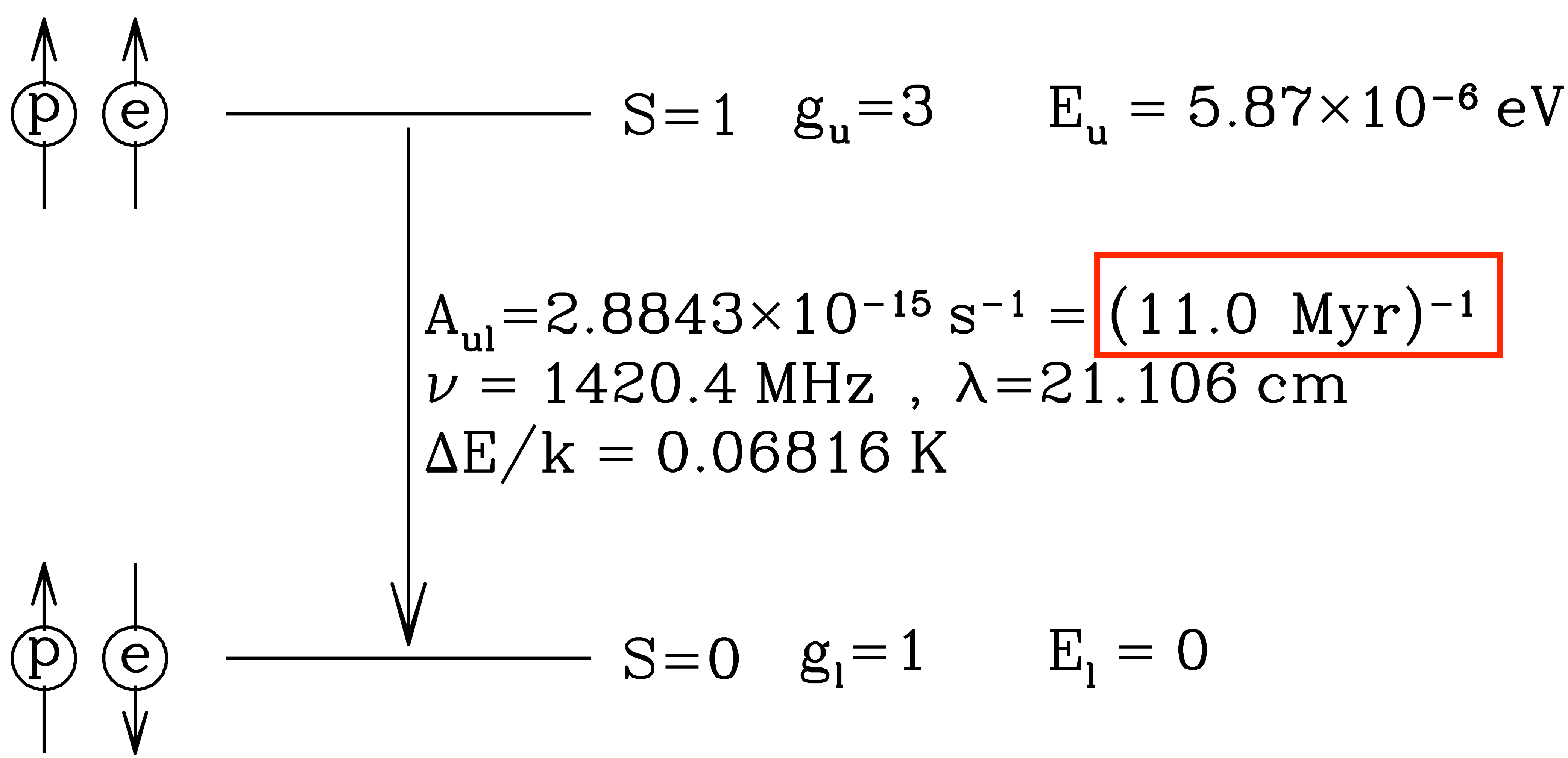

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HI 21-cm Radiative Transfer

$T_{exc} \equiv T_{spin} \gg 0.0682$ K

because cosmic microwave background can populate levels

$$\frac{n_u}{n_l} \equiv \frac{g_u}{g_l} e^{-h\nu_{ul}/kT_{exc}} = 3e^{-0.0682/T_{spin}} \approx 3$$

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Under all ISM conditions, 75% of HI is in upper level.

Emissivity is independent of T_{spin}!!

$$j_\nu = n_u \frac{A_{ul}}{4\pi} h\nu_{ul}\phi_\nu = \frac{3}{16\pi} A_{ul}\ h\nu_{ul}\ n(\mathrm{H\ I})\ \phi_\nu$$

HI 21-cm Radiative Transfer

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from last week:

$$\frac{g_l B_{lu}}{g_u B_{ul}} = 1$$

$$B_{ul} = \frac{c^3}{8\pi h\nu^3} A_{ul}$$

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to get:

$$e^{-E_{ul}/kT_{spin}} = 1 - h\nu_{ul}/kT_{spin}$$

from last week:

$$\frac{g_l B_{lu}}{g_u B_{ul}} = 1$$

$$B_{ul} = \frac{c^3}{8\pi h\nu^3} A_{ul}$$

HI 21-cm Radiative Transfer

put all this together and we find:

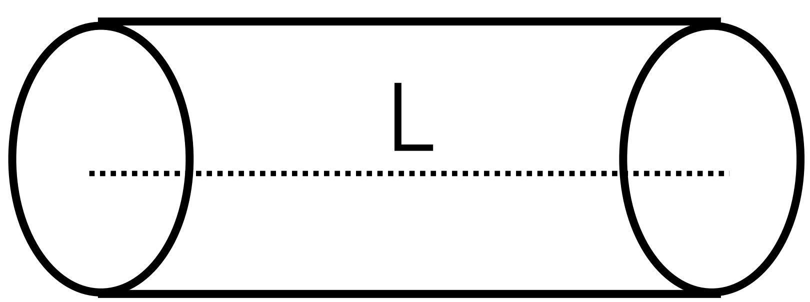
$$\kappa_\nu \approx \frac{3}{32\pi} A_{ul} \frac{hc\lambda_{ul}}{kT_{spin}} n(\mathrm{H\ I})\phi_\nu$$

absorption coefficient depends inversely on T_{spin}
this is a consequence of stimulated emission
not being negligible!

Column Density

$$N(\mathrm{H\ I}) \equiv \int ds\ n(\mathrm{H\ I})$$

for uniform volume density N = nL



n = # particles cm^{-3}

column density is
number of particles
per unit area
along a path length L

Line Profile ϕ_ν determined by two processes:

1) Natural Broadening
2) Doppler Broadening

Natural Broadening: from Uncertainty principle $\Delta E \Delta t \geq \hbar$

Δt = lifetime of state

Doppler Broadening: from spread in velocity of particles in the gas

Natural Broadening results in a Lorentz profile (approximately)

$$\phi_\nu \approx \frac{4\gamma_{u\ell}}{16\pi^2(\nu - \nu_{u\ell})^2 + \gamma_{u\ell}^2}.$$

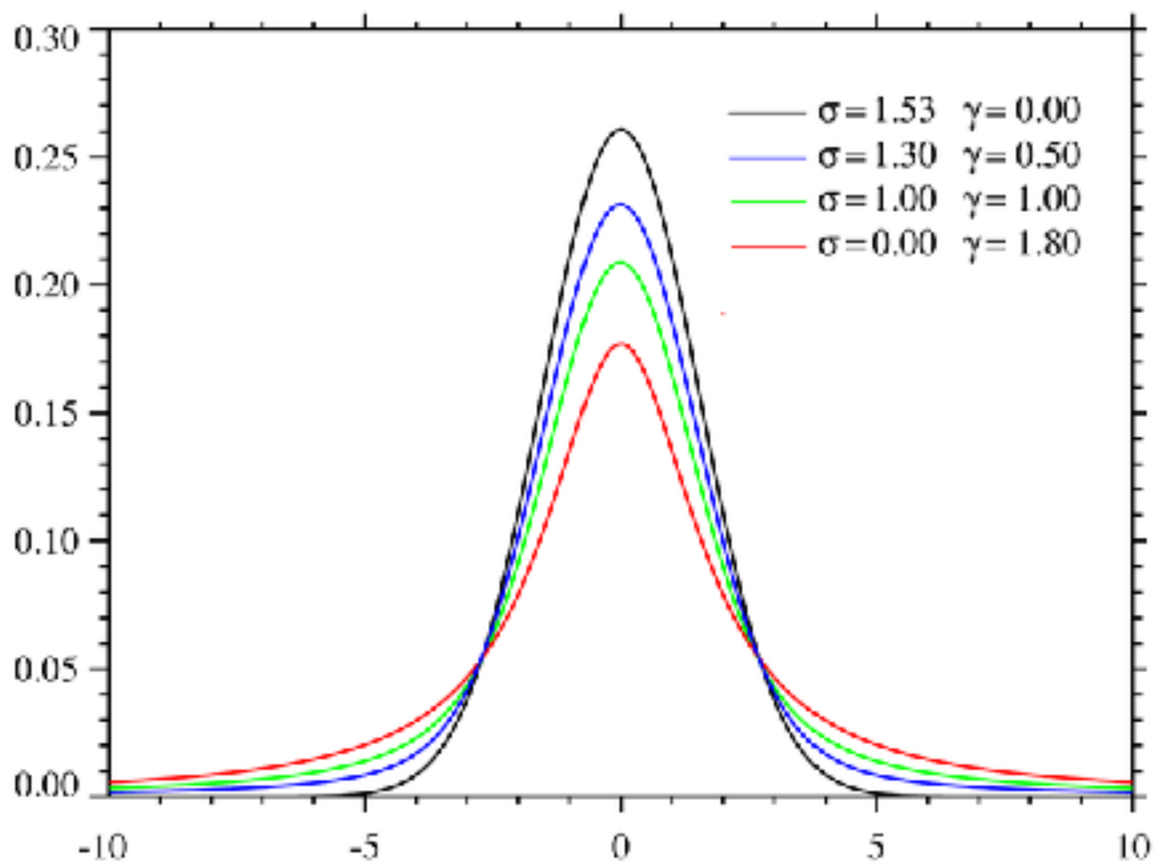
where:

$$\gamma_{u\ell} = \sum_{j<u} A_{uj} + \sum_{j<\ell} A_{\ell j}.$$

Is a sum of all of the relevant lifetimes (~1/A) for the energy levels you are transitioning between.

Doppler Broadening means that Lorentz profile is convolved with the velocity dispersion of the gas.

$$\phi_\nu = \frac{1}{\sqrt{2\pi\sigma_v^2}} \int_{-\infty}^{\infty} e^{-v^2/2\sigma_v^2} \frac{4\gamma_{u\ell}}{16\pi^2(\nu - (1 - v/c)\nu_{u\ell})^2 + \gamma_{u\ell}^2} dv$$



This gives you a "Voigt" profile.

HI 21-cm Radiative Transfer

two parts of line profile ϕ_ν:

- natural broadening
- Doppler broadening

Depends on lifetime (i.e. Einstein A value) of transition

$$(\Delta v)^{\rm intr.}_{\rm FWHM} = \frac{\gamma_{ul}}{2\pi} \sim \frac{A_{ul}}{2\pi}$$

Because lifetime of 21-cm transition is SO LONG natural broadening is negligible.

Line profile depends on velocity dispersion.

HI 21-cm Radiative Transfer

$$\tau_\nu = 2.19 \left(\frac{N(\mathrm{H\ I})}{10^{21}\mathrm{cm}^{-2}} \right) \left(\frac{T_{spin}}{100K} \right)^{-1} \left(\frac{\sigma_v}{\mathrm{kms}^{-1}} \right)^{-1} e^{-u^2/2\sigma_v}$$

u = velocity difference from line center

Lots of regions where optical depth in HI can be significant.

HI 21-cm Radiative Transfer

In the optically thin case $\tau_\nu \ll 1$

$$I_\nu = I_\nu(0) + \int j_\nu ds = I_\nu(0) + \frac{3}{16\pi} A_{ul}\ h\nu_{ul}\ \phi_\nu\ N(\mathrm{H\ I})$$

HI 21-cm Radiative Transfer

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Say $I_\nu(0) = 0$

$$\boxed{\int I_\nu\ d\nu = \frac{3}{16\pi} A_{ul}\ h\nu_{ul}\ N(\mathrm{H\ I})}$$

can get N(HI) from integral of line!

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Note: radio astronomers often convert I_ν to brightness temperature